# Competition in Law Enforcement and Capital Allocation* 

by<br>Nicolas Marceau<br>Département des sciences économiques and CIRPÉE<br>Université du Québec à Montréal<br>Case postale 8888, succursale Centre-Ville<br>Montréal, Québec, Canada H3C 3P8<br>Steeve Mongrain<br>Department of Economics, RIIM, and CIRPÉE<br>Simon Fraser University<br>8888 University Drive<br>Burnaby, British Columbia, Canada V5A 1S6<br>mongrain@sfu.ca

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#### Abstract

This paper studies interjurisdictional competition in the fight against crime and its impact on occupational choice and the allocation of capital. In a world where capital is mobile, jurisdictions are inhabited by individuals who choose to become workers or criminals. Because the return of the two occupations depends on capital, and because investment in capital in a jurisdiction depends on its crime rate, there is a bi-directional relationship between capital investment and crime which may lead to capital concentration. By investing in costly law enforcement, a jurisdiction makes the choice of becoming a criminal less attractive, which reduces the number of criminals and makes its territory more secure. This increased security increases the attractiveness of the jurisdiction for investors and this can eventually translate into more capital being invested. We characterize the Nash equilibria - some entailing a symmetric outcome, others an asymmetric one - and study their efficiency.


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## 1. Introduction

Security matters when it comes to investment decisions. Indeed, capital owners prefer to invest where crime rates are low because in such places, the likelihood they will be deprived of the return on their investment is lower. ${ }^{1}$ Local authorities, responding to those preferences, invest in crime deterrence. In this context, adjacent jurisdictions who wish to lower their respective crime rates may be competing in law enforcement, trying to make their jurisdiction relatively safer than others. Understanding the mechanics of such competition and the choice of law enforcement chosen by adjacent jurisdictions is the focus of this paper.

If different jurisdictions invest in different amounts in law enforcement, the crime rates in those jurisdictions will certainly differ. In the United States, there are many cases of "twin" cities, with similar characteristics and independent budgets, which nevertheless exhibit very different crime rates. For example, the crime rate against properties is $60 \%$ higher in Minneapolis than in St-Paul, $100 \%$ higher in Tampa than in St Petersburg, and $46 \%$ higher in Oakland than in San Francisco. ${ }^{2}$ Of course, when the crime/security levels differ, capital owners will invest in different amounts in different jurisdictions. In other words, it is possible that when crime becomes more concentrated, capital also becomes more concentrated, although obviously in different locations. ${ }^{3}$

Since Becker's (1968) seminal work on law enforcement, few economists have paid attention to the multi-jurisdictional nature of crime deterrence. ${ }^{4}$ This may explain why economists have a limited understanding of the impact of law enforcement policies on criminal activities. In this paper, we explicitly account for the multi-jurisdictional nature of the interaction between criminals and governments and show it has important consequences.

Another important feature of our analysis is that individuals make the occupational choice of becoming workers or criminals. ${ }^{5}$ For an individual, this occupational choice largely depends on

[^1]the amount of capital - a complement in production - in the jurisdiction in which he resides: more capital increases the wage a worker can earn, but more capital also translates into a higher reward to criminal activity. And since investment in capital in a jurisdiction depends on the crime rate in that jurisdiction, there is a complex bi-directional relationship between capital investment and crime.

The key mechanism we highlight in our analysis can be explained as follows. In standard models without occupational choice and in which capital must be allocated between competing jurisdictions (or uses), the unit return of capital in a given jurisdiction is a decreasing function of the stock of capital located in it. With occupational choice, it is not necessarily so because an extra unit of capital may lead to more individuals choosing to become workers (rather than criminals), and this in turn can make capital more productive. It follows that if an extra unit of capital sufficiently increases the number of workers (and decreases the number of criminals), then the unit return of capital may be an increasing function of the stock of capital located in a jurisdiction. Of course, whether the unit return of capital is an increasing or a decreasing function of capital affects the allocation of capital in an important way. As is intuitive, if the unit return declines with the stock of capital, then capital will tend to be equally distributed between jurisdictions. On the other hand, if the unit return of capital increases with the stock of capital, then capitalists will find it advantageous to concentrate their capital in a single jurisdiction.

The nature of the law enforcement game between jurisdictions is also very different under a decreasing or an increasing per unit return of capital. The equilibria we characterize are symmetric but they can result in very different outcomes for initially identical jurisdictions. For the case of an increasing per unit return of capital, we show that for the two-jurisdiction world we consider, all the capital locates in a jurisdiction which experiences low criminality, high output and a large working population, while the other jurisdiction attracts no capital and experiences high criminality with very low output.

We are also able to show that the equilibria of the law enforcement game are generally inefficient, i.e. that the levels of enforcement chosen by the jurisdictions when they act independently differs from that which would be selected by a central authority maximizing the sum of the welfare of the two jurisdictions. Of course, since enforcement is inefficient, so is occupational choice within each jurisdiction.

This paper is organized as follows. In Section 2 we present a model with mobile capital and occupational choice. Private sector behaviour is described in Section 3 and the enforcement

İmrohoroğlu et al. (2000), and Lloyd-Ellis and Marceau (2003). However, none of those papers account for capital investment and interjurisdictional competition.
policies chosen independently by the jurisdictions are characterized in Section 4 . We conclude in Section 5. All proofs can be found in the Appendix.

## 2. The Model

We examine the problem of competition in law enforcement when capital is mobile. Each jurisdiction is inhabited by a group of immobile individuals who have to choose between becoming workers or criminals. By investing in costly law enforcement, a jurisdiction makes the choice of becoming a criminal less attractive, which reduces the number of criminals and makes its territory more secure. This increased security increases the attractiveness of the jurisdiction for investors and can eventually translate into more capital being invested.

There are two jurisdictions $a$ and $b$. Each jurisdiction $i \in\{a, b\}$ is identical and is inhabited by a group of individuals who collectively own an aggregate production function $F\left(L^{i}, K^{i}\right)$, where $L^{i}$ and $K^{i}$ are the labour force and the capital located in jurisdiction $i$, respectively. As usual, it is assumed that $F_{K}\left(L^{i}, K^{i}\right)>0, F_{L}\left(L^{i}, K^{i}\right)>0, F_{K K}\left(L^{i}, K^{i}\right)<0, F_{L L}\left(L^{i}, K^{i}\right)<0$, and $F_{L K}\left(L^{i}, K^{i}\right) \geq 0$. We also assume that $F_{L}\left(L^{i}, 0\right)=0,{ }^{6}$ and that $F(0,0)>0 .{ }^{7}$

In each jurisdiction, the population consists of a continuum of agents of measure one who can choose to become a worker or a criminal. If $L^{i}$ is the number of workers in jurisdiction $i$, then the number of criminals in this jurisdiction is $C^{i}=1-L^{i}$. An individual who chooses to become a criminal appropriates for himself some of the return on capital, some of the return on the fix factor, and some of wages. Denote by $\alpha\left(d^{i}\right)$ the proportion of the total return on capital, of the total return on labour, and of the return on the fixed factor located in his jurisdiction a criminal is able to steal. The proportion $\alpha\left(d^{i}\right)$ is a decreasing function of the level of law enforcement $d^{i}$ chosen by the government of jurisdiction $i . .^{8}$ Consequently, an agent who decides to become a criminal obtains $\alpha\left(d^{i}\right)\left[F\left(L^{i}, K^{i}\right)\right]$. Alternatively, if he chooses to become a worker, he is paid according to the marginal product of labour, minus the proportion of his wage that is stolen, which amounts to a payoff given by $\left[1-C^{i} \alpha\left(d^{i}\right)\right] F_{L}\left(L^{i}, K^{i}\right)$.

A non-resident capitalist endowed with $\bar{K}$ units of capital chooses to allocate his capital between the two jurisdictions. ${ }^{9} K^{a}$ denotes the amount of capital invested in jurisdiction $a$, and $K^{b}=$
${ }^{6}$ This implies that wages are zero when there is no capital in a jurisdiction.
7 This assumption is often referred as "free lunch", and it implies that in the absence of variable input factors, there is still a fixed factor that generates some output (e.g. land).
8 The fact that the proportion $\alpha\left(d^{i}\right)$ is the same for all sorts of criminal activities is assumed to simplify the analysis. We could relax this assumption and our results below would still obtain.
${ }^{9}$ We assume that there is a single capitalist rather than a large number of them behaving competitively to avoid some technical difficulties. In particular, we will show in next sub-section that for a
$\bar{K}-K^{a}$ that invested in jurisdiction $b$. Capital is allocated by the capital owner after he has observed the level of law enforcement chosen by the government. The government is assumed to be committed to its enforcement policy. Once capital is allocated, it becomes fully immobile.

In jurisdiction $i$, the government chooses the level of law enforcement, $d^{i}$, which it can buy at a cost of one per unit. As is mentioned above, a larger $d^{i}$ affects negatively the proportion $\alpha\left(d^{i}\right)$ that is stolen by each criminal, i.e. $\alpha^{\prime}\left(d^{i}\right)<0$. We assume that the government of jurisdiction $i$ maximizes legal output (i.e. output minus what is appropriated by criminals) minus the net return on capital (because it is owned by non-residents), minus enforcement costs.

The timing is as follows. First, jurisdictions simultaneously choose their level of law enforcement. This investment if perfectly observable and is irreversible. Then, the capitalist allocates his capital between the two jurisdictions. Investments in capital are also perfectly observable and irreversible. The residents of each jurisdiction then make their occupational choice (worker or criminal). Finally, production takes place, theft takes place, and payments are awarded. The model is solved using backward induction.

## 3. Private Sector Behaviour

### 3.1 Occupational Choice

We solve for the occupational choice equilibrium of the residents of jurisdiction $i$ for given levels of enforcement $d^{i}$ and capital $K^{i}$. Since agents choose the activity entailing the largest payoff, the equilibrium number of workers in jurisdiction $i$, say $L^{i}\left(K^{i}, d^{i}\right)$, will be that which equates the return of the two occupations. Thus, $L^{i}\left(K^{i}, d^{i}\right)$ solves the following equation:

$$
\begin{equation*}
\left[1-\left(1-L^{i}\right) \alpha\left(d^{i}\right)\right] F_{L}\left(L^{i}, K^{i}\right)=\alpha\left(d^{i}\right) F\left(L^{i}, K^{i}\right) \tag{1}
\end{equation*}
$$

In other words, the number of workers must adjust so that the return to working, the wage, which is simply the marginal product of labour $\left[1-\alpha C^{i}\right] F_{L}$, is equal to the return to criminal activity $\alpha F$.

Note from equation (1) that an increase in $K^{i}$ generates an increase in the wage a worker receives only if $F_{L K}\left(L^{i}, K^{i}\right)>0$. On the other hand, an increase in $K^{i}$ translates into an increase in the total return on capital, labour and the return on the fix factor. Since the return

[^2]to criminal activity is a proportion of those total returns, an increase in $K^{i}$ also leads to an increase in the return to criminal activity. The relative size of each effect determines whether an increase in $K^{i}$ leads to more workers or to more criminals. To see this, note that from equation (1), we have:
\[

$$
\begin{equation*}
\frac{\partial L^{i}\left(K^{i}, d^{i}\right)}{\partial K^{i}}=\frac{\left[1-\alpha\left(d^{i}\right) C^{i}\right] F_{L K}\left(L^{i}, K^{i}\right)-\alpha\left(d^{i}\right) F_{K}\left(L^{i}, K^{i}\right)}{-\left[1-\alpha\left(d^{i}\right) C^{i}\right] F_{L L}\left(L^{i}, K^{i}\right)} \tag{2}
\end{equation*}
$$

\]

The denominator of this last expression is always positive, while the sign of its numerator is ambiguous. Thus, the impact of a change in the capital stock $K^{i}$ on the equilibrium employment $L^{i}$ depends on the sign of $\left[1-\alpha\left(d^{i}\right) C^{i}\right] F_{L K}\left(L^{i}, K^{i}\right)-\alpha\left(d^{i}\right) F_{K}\left(L^{i}, K^{i}\right)$. This implies that when $F_{L K}\left(L^{i}, K^{i}\right)>($ resp. $<)\left[\alpha\left(d^{i}\right) /\left(1-\alpha\left(d^{i}\right) C^{i}\right)\right] F_{K}\left(L^{i}, K^{i}\right)$, labour (resp. criminality) increases when capital increases. The incentive for a resident to participate in the legal sector will increase only if the increase in wages due to additional capital is large enough. Obviously if $F_{L K}(\cdot)=0$, an increase in capital will lead to an increase in criminal activity for the recipient jurisdiction. Note that an increase in law enforcement effort $d^{i}$ unambiguously reduces the incentive to become a criminal, and consequently increases labour supply, i.e. $\partial L^{i}\left(K^{i}, d^{i}\right) / \partial d^{i}>0$.

We now distinguish two cases differing in their implications for the magnitude of $\partial L^{i}\left(K^{i}, d^{i}\right) /$ $\partial K^{i}$.

Case I: $\partial L^{i}\left(K^{i}, d^{i}\right) / \partial K^{i}=0, \quad \forall K^{i} \in[0, \bar{K}]$.

As will be seen below, the fact that the number of workers is not affected by the level of capital will have important consequences on the nature of the equilibrium. We choose to study such a case since all situations in which $L^{i}\left(K^{i}, d^{i}\right)$ is a non-increasing function of $K^{i}$ will entail the same type of equilibrium, including when $F_{L K}(\cdot)=0$. Note that a technology for which Case I obtains is $F(L, K)=L^{\mu} K^{\nu}, 0 \leq \mu+\nu \leq 1 .^{10}$

Case II: $\partial L^{i}\left(K^{i}, d^{i}\right) / \partial K^{i}>0, \quad \forall K^{i} \in[0, \bar{K}]$.

For case II to prevail, the wage increase due to additional capital needs to be large enough, more specifically $F_{L K}(\cdot)>\left[\alpha(d) /(1-\alpha(d)) C^{i}\right] F_{K}(\cdot)$. Note that $F(L, K)=L^{\mu} K^{\nu}-K$ with $0 \leq \mu+\nu \leq 1$ is a technology for which Case II obtains. ${ }^{11}$

We now turn to the characterization of the different equilibria in capital investments in the two cases. Later, we will turn our attention to the determination of the equilibrium levels of law

[^3]enforcement. Note that it is possible for $\partial L^{i}\left(K^{i}, d^{i}\right) / \partial K^{i}$ to change sign when $K^{i}$ changes. We will briefly discuss such situations.

### 3.2 Capital Location Choice

The capitalist allocates his $\bar{K}$ units of capital between the two jurisdictions. Denote by $\rho^{i}$ the per unit return on capital invested in jurisdiction $i$. Since a proportion $\alpha\left(d^{i}\right)$ of the total return on capital is stolen by each criminal, we have that $\rho^{i}=\left[1-\alpha\left(d^{i}\right) C^{i}\left(K^{i}, d^{i}\right)\right] F_{K}\left[L^{i}\left(K^{i}, d^{i}\right), K^{i}\right]$. In standard models in which mobile capital must be allocated between regions (or uses), the per unit return on capital in jurisdiction $i$ is decreasing with the size of the investment in capital in jurisdiction $i$ because the marginal product of capital is decreasing. However, it may not be so in the current framework because the number of criminals varies with the size of the investment in capital. The impact of a change in capital on the per unit return on capital is given by:

$$
\begin{align*}
\frac{\partial \rho^{i}}{\partial K^{i}}= & \alpha\left(d^{i}\right) F_{K}\left[L^{i}\left(K^{i}, d^{i}\right), K^{i}\right] \frac{\partial L^{i}\left(K^{i}, d^{i}\right)}{\partial K^{i}}  \tag{3}\\
& +\left[1-\alpha\left(d^{i}\right) C^{i}\left(K^{i}, d^{i}\right)\right]\left[F_{K K}\left[L^{i}\left(K^{i}, d^{i}\right), K^{i}\right]+F_{L K}\left[L^{i}\left(K^{i}, d^{i}\right), K^{i}\right] \frac{\partial L^{i}\left(K^{i}, d^{i}\right)}{\partial K^{i}}\right]
\end{align*}
$$

In the first term on the right-hand side, $\rho^{i}$ is affected by a change in $K^{i}$ because changes in $K^{i}$ affects the number of workers and criminals, the change in the number of criminals itself affecting the proportion of the total return of capital which is stolen. The second term represents the more traditional impact of a change in $K^{i}$ on the per unit return, but with one difference. When capital in jurisdiction $i$ increases, the marginal return on capital decreases; this is captured by $\left.F_{K K}\left[L^{i}\left(K^{i}, d^{i}\right), K^{i}\right]\right)<0$. However, when capital increases the number of workers changes, and this impacts on the marginal return of capital in $F_{L K}\left[L^{i}\left(K^{i}, d^{i}\right), K^{i}\right]\left(\partial L^{i}\left(K^{i}, d^{i}\right) / \partial K^{i}\right)$. Thus, when a change in capital investment increases (does not decrease) the number of workers, the per unit return on capital invested in jurisdiction $i$ may be an increasing or a decreasing function of the stock of capital invested in $i$.

Below, we show that the sign of $\partial \rho^{i} / \partial K^{i}$ is a key determinant of the equilibrium allocation of capital. We focus on two simple cases: (a) $\partial \rho^{i} / \partial K^{i}<0 \forall K^{i}$; and (b) $\partial \rho^{i} / \partial K^{i}>0 \forall K^{i}$. We also briefly discuss the case in which the sign of $\partial \rho^{i} / \partial K^{i}$ varies with $K^{i}$.

Denote by $K\left(d^{a}, d^{b}\right)$ the equilibrium capital investment in jurisdiction $a$. The equilibrium capital investment in jurisdiction $b$ is then given by $\bar{K}-K\left(d^{a}, d^{b}\right)$.

Lemma 1: In Case I, $\partial L^{i}\left(K^{i}, d^{i}\right) / \partial K^{i}=0 \quad \forall K^{i} \in[0, \bar{K}]$ which implies that $\partial \rho^{i} / \partial K^{i}<0$ $\forall K^{i} \in[0, \bar{K}]$. Consequently, equilibrium capital investments $K\left(d^{a}, d^{b}\right)$ in jurisdiction a and
$\bar{K}-K\left(d^{a}, d^{b}\right)$ in jurisdiction $b$ are the solution to:

$$
\begin{array}{r}
\left.\left[1-\alpha\left(d^{a}\right) C^{a}\left(K\left(d^{a}, d^{b}\right), d^{a}\right)\right] F_{K}\left[L^{a}\left(K\left(d^{a}, d^{b}\right), d^{a}\right)\right), K\left(d^{a}, d^{b}\right)\right]=  \tag{4}\\
{\left[1-\alpha\left(d^{b}\right) C^{b}\left(\bar{K}-K\left(d^{a}, d^{b}\right), d^{b}\right)\right] F_{K}\left[L^{b}\left(\bar{K}-K\left(d^{a}, d^{b}\right), d^{b}\right), \bar{K}-K\left(d^{a}, d^{b}\right)\right]}
\end{array}
$$

In such a case, $K\left(d^{a}, d^{b}\right)$ is an increasing function of $d^{a}$ and a decreasing function of $d^{b}$.
Lemma 1 is easily understood by examination of Figure 1. The capital owner prefers to invest in the jurisdiction in which the per unit return on capital is the highest. The more the capital owner invests in a given jurisdiction, the lower is the per unit return on capital. In equilibrium, the capitalist allocates his capital so that the per unit return in both jurisdictions are equalized. Note that for a given level of enforcement chosen by the other jurisdiction, an increase in enforcement by a jurisdiction leads to an increase in capital invested on its territory. Consequently, both jurisdictions will compete to attract capital investment by offering a secure environment to the capitalist.

Because labour supply and the crime rate both depend on the amount of capital located in a jurisdiction, it is possible for the per unit return on capital to increase when capital investment increases. Indeed, when capital increases, the number of workers increases and this in turn increases the marginal product of capital. Furthermore, when the number of workers increases, the number of criminals is reduced and this also leads to an increase in the total return on capital. These effects can dominate the standard decrease in the marginal product of a factor that occurs when this factor becomes more abundant. When this happens, the per unit return on capital increases when capital investment increases.

Lemma 2, which we now introduce, deals with this possibility and describes an equilibrium in which all the capital is invested in a single jurisdiction.

Lemma 2: In Case II, if $F_{L K}(L, K)$ is sufficiently large so that $\partial \rho^{i} / \partial K^{i}>0 \forall K^{i} \in[0, \bar{K}]$, then all capital is invested in jurisdiction a $\left(K\left(d^{a}, d^{b}\right)=\bar{K}\right)$ if $d^{a}>d^{b}$, and all capital is invested in jurisdiction $b\left(\bar{K}-K\left(d^{a}, d^{b}\right)=\bar{K}\right)$ if $d^{a}<d^{b}$. If $d^{a}=d^{b}$, then $K\left(d^{a}, d^{b}\right)=\bar{K}$ with probability $p$, and $K\left(d^{a}, d^{b}\right)=0$ with probability $(1-p)$ is an equilibrium allocation for any $p \in[0,1]$; we arbitrarily assume that in such a case, $p=1 / 2$.

If the unit return $\rho_{i}$ is an increasing function of capital for all levels of investment, then the capitalist prefers to concentrate all his capital in a single jurisdiction. As can be seen in Figure 2 , if the level of enforcement is larger in jurisdiction $a$, then the capitalist prefers to concentrate all his capital in this jurisdiction. Naturally, all his capital is invested in jurisdiction $b$ if $d^{a}<d^{b}$. If both jurisdictions provide the same level of enforcement, the capitalist is indifferent between concentrating all his capital in one or the other jurisdiction.

Note that if we are in Case II it implies that $\partial L^{i}\left(K^{i}, d^{i}\right) / \partial K^{i}>0$. Also note that if $F_{L K}(L, K)$ is large enough for labour to be positively related to capital, but is not large enough to ensure that $\partial \rho^{i} / \partial K^{i}>0$, then the resulting equilibrium will be similar to that described in Lemma 1.

Lemmas 1 and 2 deal with two simple cases in which the per unit return on capital investment is monotonically decreasing or increasing in capital. The resulting equilibria are either an interior one in which some capital is invested in both jurisdictions, or one in which all capital locates in the jurisdiction with the highest level of enforcement. In fact, those two types of equilibrium also obtain in other circumstances. For example, the per unit return on capital could be a U-shaped, non-monotonic function of capital as in Figure 3. In the particular case of Figure 3, the capitalist will obviously find it profitable to invest all his capital in $a$. On the other hand, in Figure 4, where the per unit return on capital has an inverted U-shape, the capitalist will prefer to split his capital evenly between the two jurisdictions. While all those situations are interesting, the rest of the analysis will focus on the case where the per unit return of capital is a monotonic function of capital.

## 4. Enforcement Policies and Capital Allocation

We now examine the simultaneous choice of law enforcement by the two jurisdictions. Both jurisdictions are assumed to maximize legal output (i.e. output minus what is appropriated by criminals) minus the net return on capital (because it is owned by non-residents), minus enforcement costs. Thus, the problem of jurisdiction $a$ is given by:

$$
\begin{equation*}
\max _{d^{a}}\left[1-\alpha\left(d^{a}\right)\left(1-L^{a}\left(d^{a}, d^{b}\right)\right)\right]\left[\tilde{F}^{a}\left(d^{a}, d^{b}\right)-K\left(d^{a}, d^{b}\right) \tilde{F}_{K}^{a}\left(d^{a}, d^{b}\right)\right]-d^{a} \tag{5}
\end{equation*}
$$

where $L^{a}\left(d^{a}, d^{b}\right)=L^{a}\left[K\left(d^{a}, d^{b}\right), d^{a}\right]$, and where $\tilde{F}_{j}^{a}\left(d^{a}, d^{b}\right)=F_{j}\left[L^{a}\left(d^{a}, d^{b}\right), K^{a}\left(d^{a}, d^{b}\right)\right]$ for $j \in\{\emptyset, L, K, L K, K K\}$. Similarly, the problem of jurisdiction $b$ is given by:

$$
\begin{equation*}
\max _{d^{b}}\left[1-\alpha\left(d^{b}\right)\left(1-L^{b}\left(d^{a}, d^{b}\right)\right)\right]\left[\tilde{F}^{b}\left(d^{a}, d^{b}\right)-\left(\bar{K}-K\left(d^{a}, d^{b}\right)\right) \tilde{F}_{K}^{b}\left(d^{a}, d^{b}\right)\right]-d^{b} \tag{6}
\end{equation*}
$$

where $L^{b}\left(d^{a}, d^{b}\right)=L^{b}\left[\bar{K}-K\left(d^{a}, d^{b}\right), d^{b}\right]$, and where $\tilde{F}_{j}^{b}\left(d^{a}, d^{b}\right)=F_{j}\left[L^{b}\left(d^{a}, d^{b}\right), \bar{K}-K\left(d^{a}, d^{b}\right)\right]$ for $j \in\{\emptyset, L, K, L K, K K\}$.

The resulting Nash equilibrium outcomes are strikingly different depending on whether Lemma 1 (Case I) or Lemma 2 (Case II) applies. We investigate each of them in turn.
4.1 Declining Return on Capital $\left(\partial \rho^{i} / \partial K^{i}<0\right)$

When the unit return on capital declines with capital investment, we obtain the following:
Proposition 1: If $\partial \rho^{i} / \partial K^{i}<0$ (Case I), both jurisdictions choose a positive level of enforcement $\left(d^{i}>0\right)$ and there is over-deterrence in the sense that the equilibrium levels of enforcement are larger than the efficient levels.

This corresponds to the situation described in Marceau (1997). Also, this result is similar to those obtained in the literature on policy competition between governments. ${ }^{12}$ An increase in law enforcement by jurisdiction $i$ imposes a negative externality on jurisdiction $j$. Since $i$ does not take this externality into account when it makes its choice of enforcement, the laisser-faire equilibrium choice of effort is too large relative to the efficient level, i.e. that which a central authority would select if it maximized the sum of the objective functions of the two jurisdictions by choice of the level of enforcement in each of them. Recall that when the equilibrium number of workers remains constant as capital invested increases, the per unit return on capital is decreasing in capital. ${ }^{13}$ Consequently, the capitalist chooses to invest capital in both jurisdictions. By increasing enforcement, a jurisdiction attracts some capital, but it imposes a negative externality on the other jurisdiction which loses some capital. Using the terminology of Eaton and Eswaran (2002) and Eaton (2004), the actions of the jurisdictions are then plain substitutes. In such a case, both jurisdiction will choose a level of enforcement larger than the efficient level. Note that because enforcement is inefficient, so is occupational choice: there are too few criminals in this world. Note however that the allocation of capital is efficient. ${ }^{14}$

### 4.2 Increasing Return on Capital ( $\partial \rho^{i} / \partial K^{i}>0$ )

Under the conditions of Lemma 2 (Case II), the capitalist chooses to locate all his capital in a single jurisdiction. In such an environment, the nature of the equilibrium is very different. For immediate purposes, denote by $\Omega[K, d]$ the value of a jurisdiction objective function for a pair $(K, d) .{ }^{15}$ In other words, $\Omega[K, d]=[1-\alpha(d)(1-L(K, d))]\left[F(L(K, d), K)-K F_{K}(L(K, d), K)\right]-d$ is the payoff of a jurisdiction when $K$ units of capital locate on its territory and when it invests $d$ in deterrence. Let $d^{*}(\bar{K})$ denote the level of deterrence chosen by a jurisdiction when all the capital is located on its territory $(K=\bar{K}): d^{*}(\bar{K})=\arg \max _{d} \Omega[\bar{K}, d]$. Note that we assume an

[^4]interior solution $\left(d^{*}(\bar{K})>0\right)$. Similarly, $d^{*}(0)$ is defined as the level of deterrence chosen by a jurisdiction when no capital is located on its territory $(K=0): d^{*}(0)=\arg \max _{d} \Omega[0, d]$. Notice that when no capital is located in a given jurisdiction, all residents of the jurisdiction chooses to become criminals. ${ }^{16}$ Consequently, $d^{*}(0)>0$ is simply given by $\alpha^{\prime}\left[d^{*}(0)\right] F(0,0)=1$. Obviously, a jurisdiction is better off with all the capital than with no capital so $\Omega\left[\bar{K}, d^{*}(\bar{K})\right]>\Omega\left[0, d^{*}(0)\right]$. Also, let $\hat{d}$ be the level of deterrence solving $\Omega[\bar{K}, \hat{d}]=\Omega\left[0, d^{*}(0)\right]$. Clearly, it must be that $\hat{d}>d^{*}(\bar{K})>d^{*}(0)$.

Note that the following chain of inequalities must hold:

$$
\Omega\left[\bar{K}, d^{*}(\bar{K})\right]>\Omega\left[\bar{K}, d^{*}(0)\right]>\Omega\left[0, d^{*}(0)\right]=\Omega[\bar{K}, \hat{d}]>\Omega[0, d] \forall d>0
$$

Figure 5 depicts the payoffs of the jurisdictions in this law enforcement game. Recall that because $\partial \rho^{i} / \partial K^{i}>0$, all the capital locates in jurisdiction $i$ if $d^{i}>d^{j}$. If $d^{i}=d^{j}$, then all the capital locates in jurisdiction $i$ with probability $1 / 2$, and in jurisdiction $j$ with probability $1 / 2$. For the game considered, a strategy for a jurisdiction is simply a level of deterrence $d$ and the strategy sets are the positive real numbers $(d \in[0, \infty])$. A strategy profile is a pair $\left(d^{a}, d^{b}\right)$ consisting of a strategy for each jurisdiction.

We now present three useful lemmas.

Lemma 3: When $\partial \rho^{i} / \partial K^{i}>0$ (Lemma 2 - Case II), the jurisdictions never choose a strategy $d>\hat{d}$.

A jurisdiction will have no desire to invest more than $\hat{d}$ because attracting all the capital with $d>\hat{d}$ makes it worse off than investing nothing and having no capital.

Lemma 4: When $\partial \rho^{i} / \partial K^{i}>0$ (Lemma 2 - Case II), the jurisdictions never choose a strategy $d<d^{*}(0)$.

A jurisdiction will have no desire to invest less than $d^{*}(0)$ because welfare is strictly increasing in $d$ for $d<d^{*}(0)$ and $K \in\{0, \bar{K}\}$.

Lemma 5: When $\partial \rho^{i} / \partial K^{i}>0$ (Lemma 2 - Case II), the game has no pure strategy Nash equilibrium.

There is no pure strategy equilibrium because if jurisdiction $i$ chooses an enforcement level

[^5]$d^{i}<\hat{d}$, then jurisdiction $j$ will find it profitable to attract all the capital by choosing $d^{j}$ such that $d^{i}<d^{j}<\hat{d}$. As for $\left(d^{i}=d^{*}(0), d^{j}=\hat{d}\right)$, it is not an equilibrium because $d^{j}=\hat{d}$ is not a best response to $d^{i}=d^{*}(0)$.

The main result of this section is as follows: ${ }^{17}$

Proposition: 2 When $\partial \rho^{i} / \partial K^{i}>0$ (Lemma 2 - Case II), the game has a symmetric mixed strategy Nash equilibrium in which the two jurisdictions play $d \in\left[d^{*}(0), \hat{d}\right]$ according to the continuous cumulative function $H(d)$ and density function $h(d)=H^{\prime}(d)$ on $\left[d^{*}(0), \hat{d}\right]$. For $d \in\left[d^{*}(0), \hat{d}\right]$, the mixed strategy $H(d)$ is given by:

$$
H(d)=\frac{\Omega\left[0, d^{*}(0)\right]-\Omega[0, d]}{\Omega[\bar{K}, d]-\Omega[0, d]}
$$

In equilibrium, the expected payoff of the two jurisdictions is $\Omega\left[0, d^{*}(0)\right]$.

Note that given $H(d)$, we have that $H\left(d^{*}(0)\right)=0,0<H(d)<1$ for $\left.d \in\right] 0, \tilde{d}[$, and $H(\hat{d})=1$. The equilibrium described here is such that in expected terms, the two jurisdictions obtain a net surplus of zero. The intuition is simple. Suppose all the capital is invested in jurisdiction $i$ which has chosen $d^{i}>d^{j}=d^{*}(0)$ and that $\Omega^{i}\left[\bar{K}, d^{i}\right]>\Omega^{j}\left[0, d^{*}(0)\right]$. Clearly, since the two jurisdictions are otherwise identical, this situation cannot be an equilibrium because jurisdiction $j$ has an incentive to deviate to a level of enforcement $\breve{d}^{j}=d^{i}+\varepsilon$, with $\varepsilon$ small. Indeed, if jurisdiction $j$ does deviate to $\breve{d}^{j}$, the capitalist will re-locate all his capital from $i$ to $j$, and jurisdiction $j$ will now get a payoff of $\Omega^{j}\left[\bar{K}, \breve{d}^{j}\right]>\Omega^{j}\left[0, d^{*}(0)\right]$. Such an incentive to deviate will be present as long as a jurisdiction will have a positive net payoff. Therefore, in equilibrium, it must be that both jurisdictions obtain a net surplus of zero in expected terms.

The mixed strategy equilibrium described in Proposition 2 is inefficient unless $d^{i}=d^{*}(0)$ and $d^{j}=d^{*}(\bar{K})$ are drawn, an event which occurs with probability zero. The equilibrium is inefficient for several reasons. First, the jurisdiction which gets no capital spends on deterrence $d>d^{*}(0)$ with probability approaching one (an obvious case of over-deterrence). Second, the jurisdiction which gets all the capital spends too little or too much on deterrence $\left(d \neq d^{*}(\bar{K})\right)$. Finally, because enforcement is inefficient, occupational choice is distorted. ${ }^{18}$

Consider the ex post implications of such an equilibrium. First, note that all the capital locates in the jurisdiction which offers the highest level of protection. This jurisdiction will benefit from a level of welfare larger than that it would get in the no capital / low deterrence situation

17 Note that the equilibrium described in Proposition 2 is reminiscent of that discussed in Varian (1980) in another context.

18 Note however that all capital locates in a single jurisdiction, which is efficient.
$\left(\Omega\left[0, d^{*}(0)\right]\right)$. In this jurisdiction, occupational choice satisfies equation (1) and both workers and criminals co-exist. On the other hand, the jurisdiction in which no capital locates obtains a level of welfare lower than that it would get in the no capital / low deterrence situation (because the marginal benefit of deterrence effort is lower than its cost, i.e. $\alpha^{\prime}(d) F(0,0)<1$ for all $\left.d>d^{*}(0)\right)$. Note that since there is no capital in this jurisdiction, the wage is driven to zero $\left(F_{L}\left(L^{i}, 0\right)=0\right)$. Consequently, all its residents choose to become criminals. To summarize, the ex post realization of the symmetric mixed strategy Nash equilibrium entails a jurisdiction with all the capital, a relatively moderate crime rate and a relatively large output, and the other jurisdiction with no capital, an extremely high crime rate and a very low output. The simple model presented in this paper can therefore explain how two initially identical jurisdictions can experience two drastically different evolutions.

Consider now the implications of our model in terms of the relationship between inequality and crime. Imagine that in some Region $A$ (in which there are two jurisdictions), the technology is such that the unit return of capital is declining, while in some other Region $B$ (in which there are also two jurisdictions), the technology is such that it is increasing. Then, in equilibrium, capital and crime are evenly distributed in Region $A$, the crime rate being at a moderate level in the two jurisdictions, while in Region $B$, crime and capital are concentrated in different jurisdictions, and the total crime rate in $B$ is most probably larger than that in $A$. It follows that in such a world, regional inequality and regional crime rates will be positively correlated. This is consistent with the evidence reported in İmrohoroğlu et al. (2000) using 1990 U.S. state-level data. ${ }^{19}$

## 5. Conclusion

This paper has shown that in an economy with occupational choice and with jurisdictions competing in deterrence to attract mobile capital, the symmetric Nash equilibria result in an even or an uneven distribution of crime and capital across space. Those equilibria are always inefficient.

The creation of a central organization to coordinate law enforcement policies would likely be beneficial in such a context, depending on the constraints it faces and the strengths and weaknesses of centralization. For example, a central organization may be forced, by political constraints, to select a uniform level of deterrence in all jurisdictions. Also, it could be that a central agency is not as efficient at identifying criminals. To analyze the opportunity of creating such a central agency, our model would have to be extended to take these factors into account.

[^6]The current analysis assumes that labour is immobile. In our model, the occupational choice of an individual residing in a jurisdiction with a low level of capital is not a very attractive one: to obtain a relatively low wage or to become a criminal. This can be partly justified if one thinks of jurisdictions inhabited by very different individuals, say low-skilled in one and high-skilled in the other, with segregated labour markets, and with housing prices in the jurisdiction of highskilled individuals that are simply not affordable for the low-skilled individuals. Nevertheless, if individuals were identical and labour was mobile, individuals would be able to move to a region in which the labour market is more attractive than in their own. This would open a whole new set of possibilities. That our results would hold in such a context is not obvious. This is clearly the next step in our research.

In the future, we would also like to study the political economy rationale for the observed frequent arrangements in which crime deterrence falls into the hands of local authorities. To our knowledge, why this is so has not been satisfactorily answered. Certainly, the phenomena we have described in the current analysis are likely to be taken into consideration by voters, lobby groups, and politicians, and they should therefore be explicitly incorporated in a political economy model of the appropriate degree of centralization of the fight on crime.

## 6. Appendix I: Proofs

Proof of Lemma 1: In Case I, $\partial L^{i}\left(K^{i}, d^{i}\right) / \partial K^{i}=0, i=a, b$, and it follows from equation (3) that $\partial \rho^{i} / \partial K^{i}<0, i=a, b$. In such a case, the capitalist chooses to allocate his capital between the two jurisdictions until the per unit return on capital is equalized in the two jurisdictions. Consequently, $K\left(d^{a}, d^{b}\right)$ satisfies equation (4), which simply states that $\rho^{a}\left[K\left(d^{a}, d^{b}\right)\right]=\rho^{b}[\bar{K}-$ $K\left(d^{a}, d^{b}\right)$. Totally differentiating equation (4) yields that:

$$
\begin{aligned}
& \frac{\partial K\left(d^{a}, d^{b}\right)}{\partial d^{a}}=\frac{\alpha^{\prime}\left(d^{a}\right) C^{a} F_{K}(\cdot)-\left[\alpha\left(d^{a}\right) F_{K}(\cdot)+\left(1-\alpha\left(d^{a}\right) C^{a}\right) F_{L K}(\cdot)\right] \partial L^{a} / \partial d^{a}}{\left[1-\alpha\left(d^{a}\right) C^{a}\right] F_{K K}\left(L^{a}, K^{a}\right)+\left[1-\alpha\left(d^{b}\right) C^{b}\right] F_{K K}\left(L^{b}, K^{b}\right)} \\
& \frac{\partial K\left(d^{a}, d^{b}\right)}{\partial d^{b}}=\frac{-\alpha^{\prime}\left(d^{b}\right) C^{b} F_{K}(\cdot)+\left[\alpha\left(d^{b}\right) F_{K}(\cdot)+\left(1-\alpha\left(d^{b}\right) C^{b}\right) F_{L K}(\cdot)\right] \partial L^{b} / \partial d^{b}}{\left[1-\alpha\left(d^{a}\right) C^{a}\right] F_{K K}\left(L^{b}, K^{a}\right)+\left[1-\alpha\left(d^{b}\right) C^{b}\right] F_{K K}\left(L^{b}, K^{b}\right)}
\end{aligned}
$$

The denominator of these two expressions is clearly negative. Consequently, $\partial K\left(d^{a}, d^{b}\right) / \partial d^{a}$ is positive since its numerator is negative, and $\partial K\left(d^{a}, d^{b}\right) / \partial d^{b}$ is negative since its numerator is positive.

Proof of Lemma 2: Since $\partial L^{i}\left(K^{i}, d^{i}\right) / \partial K^{i}$ is an increasing function of $F_{L K}(L, K)$, inspection of equation (3) reveals that if $F_{L K}(L, K)$ is positive and sufficiently large, it is possible for $\partial \rho^{i} / \partial K^{i}$ to be positive for all values of $K^{i}$. In such a case, the capitalist invests all his capital in a single jurisdiction. Because the production functions are identical in the two jurisdictions, $\left.\rho^{a}\right|_{K^{a}=\bar{K}}>\left.\rho^{b}\right|_{K^{b}=\bar{K}}$ if and only if $d^{a}>d^{b}$. Consequently, the full $\bar{K}$ will be invested in $a$ if $d^{a}>d^{b}$, and the full $\bar{K}$ will be invested in $b$ if $d^{a}<d^{b}$. When $d^{a}=d^{b}$, then $\left.\rho^{a}\right|_{K^{a}=\bar{K}}=$ $\left.\rho^{b}\right|_{K^{b}=\bar{K}}$. The capitalist is then indifferent between investing all his capital in $a$ or investing all his capital in $b$. In such a case, the capitalist could obviously randomize and, say, choose $a$ with probability $p \in[0,1]$. For simplicity, we assume that $p=1 / 2$.

Proof of Proposition 1: We note that when $\partial \rho^{i} / \partial K^{i}<0$ and the conditions for Case I are satisfied, the capitalist allocates his capital between the two jurisdictions until the returns are equalized. Since the jurisdictions are identical, capital will not be concentrated in a single jurisdiction $\left(0<K\left(d^{a}, d^{b}\right)<\bar{K}\right)$ and the equilibrium pair of enforcement levels will satisfy the first order conditions of problems (5) and (6):

$$
\begin{aligned}
& {\left[\alpha\left(d^{a}\right) \frac{\partial L^{a}(\cdot)}{\partial d^{a}}-\alpha^{\prime}\left(d^{a}\right) C^{a}\right]\left[\tilde{F}^{a}(\cdot)-K(\cdot) \tilde{F}_{K}^{a}(\cdot)\right]} \\
& +\beta^{a}(\cdot)\left[\tilde{F}_{L}^{a}(\cdot)-K(\cdot) \tilde{F}_{L K}^{a}(\cdot)\right] \frac{\partial L^{a}(\cdot)}{\partial d^{a}}-\beta^{a} K(\cdot) \tilde{F}_{K K}^{a}(\cdot) \frac{\partial K(\cdot)}{\partial d^{a}}=1, \\
& {\left[\alpha\left(d^{b}\right) \frac{\partial L^{b}(\cdot)}{\partial d^{b}}-\alpha^{\prime}\left(d^{b}\right) C^{b}\right]\left[\tilde{F}^{b}(\cdot)-[\bar{K}-K(\cdot)] \tilde{F}_{K}^{b}(\cdot)\right]} \\
& +\beta^{b}(\cdot)\left[\tilde{F}_{L}^{b}(\cdot)-[\bar{K}-K(\cdot)] \tilde{F}_{L K}^{b}(\cdot)\right] \frac{\partial L^{b}(\cdot)}{\partial d^{b}}+\beta^{b}[\bar{K}-K(\cdot)] \tilde{F}_{K K}^{b}(\cdot) \frac{\partial K(\cdot)}{\partial d^{b}}=1,
\end{aligned}
$$

where $\beta^{i}(\cdot)=1-\alpha\left(d^{i}\right)\left[1-L^{i}(\cdot)\right]$. Note that the impact of an increase in $d^{b}$ on the objective function of jurisdiction $a$ (problem (5)) is given by:

$$
-\beta^{a}(\cdot) K\left(d^{a}, d^{b}\right) \tilde{F}_{K K}^{a}(\cdot) \frac{\partial K\left(d^{a}, d^{b}\right)}{\partial d^{b}}<0
$$

while that of an increase in $d^{a}$ on the objective function of jurisdiction $b$ (problem (6)) is given by:

$$
\beta^{b}(\cdot)\left[\bar{K}-K\left(d^{a}, d^{b}\right)\right] \tilde{F}_{K K}^{b}(\cdot) \frac{\partial K\left(d^{a}, d^{b}\right)}{\partial d^{a}}<0
$$

Since both derivatives are negative, $d^{a}$ and $d^{b}$ are said to be plain substitutes (see Eaton and Eswaran, 2002, or Eaton, 2004). In such a case, the equilibrium levels of $d^{a}$ and $d^{b}$ are both positive and they exceed the efficient levels.

Proof of Lemma 3: Since $\Omega\left[0, d^{*}(0)\right]=\Omega[\bar{K}, \hat{d}]>\Omega[\bar{K}, d] \forall d>\hat{d}$, a jurisdiction is better off when it does only deter $d=d^{*}(0)$ and have no capital $(K=0)$ than if it deters at a level larger than $\hat{d}(d>\hat{d})$ and gets all the capital $(K=\bar{K})$.

Proof of Lemma 4: We know that $d^{*}(0)$ is given by $d^{*}(0)=\arg \max _{d} \Omega[0, d]$, this implies that $\alpha^{\prime}(d) F(0,0)<1$ for all $d<d^{*}(0)$, and consequently a jurisdiction with no capital will never choose a level of deterrence $d<d^{*}(0)$. Since $d^{*}(\bar{K})>d^{*}(0)$, the same argument applies to a jurisdiction with all the capital. Consequently, $d \geq d^{*}(0)$.

Proof of Lemma 5: We first show that there is no symmetric ( $d^{i}=d^{j}$ ) pure strategy Nash equilibrium and then show that there is no asymmetric $\left(d^{i}>d^{j}\right)$ pure strategy Nash equilibrium.
(i) There is no symmetric $\left(d^{i}=d^{j}\right)$ pure strategy Nash equilibrium.

Consider a strategy profile $(d, d)$, with $d \in\left[d^{*}(0), \hat{d}\right]$ from Lemma 3 and Lemma 4.
If $d<\hat{d}$, then the payoff of each jurisdiction is $\Omega^{a}=\Omega^{b}=\frac{1}{2} \Omega[\bar{K}, d]+\frac{1}{2} \Omega[0, d]$. Clearly, this cannot be an equilibrium as any jurisdiction, say $a$, has an incentive to deviate to $d^{a^{\prime}}=d+\varepsilon$, causing all the capital to locate in $a$, an ensuring itself a payoff $\Omega^{a^{\prime}}=\Omega[\bar{K}, d+\varepsilon]>\Omega^{a}$ for $\varepsilon$ small enough (i.e. $\varepsilon<\hat{d}-d$ ).

If $d=\hat{d}$, then the payoff of each jurisdiction is $\Omega^{a}=\Omega^{b}=\frac{1}{2} \Omega[\bar{K}, \hat{d}]+\frac{1}{2} \Omega[0, \hat{d}]$. Clearly, this cannot be an equilibrium as any jurisdiction, say $a$, has an incentive to deviate to $d^{a^{\prime}}=d^{*}(0)$, ensuring itself a payoff $\Omega^{a^{\prime}}=\Omega\left[0, d^{*}(0)\right]>\Omega^{a}$.
(ii) There is no asymmetric $\left(d^{i}>d^{j}\right)$ pure strategy Nash equilibrium.

Consider any strategy profile $\left(d^{a}, d^{b}\right)$, with $d^{a}<d^{b} \leq \hat{d}$ from Lemma 3 .

If $d^{a}<d^{b}<\hat{d}$, then $\Omega^{a}=\Omega\left[0, d^{a}\right]$ and $a$ has an incentive to deviate to $d^{a^{\prime}}=d^{b}+\varepsilon$ to obtain $\Omega^{a^{\prime}}=\Omega\left[\bar{K}, d^{b}+\varepsilon\right]>\Omega^{a}$ for $\varepsilon$ small enough.

If $d^{*}(0)<d^{a}<d^{b}=\hat{d}$, then $\Omega^{a}=\Omega\left[0, d^{a}\right]$ and $a$ has an incentive to deviate to $d^{a^{\prime}}=d^{*}(0)$ to obtain $\Omega^{a^{\prime}}=\Omega\left[0, d^{*}(0)\right]>\Omega^{a}$.

If $d^{*}(0)=d^{a}<d^{b}=\hat{d}$, then $\Omega^{b}=\Omega[\bar{K}, \hat{d}]$ and $b$ has an incentive to deviate to $d^{b^{\prime}}=d^{*}(\bar{K})$ to obtain $\Omega^{b^{\prime}}=\Omega\left[\bar{K}, d^{*}(\bar{K})\right]>\Omega^{b}$.

This completes the proof.
Proof of Proposition: 2 We show that when $j$ plays according to the mixed strategy $H(d)$, $i$ has no incentive to deviate from $H(d)$.

Suppose $j$ plays the mixed strategy $H(d)$. Then, when $i$ plays $d^{\prime}$, it obtains all the capital $(K=\bar{K})$ with probability $H\left(d^{\prime}\right)$ and no capital $(K=0)$ with probability $1-H\left(d^{\prime}\right)$.

Before solving for the mixed strategies equilibrium, first note that there are no point masses in equilibrium. The intuition is simple: if a level of deterrence $d^{\prime}$ was played with positive probability, there would be a tie at $d^{\prime}$ with positive probability. Imagine then that jurisdiction $j$ decides to play $d^{\prime}+\varepsilon$ (instead of $d^{\prime}$ ) with the same probability. The cost of such a deviation would be of the order of $\varepsilon$, but if the two jurisdictions were to tie, then jurisdiction $j$ would gain a fixed positive amount. The formal proof of this is as follows. Imagine that jurisdiction $i$ plays $d^{\prime}$ with positive probability $\omega$, and that jurisdiction $j$ deviates to $d^{\prime}+\varepsilon$ with the same positive probability. The payoff for jurisdiction $j$ will change by a factor of:

$$
\begin{aligned}
& \left\{\operatorname{Pr}\left(d^{i}>d^{\prime}+\varepsilon\right)\left[F(0,0)-d^{\prime}-\varepsilon\right]-\operatorname{Pr}\left(d^{i}>d^{\prime}\right)\left[F(0,0)-d^{\prime}\right]\right\} \\
& +\left\{\operatorname{Pr}\left(d^{i}<d^{\prime}+\varepsilon\right)\left[\left[1-\alpha\left(d^{\prime}\right)(1-L)\right]\left[F(\cdot)-K F_{K}(\cdot)\right]-d^{\prime}-\varepsilon\right]\right. \\
& \left.-\operatorname{Pr}\left(d^{i}<d^{\prime}\right)\left[\left[1-\alpha\left(d^{\prime}\right)(1-L)\right]\left[F(\cdot)-K F_{K}(\cdot)\right]-d^{\prime}\right]\right\} \\
& +\left\{\omega\left[\left[1-\alpha\left(d^{\prime}\right)(1-L)\right]\left[F(\cdot)-K F_{K}(\cdot)\right]-d^{\prime}-\varepsilon\right]\right. \\
& \left.-\omega\left[\frac{\left[1-\alpha\left(d^{\prime}\right)(1-L)\right]\left[F(\cdot)-K F_{K}(\cdot)\right]-F(0,0)}{2}-d^{\prime}\right]\right\}
\end{aligned}
$$

The first terms in curly brackets represent the difference between losing with an effort level $d^{\prime}+\varepsilon$, and losing with an effort level $d^{\prime}$. As for the second terms in curly brackets, they represent the difference between winning with an effort $d^{\prime}+\varepsilon$, and winning with an effort level $d^{\prime}$. It is easy to see that the sum of those terms goes to zero when $\varepsilon$ goes to zero. Now, the last terms in curly brackets represent the difference between winning alone with $d^{\prime}+\varepsilon$, and sharing the win with $d^{\prime}$ (in expected terms). Since the sum of these terms is strictly positive when $\varepsilon$ goes to zero, it pays to deviate to $d^{\prime}+\varepsilon$ when there is a probability mass at $d^{\prime}$. This implies that $H(d)$
cannot have a probability mass. And because the cumulative function is continuous, cases in which the jurisdictions play $d^{i}=d^{j}$ (a tie) occur with probability 0.

We now solve for $H(d)$ knowing that it must be continuous on $\left[d^{*}(0), \hat{d}\right]$. When $i$ plays the mixed strategy $H(d)$, its expected payoff is:

$$
\int_{d^{*}(0)}^{\hat{d}}[H(z) \Omega(\bar{K}, z)+(1-H(z)) \Omega(0, z)] d H(z)
$$

For $(H(d), H(d))$ to be a mixed strategy Nash equilibrium, it has to be that all pure strategies played with positive probability yield the same payoff. We construct the equilibrium so that the expected payoff of the two jurisdictions is $\Omega\left[0, d^{*}(0)\right]$. Thus, it has to be that:

$$
H(d) \Omega[\bar{K}, d]+(1-H(d)) \Omega[0, d]=\Omega\left[0, d^{*}(0)\right] \forall d \in\left[d^{*}(0), \hat{d}\right]
$$

It follows that for $d \in\left[d^{*}(0), \hat{d}\right]$, the mixed strategy $H(d)$ is given by:

$$
H(d)=\frac{\Omega\left[0, d^{*}(0)\right]-\Omega[0, d]}{\Omega[\bar{K}, d]-\Omega[0, d]}
$$

When $b$ plays the mixed strategy $H(d), a$ has no incentive to deviate from $H(d)$ because increasing the probability of playing any $d \in\left[d^{*}(0), \hat{d}\right]$ would not affect its payoff as all pure strategies are equivalent by construction.

This completes the proof.

## 7. Appendix II: Figures



- is the equilibrium allocation of capital

Figure 1
Case with $\partial \rho^{i} / \partial K^{i}<0$ and $d^{a}=d^{b}$


- is the equilibrium allocation of capital

Figure 2

Case with $\partial \rho^{i} / \partial K^{i}>0$ and $d^{a}>d^{b}$


- is the equilibrium allocation of capital

Figure 3
Case in which $\rho^{i}$ is non-monotonic in $K^{i}$ and U-shaped, $d^{a}>d^{b}$


- is the equilibrium allocation of capital

Figure 4
Case in which $\rho^{i}$ is non-monotonic in $K^{i}$ and has an inverted U-shape, $d^{a}=d^{b}$


Figure 5
The Payoffs

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[^1]:    ${ }^{1}$ Besley (1995) provides empirical evidence confirming that security matters using micro-data on investment in Ghana.
    2 Other examples of "twin" cities that exhibit very different property crime rates include Kansas City (Missouri) and Kansas City (Kansas), East St-Louis (Illinois) and St-Louis (Missouri), or Los Angeles and Anaheim.
    ${ }^{3}$ Casual observation reveals that capital (real estate and productive) is indeed relatively more abundant in some jurisdictions than in others.
    ${ }^{4}$ While a large literature has focused on capital tax competition between jurisdictions (see the survey by Wilson, 1999), the literature on competition in crime deterrence is extremely limited. An exception is Marceau (1997).
    5 The interaction between crime and occupational choice has been examined in a number of papers, e.g. Baumol (1990), Murphy et al. (1993), Acemoglu (1995), Baland and Francois (2000),

[^2]:    given pair of enforcement policies $\left(d^{a}, d^{b}\right)$, there are, in principle, several possible capital location equilibria. But with a unique capitalist, we can argue that the best of them is selected. And with a unique allocation of capital given policies, we can then characterize the equilibrium choices of enforcement policies.

[^3]:    ${ }^{10}$ For such functional form, the occupational choice equation (1) becomes $\mu[1-\alpha(d) C]=\alpha(d) L$ so that $L$ is independent of $K$.
    11 Under this functional form, the sign of $\partial L / \partial K$ is the same as that of $[1-\nu] K^{-\nu}$, which is positive.

[^4]:    12 See, for example, Mintz et Tulkens (1986), Wildasin (1988), Wilson (1986, 1999), or Zodrow and Mieszkowski (1986).
    13 Recall that $\partial \rho^{i} / \partial K^{i}<0$ also obtains if $\partial L^{i}\left(K^{i}, d^{i}\right) / \partial K^{i}$ is positive but not too large.
    14 This would not hold if the supply of capital was elastic.
    15 Note that from now on, and since the problem is symmetric, we simplify notation by dropping superscript $i \in\{a, b\}$ whenever possible.

[^5]:    16 The occupational choice equation (1) becomes $[1-\alpha(d)(1-L)] F_{L}(L, 0)=\alpha(d) F(L, 0)$. Since $F_{L}(L, 0)=0$ for all level of $L$, while $F(L, 0)>0$ for all level of $L$, all residents choose to become criminals and $L=0$.

[^6]:    19 İmrohoroğlu et al. (2000) explain the observed positive correlation using an alternative mechanism in which redistribution and political economy (both absent in our model) play an important role.

