OVERCONFIDENCE AND PERCEIVED SIGNAL DEPENDENCY

Michael Theobald *

(March, 2003)

JEL Classification Numbers: G12, G14
Keywords: Behavioural finance; pricing; adjustment process; confirmatory bias; overconfidence.

* Finance Group
Department of Accounting and Finance
The Birmingham Business School
University of Birmingham
Edgbaston
Birmingham B15 2TT
U.K.

e-mail: M.F.Theobald@bham.ac.uk
fax: (44) 0121 414 6678
tel: (44) 0121 414 6540
OVERCONFIDENCE AND PERCEIVED SIGNAL DEPENDENCY

ABSTRACT

Perceived dependencies lead to time series profiles of prices in markets characterised by overconfident investors that manifest the empirically documented overreaction, momentum and mean reversion effects within a unified and robust modelling structure. Confirmatory bias is demonstrated to induce such perceived signal dependencies.
OVERCONFIDENCE AND PERCEIVED SIGNAL DEPENDENCY

I INTRODUCTION

The finance profession has increasingly focussed upon developing models which incorporate non-rational behaviour into their structures as a result of the anomalous empirical evidence that has been generated in a number of studies (Hirshleifer (2001) provides a recent discussion and analysis of a variety of behavioural issues and phenomena in the finance field). For example, DeBondt and Thaler (1985, 1987) provided empirical support for the notion that markets “overreact”, particularly in the case of negative information. Jegadeesh and Titman (1993, 2001) provided empirical support for the existence of momentum in price series. Fama and French (1988) and Poterba and Summers (1988) provided empirical support for long-run mean reversion in price series. Alternative rationalisations to irrational behaviour for these results, such as time varying risk premia, research design limitations and random efficiency departures, have been advanced (for example, Fama (1998)). However, since behavioural explanations for such anomalies, themselves, can be rather “diffuse” a number of behavioural pricing models have been advanced by, for example, Shefrin and Statman (1994), Barberis, Shleifer and Vishy (1998), Daniel, Hirshleifer and Subrahmanyam (1998, 2001), Hong and Stein (1999) and Odean (1998).

This paper focuses upon models in which investors are assumed to be overconfident (Daniel et al (1998), Odean (1998), Gervais and Odean (2001)) and, in particular, the modelling structure is extended by introducing perceived signal dependencies. That is, the actual signals within the model structure developed in this paper may have dependencies; however, overconfident investors can also introduce their own perceived
signal dependencies into the pricing structure. For example, an overconfident investor may bias upwards the perceived dependency between the received private signal (the precision of which is believed to be higher than, in fact, it is) and the subsequent public signal in pricing due to the excessive confidence that that investor has in the private signal precision. In particular, it is formally demonstrated that confirmatory biases (Fiske and Taylor (1991), Gilovich (1991)) will introduce perceived signal dependencies. The overall structure is coherent in the sense that confirmatory biases themselves will induce the overconfidence in investors (Rabin and Schrag (1999)) that is a feature of the models developed by Daniel et al (1998), Odean (1998) and Gervais and Odean (2001). The dangers of falling into “degenerative research programmes” (as articulated by, for example, Brennan (2001)) wherein widely differing behavioural rationales are advanced for differing empirical phenomena are mitigated by working within this unified confirmatory bias/overconfidence framework. The perceived signal dependencies analytically incorporated into the formal modelling structure may be considered as alternative manifestations of the biased self attribution effect included in the outcome dependent confidence models presented in Daniel et al (1998).

The resultant model incorporating the perceived signal dependencies is demonstrated to not only incorporate overreactions and mean reversion into its structure, but also to reflect momentum effects which were absent in the basic, constant confidence model developed by Daniel et al (1998) and which required the incorporation of outcome-dependent confidence by Daniel et al (1998) into their modelling structure. The simple expedient of introducing perceived signal dependencies into the constant confidence model generates these fuller results within a simpler, unified framework.
The plan of the paper is as follows. The impacts of confirmatory biases upon the dependence structure of signals are analytically demonstrated in the next section. The pricing model is then developed in the following section of the paper, followed by an analysis of the departures of observed prices from intrinsic values. The subsequent section derives expressions for price change covariances and provides numerical analysis results. Section VI discusses some of the implications of the model and Section VII provides a rationale for the mechanism whereby prices mean revert in the long run in terms of signal dependency. Conclusions are presented in the final section.

II MODELLING CONFIRMATORY BIASES AND SIGNAL DEPENDENCIES

An investor, or more generally, decision maker, who exhibits a confirmatory bias will have a tendency to interpret new information as confirmatory of prior beliefs and expectations (see, for example, Crocker (1982), Fiske and Taylor (1991), Gilovich (1991), Nisbett and Ross (1980)). In the limit, such an investor could ignore/fail to perceive signal realisations that are not confirmatory. In less extreme situations an investor will misperceive contradictory signals as confirmatory with a finite probability. Two states of the world, \( x \in [A,B] \) will be considered and it is assumed that the investor receives a (“true”) signal, \( s(t) \), in period \( t \) as \( s(t) \in [a,b] \). The probability that \( s(t) = a \) is \( \pi \) and that \( s(t) = b \) is \( 1-\pi \), \( \forall t \).

In the absence of confirmatory biases the covariance between successive signals, \( \sigma(s(t), s(t+1)) \) is easily demonstrated to be equal to zero within this simple structure. The introduction of confirmatory biases means that there is a finite probability, \( \kappa \), that an investor will misinterpret a signal as being confirmatory when, in fact, it is contradictory.
In order to focus solely upon the impacts of confirmatory biases on autocovariances, Bayesian revisions are not incorporated into the modelling structure. The analysis of confirmatory biases within the modelling structure employed in Daniel, Hirshleifer and Subrahmanyam (1998) is relatively straightforward since two signals only are incorporated into the overall scenario – a private signal, wherein the investor is overconfident (i.e. the perceived signal noise is lower than its “true” value) and a subsequent public signal wherein the noise is unbiasedly assessed. In Appendix A, the perceived signal covariance in the presence of confirmatory biases is demonstrated to be given by

\[ \hat{\sigma}(s(t), s(t+1)) = (a - b)^2 \kappa \pi (1-\pi) \] …(1)

and clearly when \( \kappa = 0 \), \( \hat{\sigma}(s(t), s(t+1)) = 0 \) and when \( \kappa \neq 0 \) but \{a = +1, b = -1\}

\[ \hat{\sigma}(s(t), s(t+1)) = 4\pi(1-\pi)\kappa. \] Effectively, then, when \( \kappa > 0 \), a positive perceived signal dependency is induced into the structure. In the pricing work conducted in Section III correlations between signals figure in the numerical analyses. If confirmatory biases do not impact upon volatilities, signal correlations will equal \( \kappa \). Since confirmatory biases lead to overconfidence, that is a perceived greater signal precision, this identity no longer holds.\(^1\) However, provided \((\kappa, \pi) < 1\), the perceived correlation coefficient will be greater than \( \kappa \).

\(^1\) In the absence of confirmatory biases the signal volatility will be given by \((a-b)^2 \pi (1-\pi)\). If, however, an investor believes that \( s(t) = a \) (i.e. \( \pi > \frac{1}{2} \)) such an investor manifesting a confirmatory bias will perceived the volatility as \( (a-b)^2(1-\pi)(1-\kappa)(\kappa+\pi(1-\kappa)) \). With \( 0 \leq \kappa \leq 1 \), the latter, perceived volatility will be less than the former volatility. That is, confirmatory biases can induce overconfidence in the sense of lower perceived signal volatilities.
Alternatively, investors who are subject to confirmatory biases may, instead of misinterpreting signals, ignore contradictory signals (Rabin and Schrag (1999). That is, investors will only incorporate \{a,a\} and \{b,b\} states into the computation of their perceived signal comovements. The perceived signal covariances in the presence of confirmatory biases which manifest themselves in this way will be given by

\[ \hat{\sigma}(s(t), s(t+1)) = 2\pi^2 (1-\pi)^2 (a-b)^2 \cdots (2) \]

In the covariance formulation at equation (2) the joint state probabilities do not sum to one. If the joint state probabilities are adjusted to sum to one, equation (2) becomes

\[ \hat{\sigma}(s(t), s(t+1)) = 2\pi^2 (1-\pi)^2 \left( \pi^2 + (1-\pi)^2 \right)^{-1} (a-b)^2 \cdots (2') \]

Again, the signal covariances will be non-zero at equations (2) and (2').

III DEVELOPMENT OF PRICING MODEL WITH PERCEIVED DEPENDENCIES

(i) Structure

The structure of the model is generally close to that developed in Daniel, Hirshleifer and Subrahmanyam (1998). The model initially consists of four periods in time, with two investor types. Overconfident, risk neutral investors receive a private signal which they perceive to have a higher precision than, in fact, the signal contains; uninformed, risk averse investors do not receive this signal. Effectively, within this structure the overconfident, risk neutral investors will determine prices. The opportunity set comprises a risky asset which generates a terminal value (i.e. at time four), \( \theta \), with \( \theta \sim N(\theta, \sigma^2(\theta)) \) and a risk-free numeraire, which without loss of generality, is assumed to generate a zero return. Informed, overconfident investors receive a noisy private signal at time one, \( s(1) \) as \( s(1) = \theta + \varepsilon \), with \( \varepsilon \sim N(0, \sigma^2(\varepsilon)) \). At time two, a noisy public signal is received by all investors (i.e. both informed and uninformed), \( s(2) \), with \( s(2) = \theta + \eta \) and,
in this case the noise element, \( \eta \), is distributed as \( \eta \sim N(0, \sigma^2(\eta)) \). As in Daniel, Hirshleifer and Subrahmanyam (1998), \( \theta \) will be assumed to be equal to zero. Informed, overconfident investors are assumed to attribute a higher precision to the noisy private signal as \( \sigma^2(c) \), with \( \sigma^2(c) < \sigma^2(\epsilon) \). All investors correctly estimate the precision of the noise element, \( 1/\sigma^2(\eta) \), in the noisy public signal.

In Daniel, Hirshleifer and Subrahmanyam (1998) it is assumed that all the signal related covariance terms are equal to zero, i.e. \( \sigma(\epsilon, \eta) = \sigma(\eta, \theta) = \sigma(c, \eta) = \sigma(c, \theta) = 0 \). Within this structure, the total signals will exhibit dependence since both contain the terminal value, \( \theta \) (i.e. in the absence of noise dependency, the covariance between \( \theta + \epsilon \) and \( \theta + \eta \) will be equal to the variance of \( \theta \)). This assumption of independence between the noise in the public and private signals will be relaxed in this paper. Overconfidence is assumed to be manifested in the residual/noise component of the total signal and it is assumed, here, that the confirmatory biases will, themselves, become manifest in the two noise terms, \( \epsilon \) and \( \eta \). As demonstrated in section II the confirmatory bias will induce perceived signal dependencies\(^2\).

\(^2\) Signal dependency may arise in a more general behavioural setting from the tendency of individuals to perceived patterns in data even when such patterns are absent (i.e. clustering illusion, ambiguity aversion (Ellsberg (1961). As a means of attempting to place a structure upon an uncertain environment, dependencies may be imposed to resolve or reduce uncertainty; the practice of “technical analysis” immediately springs to mind. The perceived and actual correlations may differ due to the well documented problems that are experienced in understanding correlations (see, for example, Kruschke and Johansen (1999)).
(ii) **Pricing**

The prices set by the risk-neutral, informed (overconfident) investors will be the same as those in Daniel et al (1998), at time zero, one and three since the perceived signal dependency will only become manifest in the time two price when the public signal occurs. That is, with \( P(0) = 0 \),

\[
P(1) = A (\theta + \varepsilon) \quad \ldots (3)
\]

and \( P(3) = \theta \) \ldots (4)

where \( A = \sigma^2(\theta)(\sigma^2(\theta) + \sigma^2(c))^{-1} \)

Using the properties of the multivariate normal distribution, the price at time two, \( P(2) \) will be

\[
P(2) = (\sigma^2(\theta)/E) \{F \theta + G \varepsilon + H \eta\} \quad \ldots (5)
\]

see the Appendix, equation (B4), where

\[
E = [\sigma^2(\theta)\sigma^2(\eta) + \sigma^2(\theta)\sigma^2(c) + \sigma^2(c)\sigma^2(\eta) - \sigma(\varepsilon, \eta)(\sigma(\varepsilon, \eta) + 2\sigma^2(\theta))] \wedge
\]

\[
F = [\sigma^2(\eta) + \sigma^2(c) - 2\sigma(\varepsilon, \eta)] \wedge
\]

\[
G = [\sigma^2(\eta) - \sigma(\varepsilon, \eta)] \wedge
\]

and \( H = [\sigma^2(c) - \sigma(\varepsilon, \eta)] \wedge \)

where \( \sigma(\varepsilon, \eta) \) is the covariance between the noise in the private and public signals as perceived by the informed investor. Naturally, when \( \sigma(\varepsilon, \eta) = 0 \), equation (5) above is identical to equation (6) in Daniel et al (1998) where noise independence is assumed.

Potentially, if investors perceive signal dependencies they may transform the subsequent signals to reflect this dependency (i.e. the noise, \( \eta \), could be partially predictable) and incorporate this effect in the price at time 1, \( P(1) \). However, with the
confirmatory biases as modelled in this paper, such transformations are not feasible ex ante (i.e. at time 1); the dependencies are perceived ex post when the signal occurs at time 2 and incorporated in the price in this period, P(2).

The price in the period when the public signal is disclosed, P(2), is a non-linear function of the signal dependency perceived by the overconfident investor. Since the price is dependent upon the signal realisations and signal precisions, insights into the nature of the pricing function are best obtained by numerical analysis. For comparability to Daniel et al (1998) the same parameters for the precisions and noise that they used in their simulations are employed here; that is, \( \sigma^2(\theta) = \sigma^2(\varepsilon) = 1 \) and \( \sigma^2(\eta) = 7.5 \). In their simulations they also assumed that \( \theta = 0 \) and \( \theta + \varepsilon = 1 \) (i.e. an “unduly favourable” private signal). Figure 1 contains the plots of P(2) against the perceived correlation\(^3\) between the noise in the public and private signals for differing levels of confidence (\( \sigma^2(c) = 0.9 \) and 0.5) and differing public signal realisations (\( \eta = 0 \) and 1). Irrespective of the public signal realisation, \( \eta \), the price is an increasing function of the perceived dependency in the range of perceived signal correlations above 0.3 and is at a minimum in the region around a perceived signal dependency of zero. As would be anticipated, the price is generally higher, the greater the overconfidence regarding the private signal precision (i.e. \( \sigma^2(c) = 0.5 \) versus \( \sigma^2(c) = 0.9 \)). Effectively, then, prices are driven by both overconfidence and perceived signal dependency and, in the positive dependency region, the price, P(2), increases with the perceived signal dependency.

\(^3\) Recall from Section II, that the perceived correlation will be an increasing function of the probability, \( \kappa \), of misinterpreting a signal as confirmatory when, in fact, it is not.
IV PRICE TIME PROFILES

In the simpler structure employed by Daniel, Hirshleifer and Subrahmanyam (1998) (that is, their basic model with constant confidence) the market price initially reflected an overreaction to the private signal about which informed investors were overconfident. The overreaction was then partially corrected in the period when the public signal was announced. As they point out, this scenario is not consistent with the momentum type property of prices documented in Jegadeesh and Titman (1993, 2001). Where, however, informed investors perceive a correlation between the noise in the private signal and the subsequent public signal, price persistency can be a feature of such a structure over the periods where the private and public signals are announced.  

The price change from \( t=1 \) to \( t=2 \) may be derived as

\[
P(2) - P(1) = \frac{\sigma^2(\theta)(\sigma^2(c) - \sigma(\epsilon, \eta))}{E(\sigma^2(\theta) + \sigma^2(c))} \left\{ (\sigma^2(c) - \sigma(\epsilon, \eta))\theta - (\sigma^2(\theta) + \sigma(\epsilon, \eta))\epsilon + (\sigma^2(c) + \sigma^2(\theta))\eta \right\}
\]

\[\ldots (6)\]

The articulation between the overconfidence, as manifested in \( \sigma^2(c) \), and that in \( \sigma(\epsilon, \eta) \) is readily apparent. Again, insights into this relationship are gleaned by numerical analyses. Using the same parameters as previously in Section II and with \( \eta=0 \), the price changes across differing perceived correlations are presented in Figure 2. As is readily apparent in Figure 2, the presence, or otherwise, of persistency in prices over the time period that includes both the private and public signals depends upon the perceived signal dependency. Where there is no perceived signal dependency, the price change is

\[\text{Note that the signal dependency as perceived by the informed investor is between the distributions of } c \text{ and } \eta \text{ and may not be fully dispelled by signal realizations differing.}\]
negative, corresponding to the correction phase in the constant confidence model proposed by Daniel et al (1998). However, where positive perceived signal dependency is present the price change from period 1 to 2 becomes positive when the perceived signal correlation is in the range from 0.4 to 1.0. That is, within this relatively simple structure, price persistency of the type documented in Jegadeesh and Titman (1993, 2001) can occur.

Further insights into the nature of the price adjustment process may be obtained from an analysis of the departures from intrinsic values induced by overconfidence. By intrinsic values in this context we mean the “unbiased” price, wherein \( \sigma^2(\varepsilon) = \sigma^2(c) \) and, where appropriate \( \sigma(\varepsilon, \eta) = \sigma(\varepsilon, \eta) \), where \( \sigma(\varepsilon, \eta) \) is the “true” covariance/dependency. At time 1, the intrinsic value departure will be

\[
P(1) - V(1) = \frac{\sigma^2(\varepsilon) - \sigma^2(c)}{\left[1 + \frac{\sigma^2(c)}{\sigma^2(\theta)}\right] \left[1 + \frac{\sigma^2(\varepsilon)}{\sigma^2(\theta)}\right]} (\theta + \varepsilon) \quad \ldots (7)
\]

It is readily apparent that the departure from intrinsic value will directly depend upon the magnitude of the overconfidence and the magnitude of the signal realisation. The analysis at time 1, equation (7), would be identical to that for the constant confidence model developed in Daniel, Hirshleifer and Subrahmanyam (1998). With intrinsic value departures at time 2 the analysis and implications will differ, however, since perceived signal dependencies are introduced in this period. The departure of actual prices from intrinsic values in this period will be given by

\[
P(2) - V(2) = \frac{\sigma^2(\theta)}{EE} \left[\alpha\theta + \beta\varepsilon + \gamma\eta\right] \quad \ldots (8)
\]

\[\text{This persistency occurs even when the signal "reverses" as in this case (i.e. } \varepsilon=1, \eta=0)\].
where \( E^1 = \sigma^2(\theta)\sigma^2(\eta) + \sigma^2(\theta)\sigma^2(\varepsilon) + \sigma^2(\eta)\sigma^2(\varepsilon) - \sigma(\varepsilon, \eta)[\sigma(\varepsilon, \eta) + 2\sigma^2(\theta)]. \)

\[ \hat{\alpha} = \sigma^4(\eta)(\sigma^2(\varepsilon) - \sigma^2(\epsilon)) - \sigma^2(\theta)(\sigma(\varepsilon, \eta)\sigma^2(\varepsilon) - \sigma(\varepsilon, \eta)\sigma^2(\epsilon)) \]

\[ + (\sigma(\varepsilon, \eta) - \sigma(\varepsilon, \eta)) [\sigma^2(\eta)(\sigma(\varepsilon, \eta) + \sigma(\varepsilon, \eta)) + 2\sigma(\varepsilon, \eta)\sigma(\varepsilon, \eta)] \]

\[ + \sigma^2(\varepsilon, \eta)\sigma^2(\epsilon) - \sigma^2(\varepsilon, \eta)\sigma^2(\epsilon) \]

\[ \beta = \sigma^2(\eta)(\sigma^2(\theta) + \sigma^2(\eta))(\sigma^2(\epsilon) - \sigma^2(\epsilon)) \]

\[ + \sigma^2(\eta)[\sigma^2(\theta) + \sigma(\varepsilon, \eta) + \sigma(\varepsilon, \eta)](\sigma(\varepsilon, \eta) - \sigma(\varepsilon, \eta)) \]

\[ - (\sigma^2(\theta) + \sigma^2(\eta))(\sigma(\varepsilon, \eta)\sigma^2(\epsilon) - \sigma(\varepsilon, \eta)\sigma^2(\epsilon)) \]

and \( \gamma = \sigma^2(\theta)\sigma^2(\eta)(\sigma^2(\epsilon) - \sigma^2(\epsilon)) - (\sigma^2(\theta) + \sigma^2(\eta))(\sigma^2(\epsilon)(\sigma(\varepsilon, \eta) - \sigma(\varepsilon, \eta)) \sigma^2(\epsilon)) \]

\[ - \sigma^2(\theta)\sigma^2(\eta)(\sigma(\varepsilon, \eta) - \sigma(\varepsilon, \eta)) \]

\[ - (\sigma^2(\varepsilon, \eta)\sigma^2(\epsilon) - \sigma^2(\varepsilon, \eta)\sigma^2(\epsilon)) \]

When there is no overconfidence regarding signal precisions (i.e. \( \sigma^2(c) = \sigma^2(\varepsilon) \)) and no actual (\( \sigma(\varepsilon, \eta) = 0 \)) or perceived (\( \sigma(\varepsilon, \eta) = 0 \)) noise dependencies then \( P(2) = V(2) \).

Insights into the departures from intrinsic values in this period are best provided by numerical analyses. Using the parameters already used in this section and with signals \( \{\varepsilon=1, \eta=0\} \) two difference functions are plotted in Figure 3 for the two differing levels of precision overconfidence (i.e. \( \sigma^2(c) = 0.9 \) and \( 0.5 \)) against the perceived signal dependency assuming an intrinsic value where \( \sigma(\varepsilon, \eta) = 0 \). The price/intrinsic value difference is always strictly positive and the departures from intrinsic values increase as the perceived signal dependency increases.
V  PRICE CHANGE COVARIANCES

i) Covariance between P(2) – P(1) and P(1) – P(0)

The covariance between the price changes in the first two periods is derived in the
Appendix, equation (C3), as

\[
\text{cov}\{P(2) - P(1), P(1) - P(0)\} = \frac{\sigma^4(\theta)}{E(\sigma^2(\theta) + \sigma^2(c))^2} \left[ \sigma^2(\theta)(\sigma^2(c) - \sigma^2(\varepsilon))(\sigma^2(c) - 2\sigma(\varepsilon, \eta)) + (\sigma^2(c) + \sigma^2(\theta) - \sigma(\varepsilon, \eta))\sigma(\varepsilon, \eta)\sigma^2(c) - \sigma(\varepsilon, \eta)\sigma^2(\varepsilon)) + \sigma(\varepsilon, \eta)\sigma^2(\theta)(\sigma(\varepsilon, \eta) - \sigma(\varepsilon, \eta)) \right] \quad \ldots(9)
\]

where \( \sigma(\varepsilon, \eta) \) is the actual or “true” covariance between the noise in the private and public signals.

When there is no perceived or actual signal dependency (i.e. \( \hat{\sigma}(\varepsilon, \eta) = \sigma(\varepsilon, \eta) = 0 \)) equation (7) becomes identical to the covariance expression developed in Daniel, Hirshleifer and Subrahmanyam (1998)). That is,

\[
\text{cov}\{P(2) - P(1), P(1) - P(0)\} = \frac{\sigma^6(\theta)\sigma^2(c)(\sigma^2(c) - \sigma^2(\varepsilon))}{E(\sigma^2(\theta) + \sigma^2(c))^3} \quad \ldots(10)
\]

where \( E = \sigma^2(\theta)\sigma^2(c) + \sigma^2(\theta)\sigma^2(\varepsilon) + \sigma^2(\eta)\sigma^2(c) \) in this case. With overconfident investors receiving the private signal this covariance term will be negative since \( \sigma^2(c) < \sigma^2(\varepsilon) \) in this structure. That is, in the “correction” phase of the model, the correction corresponds to a negative comovement or reversal. Note, however, that this negative covariance essentially derives from the market pricing ensuring that the observed prices P(2) and
P(1) are priced such that the observed covariance between prices reflects $\sigma^2(\varepsilon)$ not $\sigma^2(c)$. That is, $\text{cov} \{P(2) - P(1), P(1) - P(0)\}$ is a function of $\text{cov} \{E^{-1}\sigma^2(\theta)\sigma^2(\eta)\varepsilon, A\varepsilon\}$ and this covariance equals $A E^{-1} \sigma^2(\theta)\sigma^2(\eta)\sigma^2(\varepsilon)$ not $A E^{-1} \sigma^2(\theta)\sigma^2(\eta)\sigma^2(c)$; if this were the case (i.e. involving $\sigma^2(c)$, not $\sigma^2(\varepsilon)$), then there would be no correction, the covariance of price changes in this phase being zero. Figure 4 contains the corresponding plots of equation (9) for various values of the perceived correlations and precision overconfidence with actual correlations equal to zero and using similar model parameters to previous analyses. As the perceived dependency increases so the persistence/momentum increases as would be anticipated from the previous analyses. When $\sigma(\varepsilon, \eta) = 0$ the covariance is negative, corresponding to the Daniel et al (1998) constant confidence model.

(ii) **Covariance between P(3) – P(2) and P(2) – P(1)**

The covariance for this time interval, derived in outline in Appendix C, has the form

$$\text{cov} \{P(3) - P(2), P(1) - P(0) =$$

$$\frac{\sigma^4(\theta)}{E^2(\sigma^2(\theta) + \sigma^2(c))} \left\{ \begin{array}{c}
\sigma^2(\theta)\sigma^2(\eta)(\sigma^2(\varepsilon) - \sigma^2(c))(\sigma^2(c) - \sigma(\varepsilon, \eta)) + \sigma(\varepsilon, \eta)
\end{array} \right\} + \sigma(\varepsilon, \eta)$$

$$+ \sigma^4(\varepsilon, \eta) + \sigma^2(\varepsilon, \eta) \sigma^2(c) + \sigma(\varepsilon, \eta) \sigma^2(c) + \sigma^2(c) \sigma^3(\varepsilon, \eta)$$

$$\text{where } S = \sigma^2(c) \{\sigma^2(\varepsilon)\sigma^2(\eta) + \sigma^2(\theta)\sigma^2(\eta) - \sigma^2(\theta)\sigma^2(c)\}$$

$$T = -\sigma^4(\varepsilon) - \sigma^2(\theta)\sigma^2(\eta) + \sigma^2(\theta)\sigma^2(\eta) - \sigma^2(\varepsilon)\sigma^2(c) - \sigma^2(\theta)\sigma^2(\eta)$$

$$U = 2\sigma^2(c) + \sigma^2(c)$$

$$V = \sigma^2(c) \{\sigma^2(\theta)\sigma^2(\eta) - \sigma^2(\eta)\sigma^2(c) - \sigma^2(\theta)\sigma^2(\eta)\}$$

$$W = 2\sigma^4(c) + \sigma^2(\theta)\sigma^2(\eta) + \sigma^2(\varepsilon)\sigma^2(c) - \sigma^2(\theta)\sigma^2(c)$$
and $X = -3\sigma^2(c)$

When $\sigma(\epsilon, \eta) = \sigma(\epsilon, \eta) = 0$ we obviously have the same situation as in the constant confidence model developed in Daniel, Hirshleifer and Subrahmanyam (1998) – that is, a positive covariance corresponding to a continuation of the correction that started in the time two prices in the absence of perceived signal dependency. The covariance deriving from equation (11) for differing perceived covariances are plotted in Figure 5 for the cases where $\sigma^2(c) = \sigma^2(\epsilon)$ and where $\sigma^2(c) = 0.9$, with $\sigma(\epsilon, \eta) = 0$ for both plots. With $\sigma^2(c) = 0.9$, the covariance is negative across the whole positive correlation range and with zero precision overconfidence the covariance is negative over the positive perceived correlation range from 0.4. That is, the persistency manifested in time two prices is corrected in moving to the period three price. Effectively, then, the overreaction to the private signal impounded in $P(1)$ can persist in the price when the public signal occurs, $P(2)$, in the presence of perceived signal dependencies; mean reversion occurs subsequently towards the terminal value, $P(3)$ in such a structure.

V IMPLICATIONS OF SIGNAL DEPENDENCY

This paper has demonstrated that momentum effects will be parsimoniously introduced into the static confidence model developed in Daniel, Hirshleifer and Subrahmanyam (1998) by signal dependencies and that confirmatory biases are potential causes. The new implications of the modelling developed in this paper will derive from the momentum/confirmatory bias articulations. That is, momentum effects will increase as signal dependencies increase which, in turn, increase with the incidence of confirmatory biases.
Confirmatory biases are likely to occur where signal ambiguity is present, where correlations that are displaced through time are evaluated and where information is selectively collected or scrutinished (Rabin and Schrag (1999)). Signal ambiguity will increase as the signal noises, $\sigma^2(\varepsilon)$ and $\sigma^2(\eta)$, increase. Signal noise would be anticipated to be higher with growth stocks rather than value stocks since in the former case growth options have to be incorporated into the analysis. The ambiguity of signals will increase where the interpretation of financial statements is difficult. For example, momentum effects would be anticipated to be larger in jurisdictions with lower accounting disclosure requirements. Similarly, since the financial information relating to dotcom stocks is difficult to interpret, momentum effects would be strong in this stock category. Momentum effects would be expected to be higher in more “volatile” periods since there is the potential for greater ambiguity in such periods (and, indeed, the higher volatility would lead to higher $\sigma(\varepsilon,\eta)$ terms).

When investors view signals as “linked” in some way, investors may perceive signals as reinforcing prior beliefs and hence increasing perceived signal correlations and momentum effects. For example, quarterly earnings disclosures may lead to greater price persistency than the disclosure of interim and final earnings disclosures due to their greater frequency and the necessary approximations inherent in their generation.

Investors are more likely to selectively collect or scrutinise information in situations where investor education and knowledge is relatively poor or where limited information is made available. Momentum would be anticipated to be greater in countries such as the emerging markets or asset classes which are neglected by larger, sophisticated investors such as small caps.
VI LONG TERM MEAN REVERSION MECHANISM

The price profile developed to this point has focussed upon generating the impacts of signal dependency upon medium term price persistence (or momentum). The previous sections demonstrated that, in the presence of perceived positive signal dependence, price persistence could occur in the \{P(1), P(2)\} variables. Since the price at time three equals \(\theta\), price divergences will of necessity have to mean revert in moving from P(2) to P(3) where price persistency has occurred. However, to this point, a mechanism has not been advanced for this reversion. The biased self attribution rationale advanced in Daniel et al (1998) may be invoked to provide a behavioural mechanism for such price reversions. An alternative rationale may be provided in the context of perceived signal dependencies, however.

An additional public signal, S(2a), with \(S(2a) = \theta + \emptyset\), where \(\emptyset \sim N(0, \sigma^2(\emptyset))\) is now assumed to occur in period 2(a), immediately after the public signal, \(\eta\), but prior to the final period of the model. The price, P(2a), may be determined in an analogous fashion to that employed in the appendix in the determination of P(2). To keep the formulae in manageable terms, the price at P(2a) will be determined for the special case where \(\sigma^2(c) = \sigma^2(\epsilon) = \sigma^2(\theta) = 1\), \(\sigma^2(\eta) = \sigma^2(\emptyset)=7.5\) and \(\theta = \eta = \emptyset = 0\), with \(\epsilon = 1\). Within this simplified scenario, the price at time 2a, P(2a), will be given by

\[
P(2a) = \left(7.5(1+\sigma(\epsilon,\eta)) - 8.5(1+\sigma(\epsilon,\emptyset)) + 71.25\sigma(\epsilon,\emptyset)\right)J^{-1} \quad \ldots(12)
\]

where \(J = (8.5(1 + \sigma(\epsilon,\eta)) - (1+\sigma(\epsilon,\emptyset)))(1+\sigma(\epsilon,\eta)) - 8.5(1+\sigma(\emptyset,\epsilon))(2 - (1 + \sigma(\epsilon,\eta))(1+\sigma(\epsilon,\emptyset)))\)

with \(\sigma(\epsilon,\emptyset)\) the perceived signal dependency between signals \(\epsilon\) and \(\emptyset\). To keep the
formulation simple $\sigma(\eta, \emptyset)$ has been assumed to be equal to zero. Figure 6 contains the plots of the $P(2a)-P(2)$ difference as a function of $\rho(\varepsilon, \eta)$ with $\rho(\varepsilon, \emptyset) = 1.0$ and 0.9. As can readily be seen, the price declines with the second public signal announcement when $\rho(\varepsilon, \eta) \neq 0$. The decline is particularly strong for high values of $\rho(\varepsilon, \eta)$ (where $P(2)$ is high) and when the perceived dependency between the private and second public signal, $\rho(\varepsilon, \emptyset)$, is lower. That is, $P(2)$ and $P(2a)$ will be positive functions of $\rho(\varepsilon, \eta)$ and $\rho(\varepsilon, \emptyset)$, respectively. Then, as $\rho(\varepsilon, \eta) \to 1$ and $\rho(\varepsilon, \emptyset) \to 0$, so the negativity of $P(2a)-P(2)$ will be the larger.

VI CONCLUSIONS

When perceived signal dependencies that can be induced by confirmatory biases are included within the constant confidence modelling structure originally proposed in Daniel, Hirshleifer and Subrahmanyam (1998), the pricing formulation for their “correction” period is modified somewhat. These modifications lead to changes in the time series profile of prices and price change comovements. The extended model implies that the overreaction to the private signal caused by overconfidence with regard to its precision continues into the period where a public signal is received where overconfident investors manifesting confirmatory biases perceive a signal dependency. Correction (mean reversion) then wholly takes place in the final period of the model. Effectively, then, an alternative mechanism is provided for incorporating momentum effects in the original constant confidence model developed analytically by Daniel, Hirshleifer and Subrahmanyam (1998). The overall modelling structure is robust and coherent in that confirmatory biases, which induce investor overconfidence, also lead to investors perceiving signal dependencies.
Figures 1-6 here
Key to Figures:

Figure 1: \( P(2), \{x,y\} \) denotes the price at period 2 with \( \sigma^2(c) = x \) and \( \eta = y \).

Figure 2: \( P(2)-P(1), \{x\} \) denotes the price change when \( \sigma^2(c) = x \).

Figure 3: \( P(2), \{x\} \) denotes the price/intrinsic value difference when \( \sigma^2(c) = x \).

Figure 4: \( \text{cov}(P2-P1, P1-P0), \{x\} \) denotes the price change covariance when \( \sigma^2(c) = x \).

Figure 5: \( \text{cov}(P3-P2,P2-P1), \{x\} \) denotes price change covariance with \( \rho(\varepsilon,\varnothing) = x \).

Figure 6: Series 1 denotes \( P(2a)-P(2) \) with \( \rho(\varepsilon,\varnothing) = 0.9 \)

Series 2 denotes \( P(2a)-P(2) \) with \( \rho(\varepsilon,\varnothing) = 0.5 \).
## APPENDIX

### A: CONFIRMATORY BIASES AND SIGNAL DEPENDENCIES

The joint signal probabilities in the presence of confirmatory biases will be as follows:

<table>
<thead>
<tr>
<th>Signal type</th>
<th>Joint Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s(t)$</td>
<td>$s(t+1)$</td>
</tr>
<tr>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>b</td>
<td>a</td>
</tr>
<tr>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>b</td>
<td>a</td>
</tr>
</tbody>
</table>

The perceived covariance, $\sigma(s(t), s(t+1))$ will then equal

\[
\pi^2 (1-\pi)^2 (a-b)^2 + (1-\pi)^2 \pi^2 (a-b)^2 + \kappa\pi(1-\pi)^3 (a-b)^2
- (1-\kappa)\pi^2 (1-\pi)^2 (a-b)^2 + \kappa\pi^3 (1-\pi)(a-b)^2
- (1-\kappa)\pi^2 (1-\pi)^2 (a-b)^2
\]

\[
= (a-b)^2 \pi(1-\pi) \{2\pi (1-\pi)(1-\kappa) + \kappa((1-\pi)^2 + \pi^2)\}
= (a-b)^2 \pi(1-\pi) \kappa
\]

\[\text{\ldots(A1)}\]
B: PRICING

Since the overconfident investors are the price setters,

\[ P(2) = E \{ \theta / \theta + \epsilon, \theta + \eta \} \quad \text{...(B1)} \]

where \( E \) equals the conditional expectation based upon the overconfident investors’ estimates of the precision of the noise in the private signal. Via the multivariate normal distribution, then,

\[ P(2) = \frac{\sigma^2(\theta+\epsilon), \sigma(\theta+\epsilon, \theta+\eta), \sigma(\theta+\epsilon, \theta+\eta)}{\sigma^2(\theta+\epsilon) \cdot \sigma^2(\theta+\eta) - (\sigma(\theta+\epsilon, \theta+\eta))^2} \]

\[ + \frac{\sigma^2(\theta+\epsilon), \sigma(\theta+\eta), \sigma(\theta+\epsilon)}{\sigma^2(\theta+\epsilon), \sigma^2(\theta+\eta) - (\sigma(\theta+\epsilon, \theta+\eta))^2} (\theta+\epsilon) (\theta+\eta) \quad \text{...(B2)} \]

\[ = \frac{\sigma^2(\theta)}{E} \left[ (\sigma^2(\eta) - \sigma(\epsilon, \eta)) (\theta+\epsilon) + (\sigma^2(c) - \sigma(\epsilon, \eta)) (\theta+\eta) \right] \quad \text{...(B3)} \]

where \( E = [\sigma^2(\theta) \sigma^2(\eta)+\sigma^2(\theta) \sigma^2(c)+\sigma^2(c) \sigma^2(\eta) - \sigma(\epsilon, \eta) (\sigma(\epsilon, \eta) + 2 \sigma^2(\theta))] \)

\[ = \frac{\sigma^2(\theta)}{E} \left[ (\sigma^2(\eta) + \sigma^2(\epsilon) - 2 \sigma(\epsilon, \eta) \theta + (\sigma^2(\eta) - \sigma(\epsilon, \eta)) \epsilon + (\sigma^2(c) - \sigma(\epsilon, \eta)) \eta \right] \quad \text{...(B4)} \]
C: PRICE CHANGE COVARIANCES

\[ \text{cov} \{ P(2) - P(1), P(1) - P(0) \} = \text{cov} \left( \frac{\sigma^2(\theta) \{ F\theta + G\varepsilon + H\eta \} - A(\theta + \varepsilon), A(\theta + \varepsilon) }{E} \right) \ldots (C1) \]

\[ = \left[ \frac{\sigma^4(\theta)}{\sigma^2(\theta) + \sigma^2(c)} \right] \left\{ \frac{X}{E} - \left[ \frac{\sigma^2(\theta) + \sigma^2(\varepsilon)}{\sigma^2(\theta) + \sigma^2(\varepsilon)} \right] \right\} \ldots (C2) \]

where \( X = \sigma^2(\theta) \sigma^2(c) + \sigma^2(\theta) \sigma^2(\eta) - 2\sigma(\varepsilon, \eta) \sigma^2(\theta) + \sigma^2(\varepsilon) \sigma^2(\eta) \)

\[ - \sigma(\varepsilon, \eta) \sigma^2(\varepsilon) + \sigma(\varepsilon, \eta) \sigma^2(c) - \sigma(\varepsilon, \eta) \sigma(\varepsilon, \eta) \]

and, with some rearrangement,

\[ = \frac{\sigma^4(\theta)}{E(\sigma^2(\theta) + \sigma^2(c))^2} \left\{ \frac{\sigma^2(\theta)(\sigma^2(c) - \sigma^2(\varepsilon))}{E} \left( \sigma^2(c) - 2\sigma(\varepsilon, \eta) \right) \right\} \]

\[ + (\sigma^2(c) + \sigma^2(\theta) - \sigma(\varepsilon, \eta)) (\sigma(\varepsilon, \eta) \sigma^2(\varepsilon) - \sigma(\varepsilon, \eta) \sigma^2(\varepsilon)) \]

\[ + \sigma(\varepsilon, \eta) \sigma^2(\theta)(\sigma(\varepsilon, \eta) - \sigma(\varepsilon, \eta)) \ldots (C3) \]

Moving on to the covariance between \( P(3) - P(2) \) and \( P(2) - P(1) \), this covariance is defined as

\[ \text{cov} \{ P(3) - P(2), P(2) - P(1) \} = \text{cov} \left( \frac{\theta - \sigma^2(\theta) \left( F\theta + G\varepsilon + H\eta \right)}{E} \right) \]

\[ \left\{ \frac{\sigma^2(\theta) \left[ F\theta + G\varepsilon + H\eta \right]}{E} \right\} - A(\theta + \varepsilon) \ldots C(4) \]

\[ = \frac{\sigma^4(\theta)}{E^2(\sigma^2(\theta) + \sigma^2(c))} \left\{ F(\sigma^2(\theta) + \sigma^2(c)) E - E^2 - (\sigma^2(\theta) + \sigma^2(c)) \alpha + E\beta \right\} \]

with \( \alpha = F^2 \sigma^2(\theta) + G^2 \sigma^2(\varepsilon) + H^2 \sigma^2(\eta) + 2GH\sigma(\varepsilon, \eta) \)
and $\beta = F\sigma^2(0) + G\sigma^2(\varepsilon) + H\sigma(\varepsilon, \eta)$

After expanding and with some manipulation the expression at equation (9) is obtained.
REFERENCES


