Dynamic Information Disclosure

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Job Market Paper

This draft: Nov 21, 2002

Abstract

We study information disclosure as a tool to improve the informativeness of the stock price. Introducing a dynamic financial market changes the economics of the disclosure process and overturns the current intuition about the desirability of immediate disclosure. In our model, the firm’s manager maximizes the present value of the firm by ex-ante maximizing the informational efficiency of the stock price. Information can flow into the stock price through both voluntary disclosure and costly private information acquisition. While the manager can improve the trading opportunities of informed speculators by withholding his information, the arbitrage capacity of these speculators is limited by the amount of risk they have to bear. Thus, in the static benchmark case, full managerial disclosure leads to maximal informational efficiency, confirming conventional intuition. In a dynamic setting, however, we find that immediate disclosure is not always optimal. By delaying disclosure until an intermediate point in time, the firm can offer opportunities for informed trading without exposing speculators to the substantial risks of holding the asset until liquidation. This strengthens the benefits of delaying disclosure while the information is only withheld for a short time. We characterize situations in which it will be optimal to disclose information immediately and situations in which delaying disclosure until an intermediate point in time is preferable. This gives us empirical predictions that help us understand the significant differences in the timing of earnings announcement across firms. Further applications include IPOs and earnings pre-announcements.

*The author wants to thank the finance Ph.D. program at UCLA’s Anderson School for their support and a great education. I am extremely grateful to the members of my committee: Avanidhar Subrahmanyam (chair), Antonio Bernardo, Michael Brennan, Jack Hughes, and Bill Zame. I also benefitted much from Francis Longstaff and Walter Torous. All errors are my responsibility. Comments are highly welcome. Corresponding Address: Anderson School at UCLA, Room C4.01, Los Angeles, CA 90095-1481. Email: mdierker@anderson.ucla.edu
1 Introduction

The purpose of the disclosure process is to provide investors with useful information. Timeliness of information is considered a key factor for its usefulness. Information, it is widely believed, should be disclosed as early as possible, since this results in more informative stock prices and the removal of information asymmetries. This is, for example, the rationale underlying a current S.E.C. proposal to speed up the filing dates of annual and quarterly 10-K/Q reports:

"Shortening the due dates for quarterly, annual and transition reports would ... accelerate the delivery of information to investors and the capital markets, enabling them to make more informed investment and valuation decisions. This helps the capital markets function more efficiently, which means more efficient valuation and pricing."


This paper challenges the above notion. We find that some firms maximize the informational efficiency of the stock price by withholding information for short periods of time. We characterize situations in which it will be optimal to disclose information immediately and situations in which delaying disclosure until an intermediate point in time is preferable. This helps us to better understand the significant differences in the timing of earnings announcement across firms observed in the data.

To address issues of disclosure timing, clearly we need a dynamic model. We use a model of a dynamic financial market with noisy rational expectations along the lines of Hirshleifer, Subrahmanyam, and Titman (1994). In addition, we also need a clear economic rationale why disclosure is desirable. Better information leads to more efficient investment decisions. In this paper, we assume that a more informative stock price improves the firm’s access to outside capital, which increases the net present value of its operations. The dynamic feature of the model allows a more realistic description of the trading environment, which in turn affects the incentives for private information acquisition. Obviously, if information is publicly revealed before informed agents have the opportunity to trade, disclosure acts as a substitute for private information acquisition, while not disclosing the information gives investors an opportunity for informed trading.
Incentives to acquire information are important, since we assume that the firm’s manager and costly informed outside investors have overlapping, but not identical, information about the firm’s value. Since part of the information is unique to informed investors, informed trading is the only way this information can flow into prices. The manager can choose ex ante if and when to disclose his signal. Previous research has assumed a single round of informed trading. In such a static setting, informed traders have to bear the substantial amount of risk of holding the asset until the liquidation date. This limits their asset holdings, and thus reduces the advantages of non-disclosure. We prove that in this case informational efficiency is always maximized if the firm discloses all available information, confirming the common intuition based on static models. In a dynamic setting, however, the manager gains the option to disclose at an intermediate date. Because the manager’s signal is correlated with informed agents’ information, this gives informed agents the opportunity to speculate on the disclosure (e.g., an earnings announcement). They can then reverse their positions after the announcement is made, thus avoiding the substantial risks of holding the asset until the liquidation date. This strengthens the benefits of withholding information, while at the same time lowering the “informational cost” (since the information is only temporarily withheld). Thus it can be optimal to the firm to delay disclosure, but never to withhold information forever.

We characterize both situations in which it will be optimal to disclose information immediately and situations in which delaying disclosure until an intermediate point in time is preferable. The informational structure is the most important determinant of optimal disclosure timing. Only if part of the information (such as required market discount rates - some concrete examples are provided later) is unique to investors can disclosure have a positive effect. In fact, a back-of-the-envelope calculation shows that this investor-specific information has to be sufficiently important as compared to information that is known to the manager only. This firm-specific information discourages disclosure at an intermediate point in time, since it acts as a source of risk for informed investors and cannot be incorporated into price unless disclosed.

The most interesting role is played by the informational overlap between manager and investors. Only if it is sufficiently large does delayed disclosure offer sufficient extra trading opportunities to outside investors. However, the marginal
benefit of more overlap are decreasing, and at some point the “informational cost” of delaying the disclosure starts to dominate again. Thus, it is for intermediate values of informational overlap that delayed disclosure is the most beneficial. In addition, the more risky it is to hold the asset until liquidation, or the higher the risk aversion, the more beneficial it is to delay disclosure. Finally, the more costly it is to acquire information, or the less noise there is in the market, the more beneficial it is to delay disclosure.

Proponents of immediate disclosure believe that firms should reveal their information as early as possible. This, they claim, leads to the immediate incorporation of the information into the share price, which in turn leads to more efficient production and investment decisions (Kunkel 1982). In addition, public disclosure “levels the playing field”, eliminating information asymmetries among investors (Diamond 1985). This suggests that immediate disclosure should preempt private information acquisition, and security analysis and informed trading around earnings announcements should not take place.

Alternatively, people approach disclosure from studying its effects on capital markets, often employing an event study methodology. Empirical research following the classic paper by Ball and Brown (1968) views disclosures as exogenous events and finds some results that are at odds with the first perspective. In reality, earnings announcements are accompanied by extensive security analysis and speculative trading, as evidenced by increased bid-ask spreads and trading volume.

This raises a number of questions and concerns. If disclosure leads to increased information acquisition, then disclosure could, for example, lead to lower liquidity and a higher cost of capital, thus overturning the result of Diamond and Verrecchia (1991). In this situation, it also becomes unclear what the economic purpose of disclosure is. We suggest that it may be beneficial for the firm to attract informed investors, since their speculative trades may provide information that is otherwise unobtainable. In particular, more informativeness and less information asymmetry need not go hand in hand. Event studies around earnings announcements, on the other hand, can be misspecified if they view disclosure as an exogenous event. We show how introducing efficiency considerations can and will change the market behavior around announcement dates. Our results suggest that empirical predictions

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1 This tradeoff has been generalized, among others, in Bushman (1991), Indjejikian (1991), Lundholm (1991), and Alles and Lundholm (1993).

2 This has been modeled, among others, by Kim and Verrecchia (1991) as well as Demski and Feltham (1994)
should only be derived from models of endogenous disclosure, and careful controls are necessary in empirical work.

We provide some applications and empirical predictions of our model. Our model implies that firms with more firm-specific information will time their disclosures earlier. Since asymmetric information is more pronounced for young firms or growth firms (high market-to-book ratio), or if analysts’ forecasts have low predictive power, and we predict that these firms will have earlier earnings announcements. On the other hand, investor-specific information will be more important in young industries or growth industries, or when the market if experiencing high volatility or regime shifts. In these cases, we expect firms to time their disclosures later.

Initial Public Offerings are good examples of our information structure. Here, we clearly have a situation in which the manager has some unique information about the details of his firm, while large institutional investors with their expertise in valuation are required to find the market clearing price. Companies that go public are frequently cited as examples for insiders not disclosing their asymmetric information in order to expropriate uninformed market participants. In contrast, our model suggests that these firms may not want to disclose their information right away as to attract informed investors who will help in valuing the new issue. More generally, young or small firms who often find it hard to attract analyst coverage may be able to do so by delaying disclosure of their information.

Furthermore, some companies pre-announce information before an actual earnings disclosure. Baginski, Hassell, and Waymire (1994) report that this is typically the case for the company that has important news which shareholders are unaware of. Consistent with this observation, our model predicts that firms that have an unusual amount of firm-specific information will want to disclose this information earlier.\(^3\)

This paper is organized as follows. Section 2 sets up the model. In section 3 we show that disclosure is always optimal in the static setting. The steps to solving the dynamic case are outlined in section 4, while results on disclosure timing are presented in section 5. Finally, section 6 explores applications and empirical predictions, before section 7 concludes.

\(^3\)Here, economic benefits may include the avoidance of legal action. Admittedly, preannouncements are related to the news realization, the study mentioned above finds that two thirds of preannouncements contained negative news.
2 The Model

2.1 Agents and Timing

Consider an economy with four dates $t = 0, 1, 2, \text{and } 3$. These dates represent the different stages in the life cycle of the single firm in the economy. The company is set up as an equity-financed firm at time 0. Time 1 represents the early stage in the life cycle of the firm. Its shares are publicly traded, and the firm can expand its operations by investing in a growth opportunity. This growth opportunity acts as a source of benefits from price informativeness. Specifically, we assume that the firm does not have the funds to invest itself, and therefore has to sell the growth opportunity to outside investors. A more informative stock price leads to a higher value of the growth opportunity, since it will enable outside investors to allocate capital more efficiently. Time 2 represents the intermediate stage in the firm’s life-cycle. Again, shares are publicly traded, and the firm has access to another growth opportunity. Finally, time 3 represents the end of the firm’s life. Operation is disbanded, assets are liquidated, and final payoffs are realized to the shareholders.

Agents in the model consist of one of the following types.

- An infinite number of risk-averse potentially informed investors who can obtain information about the asset’s liquidation value at a cost. Their utility is given by:

$$u(w) = -\exp(-R \cdot w).$$

(1)

- An unmodeled group of noise traders who trade for liquidity purposes. For simplicity, their demands are unmodeled and assumed to be an inelastic and random amount $z \sim N(0, \sigma_z^2)$.

- The firm’s risk neutral manager. From a modeling perspective, his purpose is to maximize the ex-ante value of the firm by maximizing informational efficiency of the stock price. This is done by committing to an optimal dynamic disclosure policy, which determines if and when information is publicly revealed. Commitment devices are the manager’s reputation in the repeated interaction of quarterly earnings announcements, or a choice of accounting principles. For example, Aboody (1996) shows how footnote disclosures have
a much smaller effectiveness than explicit recognition in financial statements. A lower effectiveness is all we need in our model.

- A competitive risk neutral uninformed investor ("market maker") who sets asset prices as expected liquidation value conditional on his information set, which consists of information disclosures and observing net order flow.

2.2 Informational Structure

In our model, the firm’s manager is endowed with information about his firm’s value, which he needs for running the firm. In addition, information can be obtained by investors through costly research. Information can be incorporated into the price in two different ways. The manager can choose to disclose his information, while trader’s information is partly revealed through their trades. We assume that these two types of information are correlated, but not identical. The manager, by his intimate knowledge of the firm and better access to the books, is likely to have information that outsiders cannot obtain. Investors, on the other hand, may be more familiar with the stock market and, therefore, may have a superior understanding of the pricing implications of accounting information.\footnote{e.g. better knowledge of market discount rates. See Dow and Gorton (1997) for a model in which managers and shareholders have different information}

Let us write the asset’s payoff as

\[ F = \eta + \theta + \delta + \epsilon, \]

and assume that the manager’s signal is given as \( s_m = \eta + \theta \), while the informed investors’ signal is \( s_i = \theta + \delta \). Thus, \( \theta \) represents the overlap between the manager’s and the investors’ information. \( \eta \) is known to the managers exclusively, and we refer to it as firm-specific information. Finally, \( \delta \) denotes information that is exclusively known to investors, and we refer to it as investor-specific information. In addition, the asset payoff is subject to an unpredictable shock \( \epsilon \). We make the usual microstructure assumptions that all random variables are jointly independent and normally distributed with mean zero. The variance of a random variable \( r \) is denoted by \( \sigma_r^2 \) (e.g., \( \eta \) has variance \( \sigma_\eta^2 \)).

The presence of investor-specific information is central to our model. We argued above that informed market participants, such as institutional investors with their expertise, have important valuation information to contribute. Since their
survival directly depends on their skills in security analysis, they are likely to perceive the pricing implications of accounting numbers much better than the firm’s management. Dye and Sridhar (2002) study how a firm may publicly disclose a business strategy in order to learn from the capital market’s reaction. As an example, United Airlines abandoned a plan to consolidate airline, hotel and rental car business into a planned “Allegis” conglomerate after a negative stock market reaction to the announcement. Alternatively, consider Roll’s (1984) result on how price spikes in orange juice futures help predict a freeze in Florida’s orange farms. Thus, price informativeness can be directly beneficial to orange growers, who may take better precautions against a freeze if they can predict it better.

In our model, the manager is prohibited from insider trading in his firm’s shares, but can choose to disclose information. We assume that disclosure is costless, credible, and truthful. Before signals are realized, the manager can commit to a dynamic disclosure policy. This means that he can decide if he wants to disclose his signal, respectively, at time 1, at time 2, or not at all (which is equivalent to disclosure at time 3).  

Besides disclosure, information can flow into prices through informed trading. Informed agents can do costly research to learn about the asset’s payoff. Investors can learn the realization of the signal \( s_i = \theta + \delta \) at a cost \( c \).  

Let \( M \) denote the mass of informed agents. The main focus of the paper is on the case where \( M \) is determined endogenously within the model. As mentioned above, we assume that informed agents have CARA utility with risk aversion parameter \( R \), which limits the size of their arbitrage positions.

Prices in our model are set by a risk-neutral, competitive market maker. This market maker sets price as the expectation of future payoffs conditional on his own information, which consists of publicly disclosed signals and observed net total order flow. However, the market maker cannot perfectly infer the asset value, since he cannot distinguish which part of the order flow is due to informed agents and which part is due to random liquidity shocks \( z_t \sim N(0, \sigma_z^2) \) that arrive at each period \( t = 1, 2 \). This implies that there are abundant amounts of liquidity that market makers are willing to provide freely. Thus, to the econometrician, prices

\[ \text{Previous drafts of this paper allowed the manager to choose to reveal a certain fraction of his information at each point in time. However, we restrict ourselves to the more simple case mentioned above.} \]

\[ \text{Earlier drafts also allowed the time of information arrival to depend on the amount of resources spent. Specifically, assumed that it costs an amount } c_E \text{ to obtain the information early (at date 1), while it costs } c_I \leq c_E \text{ to obtain information at an intermediate time (at date 2). The tension between early and intermediate informed trades is an important aspect of the model. However, we abstract from this issue in our current draft for the sake of simplicity.} \]
appear as a martingale, and we can abstract from issues of discounting in order to focus on the informational role of prices.

2.3 Growth Opportunities

We view price informativeness as a desirable good. For this purpose, we explicitly model economic benefits from informational efficiency. In our setup, a more informative stock price leads to more efficient investment decisions. We assume that at both periods 1 and 2, the company has access to a growth opportunity. We assume that the firm can change its scale of operation by \( \kappa \) units of capital which will increase the liquidation value by \( \kappa F = \kappa (\eta + \theta + \delta + \epsilon) \). Adding \( \kappa \) units of capital cost a dollar amount of \( \frac{1}{2}\kappa^2 \). Thus, the NPV of adding \( \kappa \) units of capital is given by

\[
\kappa (\eta + \theta + \delta + \epsilon) - \frac{1}{2}\kappa^2
\]

Consider the first order condition for the optimal investment decision. Differentiating the above equation yields

\[
\kappa^* = E [F|\text{available information}]
\]

We want to capture the intuition that information conveyed by the stock price improves the access to outside capital. For this purpose we want the investment decision to be made conditional on market information only. To convey this intuition, we assume that the firm does not possess the required resources to finance an expansion. Instead, we assume it has to spin off the growth opportunity and sell it to the market maker, who then proceeds to decide how much capital to devote to the project. Competitive market making ensures that the company receives the “fair” value of the growth opportunity based on the market evaluations of the project. To preserve the linear structure of the model, we assume that the growth opportunities are sold ex-ante, before any trade takes place, but after the manager has credibly committed himself to a disclosure policy.

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7The current formulation allows the amount of capital invested to be negative. This is an undesirable but typical problem of the normal-exponential framework we use in our model. However, this problem could easily be mitigated by using truncated normal distributions and/or positive mean payoffs instead.

8We could also consider the case of the firm’s manager using both market information and his private signal to make a decision, but have not done so as we want to focus on the information content of the market price.

9Since the mapping from one situation to the other is deterministic, so this assumption does not change the intuition behind the model.
These assumptions assure that the optimal investment decision is given by

$$\kappa^* = E[F|P_t] = P_t$$

Taking the ex-ante expectation yields a value of the growth opportunity of

$$GO_t = E\left[F \cdot P_t - \frac{1}{2} P_t^2\right] = \frac{1}{2} \left(\text{Cov}(F, P_t) + \text{Cov}(F - P_t, P_t)\right)$$

However, by construction, prices are martingales, and thus \(\text{Cov}(F - P_t, P_t) = 0\).

We again use the martingale property of prices to find alternative formulations of the NPV of the growth opportunity below.

**Proposition 1.** Under these assumptions, the net present value of the growth opportunity in period \(t\) equals to one half times the amount of ex-ante uncertainty resolved by price (variance of \(F\) explained by \(P_t\)):

$$GO_t = \frac{1}{2} \text{Cov}(F, P_t) = \frac{1}{2} \left[\text{Var}(F) - \text{Var}(F|P_t)\right]$$

Thus, it follows that maximizing the present value of growth opportunities is equivalent to minimizing payoff uncertainty remaining after trade, measured by \(\text{Var}(F|P_t)\).

The growth opportunities in our model act as a source of economic gains from informational efficiency, since informational asymmetry between the manager and the market limits the efficient use of the growth option. Other examples of economic benefits from informational efficiency in the literature are a mitigation of the principal-agent problem (Holmström and Tirole, 1993) and better communication of investment decisions to the shareholders (Fishman and Hagerty, 1989). Our model is very similar to Subrahmanyam and Titman (1999), albeit with a twist. In their model, informational efficiency helps the manager to make better investment decisions, while in our model benefits come from improved access to additional capital.

From a welfare perspective, we should compare the value of growth opportunities to the cost of information production. The losses of uninformed traders can be easily calculated in our model. To get a welfare statement, we would then have to compare the cost of information acquisition (which equals the losses of the uninformed) to the productive gains from investing in the growth opportunities. However, the relative size of these two effects is arbitrary, so that a simple scaling of the value of the growth opportunity can overwhelm the cost of information production. Since it is well-known from Diamond (1985) how disclosure can reduce
costly private information production, we abstract from the issue here to focus on the novel aspects of the dynamic model. First, however, we solve the static benchmark case so we can draw meaningful comparisons.

3 The Static Benchmark Case

The main goal of the paper is to study the effects of disclosure in a dynamic financial market. For this purpose, we first need to examine the predictions of the static benchmark case. We confirm existing notions that in a static setup, disclosure always maximizes informational efficiency. Thus, we can prove that our new insights are completely driven by the dynamic feature of our model. Let us assume there is only one period of trade. For notational convenience, let us denote this period as time $t = 1$. At time $t = 2$ the asset is liquidated and pays an amount of

$$F = \eta + \theta + \delta + \epsilon$$

and we assume the asset price to be a linear function of the state variables:

$$P = a_i \cdot s_i + a_m \cdot s_m + b \cdot z_1$$

In a one-period model, the demand functions for the informed traders and their expected utility simplify significantly. This makes it possible to analytically solve the model with endogenous information acquisition, and prove the following statement.

Proposition 2. In a static setup, there exists a unique linear equilibrium for both the case of managerial disclosure and the case of the manager withholding his information. This statement is true for both exogenous and endogenous information acquisition. In all of the above situations, the equilibrium can be solved in closed form. The solutions are presented in the appendix.

Furthermore, informational efficiency is independent of the amount of liquidity trading, $\sigma_z$. This allows a meaningful comparison between the static and the dynamic model independent of specific assumptions about noise trading.

Proposition 3. In a static model with endogenous information acquisition, informational efficiency is always maximized when the manager discloses his signal $s_m$. Potentially, there are three different cases to consider:
1. The number of informed agents is greater than zero both with and without disclosure. In this case, disclosure reduces the payoff variance remaining after trade by

\[ \frac{e^{2Rc}\sigma_\eta^4(\sigma_\delta^2 + \sigma_\theta^2)}{\sigma_\eta^2\sigma_\delta^2 + \sigma_\theta^2(\sigma_\eta^2 + \sigma_\theta^2)} \]  

(10)

In particular, informational efficiency remains the same if the manager does not have firm-specific information, i.e. whenever \( \sigma_\eta^2 = 0 \).

2. The number of informed agents is zero in the case of disclosure, and greater than zero otherwise. In this case, disclosure reduces the conditional payoff variance by at least

\[ \frac{(\sigma_\delta^2 + \sigma_\epsilon^2)\sigma_\eta^4(\sigma_\delta^2 + \sigma_\theta^2)}{\sigma_\epsilon^2\sigma_\eta^2\sigma_\delta^2 + \sigma_\delta^2(\sigma_\eta^2 + \sigma_\theta^2)} \]  

(11)

3. The number of informed agents is zero without disclosure. In this case, disclosure reduces the conditional payoff variance by at least \( \sigma_\eta^2 + \sigma_\theta^2 \).

We can see from the above results how disclosure is always beneficial in a static setting. It is well known from Grossman and Stiglitz (1980) that voluntary disclosure does not change information efficiency if the manager and outside investors have identical signals. This means that the increase in informed trade due to withholding of information exactly offsets the informational cost of non-disclosure. This result holds as a special case of our model.

In the presence of firm-specific information (\( \sigma_\eta > 0 \)), however, we find that disclosure is strictly beneficial. There are two reasons for this effect. Firstly, there is no way this information can flow into prices besides disclosure. Secondly, it acts as a source of additional risk to the informed agents. In addition to maximizing informational efficiency and thus firm value, disclosure in general also reduces the cost of private information production, as in Diamond (1985). We abstract from this issue here, since they are well understood in previous literature. Note that while the benefits of disclosure seem intuitive, they are not entirely obvious. For example, disclosure can reduce the amount of information asymmetry just enough to “crowd out” informed agents completely (i.e. \( M = 0 \)). For this to be the case, however, information has to be sufficiently costly. And that in turn implies that the benefits of disclosure are quite high.
In the proposition we point out how our results allow a meaningful comparison between the static and the dynamic model. The reason is that going to the static case requires an extra assumption on the treatment of noise. Do we just assume one trading round instead of two, thus liquidity demand \( z \sim N(0, \sigma^2_z) \)? Or do we lump the two trading rounds of the dynamic model together, which would yield a liquidity demand of \( z \sim N(0, 2\sigma^2_z) \)? Fortunately, our results do not depend on any specific assumptions.

4 Equilibrium in the Dynamic Model

4.1 Equilibrium

An equilibrium with endogenous information acquisition is defined by the following three conditions

1. For each competitive informed trader, his demand function at each date maximizes his expected utility conditional on information available to him at that date.

2. Competitive market making insures that prices at each date are given as expected asset payoffs conditional on all information available to market makers at that date.

3. The certainty equivalent of informed trader’s payoffs equals the cost of information. (For lack of interest, we abstract from corner equilibria in which there are no informed agents.)

As is common in informational models of this type, we restrict ourselves to equilibria in which prices are given as a linear function of the underlying random variables. In particular, we conjecture pricing functions of the type

\[
P_1 = a_i \cdot s_i + a_m \cdot s_m + b \cdot z_1
\]

\[
P_2 = d_i \cdot s_i + d_m \cdot s_m + e \cdot z_1 + f \cdot z_2
\]

This allows us to apply the powerful theory of regressions to our model. In the following part, we hold the number of informed agents, \( M \), fixed as we derive expressions for their optimal dynamic trading strategy. For this purpose, let \( \Psi_t \) denote the informed agents information set at time \( t \).
4.2 Informed Agents Demand

The driving force behind our results is the trading strategy of informed agents. Introducing a dynamic feature into the model results in the presence of hedge demands at time 1. As a consequence, the intuition underlying static models can be overturned. At date \( t = 2 \), agents demands are simply found by mean-variance analysis. Informed agents demand

\[
x_2(P_2) = \frac{E(F|\Psi_2) - P_2}{R \cdot \text{Var}(F|\Psi_2)}
\]

(14)

This implies an indirect utility of wealth at time 2 of

\[
u_2(W) = E[- \exp(-RW)|\Psi_2]
\]

\[= E \left[ - \exp \left( -R \left( x_1(P_2 - P_1) + \frac{(E(F|\Psi_2) - P_2)(F - P_2)}{R \cdot \text{Var}(F|\Psi_2)} \right) \right) \right] \]

\[= - \exp \left( -R \left( x_1(P_2 - P_1) + \frac{(E(F|\Psi_2) - P_2)^2}{2R \cdot \text{Var}(F|\Psi_2)} \right) \right) \]

(15)

Thus, at time 1, the informed agent maximizes the expectation of period-2 indirect utility. The appendix shows that the solution is of the form

\[
x_1(P) = \frac{E(P_2|\Psi_1) - P_1}{RS} + \frac{E(F - P_2|\Psi_1)}{R \text{Var}(F|\Psi_2)} \cdot \frac{S - T}{S}
\]

(16)

Thus, first period demand is more complex than second period demand. It consists of two terms. The first expression is to take advantage of the expected price appreciation between periods 1 and 2, which makes up the numerator of the first term. The denominator denotes the risk premium, consisting of the risk aversion parameter \( R \), and a risk measure \( S \) that can, in general, be fairly complex. The second term denotes the hedge demand to take advantage of anticipated price changes between periods 2 and 3. The first fraction equals the expected share holdings from period 2 to 3. However, these share holdings cannot be perfectly hedged. In fact, the expression \( \frac{S - T}{S} \in [0, 1] \) denotes the extent to which these share holdings can be anticipated in advance.

In the general case, \( S \) can be found as

\[
S = \frac{- (\text{Cov}(P_2, E(F|\Psi_2)|\Psi_1))^2 + \text{Var}(P_2|\Psi_1) (\text{Var}(F|\Psi_2) + \text{Var}(E(F|\Psi_2)|\Psi_1))}{-2 (\text{Cov}(P_2, E(F|\Psi_2)|\Psi_1)) + \text{Var}(F|\Psi_2) + \text{Var}(P_2|\Psi_1) + \text{Var}(E(F|\Psi_2)|\Psi_1)}
\]

(17)

and

\[
T = \frac{\text{Cov}(P_2, E(F|\Psi_2)|\Psi_1)) (\text{Var}(F|\Psi_2) - (\text{Cov}(P_2, E(F|\Psi_2)|\Psi_1)) + \text{Var}(P_2|\Psi_1) \text{Var}(E(F|\Psi_2)|\Psi_1))}{-2 (\text{Cov}(P_2, E(F|\Psi_2)|\Psi_1)) + \text{Var}(F|\Psi_2) + \text{Var}(P_2|\Psi_1) + \text{Var}(E(F|\Psi_2)|\Psi_1)}
\]

(18)
The various conditional moments can be easily calculated given the conjecture of a linear equilibrium. Details are found in the appendix.

Fortunately, the math simplifies significantly if either one of the following two cases hold. First, if the manager discloses his signal before trading in period 1, or second, if he discloses his information between dates 1 and 2, but does not have any information beyond that of informed agents \( \sigma^2 = 0 \). In this case \( T = 0 \), period 2 demand can be perfectly hedged, and the first period risk measure \( S \) simplifies to \( S = \left( \frac{1}{\text{Var}(F|\Psi_2)} + \frac{1}{\text{Var}(P_2|\Psi_1)} \right)^{-1} \). Again, we refer the reader to the appendix for more details.

### 4.3 Market Prices

Prices in our model are set by a competitive risk-neutral market maker. This condition ensures that prices at each point in time are expectations conditional on available information, and that the time series of prices follows are martingale. In period 1, the market maker observes net order flow of

\[
\tau_1 = M x_1(P_1) + z_1 = M \left[ \frac{E(P_2|\Psi_1) - P_1}{RS} + \frac{E(F - P_2|\Psi_1)}{R \text{Var}(F|\Psi_2)} \cdot \frac{S - T}{S} \right] + z_1
\] (19)

Thus, period 1 price is found by regressing payoff \( F = \eta + \theta + \delta + \epsilon \) on \( \tau_1 \). In the case of managerial disclosure at time 1, the price is found by regressing \( F \) on the vector \( (\tau_1, s_m) \). These regressions can be simplified by first removing all terms involving \( P_1 \) from \( \tau_1 \) (which yields an observationally equivalent signal). In either case, we obtain analytical expressions for period 1 price as a function of period 2 price coefficients.

In the second period, the market makers observe another signal in the form of second period net order flow, which is affected by the arrival of another liquidity (supply) shock, \( z_2 \).

\[
\tau_2 = M \frac{E(F|\Psi_2) - P_2}{R \cdot \text{Var}(F|\Psi_2)} + z_1 + z_2
\] (20)

Again, the market maker determines the price \( P_2 \) by regressing payoff \( F \) on the vector of signals that is available to him, i.e. \( (\tau_1, \tau_2) \), but also the manager’s signal \( s_m \), if this has already been disclosed. This leads to a system of four equations in the four unknowns, \( d_i, d_m, e \) and \( f \). In general, this system is highly non-linear and cannot be solved analytically. However, there are exceptions, for example when
the manager discloses his information before trade in period 1, thus simplifying the expression for informed trader’s hedge demand. Holding the number of informed agents, $M$, fixed, the solution to the equilibrium equations is provided in the appendix.

4.4 Multiplicity of Equilibria

The non-linear nature of the equations for period 2 price implies the potential for multiple equilibria, an issue that is well known since the early work of Brown and Jennings (1989) or Hirshleifer, Subrahmanyam, and Titman (1994). Equilibria fall into one of two different categories. Typically, there always exist equilibria in which prices do not move between time 1 and time 2. We feel justified in ignoring these equilibria, since they essentially lead to results that could also be (and have been) obtained in static models. Since the focus of our paper is the dynamic interaction between trading and information disclosure, we restrict ourselves to equilibria in which $P_1 \neq P_2$. Typically, the number of such equilibria depends on the sign of discriminant of a quadratic equation. Thus, for some combination of parameter values it can happen that no such equilibrium exists. On the other hand, if such an equilibrium exists at all, there also exists a second one. This result again is representative of the prior literature cited above. Similarly to the asset pricing model of Spiegel (1998), one of the two equilibria typically displays counterintuitive or surprising behavior. Thus, we feel justified in ruling this equilibrium out, and focusing on the one remaining equilibrium. We have been able to obtain a few result on the counterintuitive behavior of one of the equilibria, but do not include these results in this paper.\footnote{Please contact the author on these results, if interested.}

4.5 Utility of Informed Agents

Due to the linear nature of the model, both demands and asset prices are linear in the state variables. This implies that wealth is a quadratic function of the normally distributed random variables in our model. We use standard results of “completing squares” for quadratics of normal random variables to compute the expected utility of informed agents. For details of these tedious calculations, see the appendix. Expected utility is basically determined by the determinant of a
6-by-6 matrix, which is far beyond hope for any kind of analytical tractability. Thus, we have to rely on numerical simulations for our results.

5 Optimal Disclosure Timing

In this section, we explore the factors that determine the optimal timing of managerial disclosure. We characterize situations in which it will be optimal to disclose information immediately and situations in which delaying disclosure until an intermediate point in time is preferable. Numerical simulations show that it will never be optimal to withhold information until the liquidation date. The positive effect of attracting privately informed agents by withholding information is always stronger when we delay disclosure only until an intermediate point in time, as we explain below.

5.1 Informed Trading Behavior and the Suboptimality of Late Disclosure

To understand the key findings of our paper, it is important to understand the trading behavior of informed agents. As in Hirshleifer, Subrahmanyam, and Titman (1994), informed agents typically engage in short term profit taking. Numerical simulations show that the price move between dates 1 and 2 is correlated positively with their date-1 asset holdings and negatively with their trades at date 2: \( \text{Cov}(x_1, P_2 - P_1) > 0, \text{Cov}(x_2 - x_1, P_2 - P_1) < 0. \) The reason for this is that information is incorporated into prices gradually. The market maker has more information when setting period 2 stock price, since time 2 order flow provides him another signal. On average, this moves stock price closer to the true fundamental value of the asset. This diminishes the returns for holding the asset another period, which in turn leads informed agents to partially reverse their risky positions. These effects hold no matter when the firm discloses information. If, however, the manager chooses to reveal his signal at the intermediate date \( t = 2 \), the news arrival at time 2 results in a large price move \( |P_2 - P_1| \text{ large} \), which leads to much more pronounced short term reversal trading by informed agents.

Figure 1 shows the behavior of our model under the assumption of intermediate (time 2) disclosure. We proceed to compare these results with the behavior of the
model should the manager late (time 3), which is shown in figure 2. The top left plots display the amount of payoff uncertainty remaining after trade, measured by \( \text{Var}(F|P_t) \). In our model, this is equal to the dollar loss the firm is suffering due to the asymmetric information problem. We can clearly see how it is never optimal to delay disclosure until time 3. It comes as no surprise that date 2 stock price, \( P_2 \), is more informative with intermediate disclosure, because this means that firm-specific information can be incorporated into price. Perhaps more surprisingly, we find that even in the first period, stock price \( P_1 \) is more informative with intermediate disclosure than with late disclosure. Thus it is not true that by withholding his signal longer, the manager can achieve a more informative signal from informed traders.

The plots in the top right corner indicate how the expected price move between periods 1 and 2 is small with late disclosure, and much more pronounced with intermediate disclosure. In the latter case, it is still significantly smaller than the amount of information contained in the corporate disclosure, which is again evidence that a large part of this information has already been incorporated into prices before the announcement. This relates to the number of shares held by informed agents, which is plotted in the bottom left corner. Figure 1 shows how, in the case of intermediate disclosure, informed agents hold large positions at time 1, which they largely reverse at time 2. Note that as the amount of investor-specific information, \( \sigma_\delta \), increases, their period 1 shareholdings, \( x_1 \), actually decrease. Normally, we would expect informed agents to hold more shares as their informational advantage increases. With intermediate disclosure, however, more investor-specific information makes it harder to forecast the corporate news announcement, and in turn makes short term trading less attractive. In contrast, period 2 shareholdings, \( x_2 \), increase in \( \sigma_\delta \) as expected. Thus, we find that with intermediate disclosure, informed agents engage in substantial short term trading around the earnings announcement.

This is further substantiated by the plots on the bottom right corner. With intermediate disclosure, the correlation between the initial informed shareholdings, \( x_1 \), and the intermediate price move, \( P_2 - P_1 \), is very close to +1, while the correlation between period 2 trades, \( x_2 - x_1 \), and the price move \( P_2 - P_1 \), is almost −1. Thus, informed agents buy shares in the first period in clear anticipation of unwinding their position as soon as the announcement is made.
With late disclosure, however, we find little evidence of interesting dynamic behavior (figure 2). Stock positions at dates 1 and 2 are similar, albeit mildly increasing in the amount of investor specific information. Period 1 informed shareholdings are hardly correlated with the intermediate price move. And while informed position reversal is significantly negatively correlated with this price move, the magnitudes of both are small. This explains why price informativeness hardly increases between dates 1 and 2, as seen on the top left plot of figure 2.

5.2 Early vs. Intermediate Disclosure

For the remainder of the paper, we will focus on the distinction between early \((t = 1)\) and intermediate \((t = 2)\) disclosure. The most important and interesting determinant of disclosure timing is the informational setup. Note that if the manager and investors have identical information \((\sigma_\eta = \sigma_\delta = 0)\), full informational efficiency can be achieved by immediate disclosure. This still holds if the manager has some unique information, i.e. \(\sigma_\eta > 0\). The only reason to delay disclosure is to encourage informational trade. Only when \(\sigma_\delta > 0\), part of the information is unique to informed investors, and private information collection can be beneficial for the company. It seems intuitive that the amount of investor-specific information has to be sufficiently large for delayed disclosure to be optimal. In the extreme case, prices do not reflect \(\delta\) at all under immediate managerial disclosure, and perfectly under delayed disclosure. Thus, the maximum gain in informational efficiency from delaying disclosure is \(\sigma_\delta^2\) in both periods 1 and 2, for a total of \(2\sigma_\delta^2\). On the other hand, if the manager withholds his information until the intermediate date, there is no way how the firm specific information, \(\eta\), can flow into prices in the first round of trading. As a consequence, the minimal loss of delaying disclosure is given by \(\sigma_\eta^2\). Thus, we have shown the following.

**Proposition 4.** Immediate disclosure is always optimal when the amount of firm-specific information is sufficiently large compared to the amount of investor-specific information. Formally, immediate disclosure is optimal whenever

\[
\sigma_\eta^2 \geq 2 \sigma_\delta^2
\]  

(21)

The nature of information is important in determining the optimal timing of its release. \(\theta\) measures the correlation between the manager's and the investors'
signals. Thus the higher $\sigma_\theta$, the more does delayed disclosure offer profitable low-risk trading opportunities for the informed agents. They will take on large arbitrage positions which they reverse after the news announcement, since holding the asset until liquidation is highly risky. The more unique information investors can contribute, i.e. the higher $\sigma_\delta$, the higher are the possible advantages of delaying disclosure. This can be seen in figure 3. On the other hand, delaying disclosure also has an informational cost, since $\theta$ has to be noisily inferred from net order flow instead of observed as part of the manager’s signal.

Interestingly, the positive and negative effects have different marginal strengths. While the potential negative effects of delaying disclosure are somewhat linear in the amount of information, $\sigma_\theta$, the marginal positive effects are the strongest for small amounts.\(^{11}\) When informational overlap between the manager and investors is small, delaying disclosure may not provide enough extra incentives to acquire information. When the informational overlap is large, however, the negative effects of delaying disclosure dominate the increase in informational trade due to disclosure. Thus, it is for intermediate values of informational overlap that delaying disclosure is the most beneficial. Figure 4 illustrates this effect.

Firm-specific information, $\eta$, on the other hand, cannot be obtained through private research. Thus, delaying the disclosure of $\eta$ has a detrimental effect. Firstly, the loss in informational efficiency due to delayed disclosure is significant, since there is no other way this information can flow into prices. Secondly, there is no benefits for informed agents from the opportunity to trade on this information. Instead, the announcement of this information at the intermediate trading round acts as risk for the informed agents, therefore limiting their arbitrage positions. To summarize, let $\rho = \sigma_\eta/\sigma_\theta$ denote the specialness of the manager’s information. Then the more special the manager’s information is, the less likely he is to disclose his information. This is shown in figure 5.

In figure 5, we also break up the benefits from delaying disclosure according to period. We have seen before how delaying disclosure can stimulate private information acquisition, at the expense of not incorporating the manager’s information into prices immediately. Thus it comes as no surprise that in period 2, after the delayed announcement, stock price is always more informative than with immedi-

\(^{11}\)In previous drafts we identified cases where it will be optimal to delay part of the information to attract outside analysts, while we want to disclose the remainder of the information immediately to prevent the negative effects. Presently, we do not allow for this strategy, but instead find interesting effects in optimal disclosure timing.
ate disclosure. Surprisingly, this effect is even more pronounced if there is a lot of firm-specific information. On the other hand, it is not true that delaying disclosure always leads to a less informative price in the first period. Instead, the effect of delaying disclosure on period 1 informativeness critically depends on the amount of firm-specific information, $\sigma_\eta$. If it is low, it can be slightly positive, while it becomes highly negative as $\sigma_\eta$ increases. Optimal disclosure timing then depends on the comparison of loss in the first period to gains in the second. For simplicity, we have assumed that the growth opportunities of the firm are of equal size in both periods. If the firm has some discretion in the timing of its growth opportunities, it will generally decide to invest more in the second period, which makes a delayed disclosure even more desirable.

Finally, $\sigma_\epsilon$ measures the unpredictable part of asset payoff. This acts as a source of risk when holding the asset until liquidation. Consequently, the higher $\sigma_\epsilon$, the more beneficial it becomes for agents to have short term trading opportunities provided by intermediate disclosure. This simple intuition is verified in our numerical simulations, see for example the plot in figure 6.

Increasing the risk aversion parameter, $R$, has essentially the same effect as increasing the amount of risk, $\sigma_\epsilon$. It increases the advantages of delayed disclosure. When information is disclosed early, informed agents have fewer short term trading opportunities, and are therefore more affected by the risk of holding the asset until liquidation. When information is disclosed at an intermediate point in time, the opportunity to reverse their position after the news announcement enables the informed to speculate on the nature of the announcement without bearing the risk of holding the asset until liquidation. This opportunity becomes relatively more valuable as risk aversion increases.

Noise in our model, captured by the parameter $\sigma_z$, enables the informed agents to take on disguise their trading positions, thus making price discovery more difficult for market makers. Therefore, it comes as no surprise that more noise makes immediate managerial disclosure more valuable. In addition, noise in the second trading round also acts not only as disguise, but also as a source of risk for informed traders, and thus reduces the relative advantage of delayed disclosure.

Finally, informational efficiency is also affected by the cost of information acquisition. The more costly information is, the more difficult it becomes to attract informed private investors without offering them the additional trading opportu-
nities of delayed disclosure. In particular, at higher levels of information costs, agents may cease to collect information unless delayed disclosure provides them with lucrative trading opportunities. In these cases, delayed disclosure can have a very pronounced positive effect.

To conclude, we find that low risk aversion, small payoff uncertainty, a low cost of information, a high level of noise, high levels of firm-specific information as opposed to investor-specific information, and extreme values (high or low) of informational overlap all favor early disclosure, while the opposite is true for intermediate disclosure.

6 Applications and Empirical Predictions

6.1 Regulatory Action

We believe it is important to allow firms discretion as to timing their disclosures. The section above outlines how this may be beneficial to the firm, and improve the efficiency of production. Thus, our model suggests that planned regulatory action may be problematic. Admittedly, there may be factors outside our model that justify the regulation. In our analysis we have disregarded the welfare of uninformed traders. Delayed disclosure may harm these investors, since they on average lose to informed agents. As one of the central premises of the disclosure process is to protect uninformed investors, this protection may be important enough to offset efficiency gains from more informative stock prices (Incorporating the losses of informed investors into our analysis suggests that this can sometimes be the case). Secondly, since the early work of Chambers and Penman (1984) and Givoly and Palmon (1982), it is widely known that disclosure timing is news-dependent. If it can be shown empirically that late disclosures are typically accompanied by negative market reactions, this can be viewed as evidence of harmful consequences of delaying the S.E.C. filings. Finally, given evidence of strategic timing of disclosures with respect to executive stock option plans (Aboody and Kaznik 2001, Yermack 1997), we will have to consider if firms really choose their accounting principles in order to maximize informational efficiency.
6.2 Applications

6.2.1 Earnings Pre-Announcements

Firms with knowledge of significant news often pre-announce this information before the schedules announcement date, as reported by Baginski, Hassell, and Waymire (1994).\(^\text{12}\) We can rationalize this by acknowledging that the information structure changes over time. A firm that typically prefers to disclose at an intermediate date may find it optimal to reveal its information at an early date, if there is an unusually important firm-specific information. Pre-announcements are typically made when the firm has knowledge of a significant news item unknown to shareholders, which leads to an earlier optimal disclosure according to our model.

Before enactment of regulation Fair Disclosure, this information could be conveyed to informed agents by selective communication such as conference calls, which means that it was not necessary to time the public announcement earlier. Since this is no longer possible under regulation FD, it comes as no surprise that the frequency of pre-announcements has increased approximately fivefold.\(^\text{13}\)

6.2.2 Initial Public Offerings

We believe that our correlated information structure is particularly relevant to Initial Public Offerings. Clearly, the owner of the firm has much more detailed knowledge of the firm details and is closely familiar with the books. However, valuing companies takes years of expertise, knowledge of the whole industry and current market conditions. It is doubtful if a firm that is going public really discloses all the information available to the management. One obvious reason might be that the owners want to maximize the offering price by presenting the company in a positive light. Alternatively, our model suggests that firms may delay disclosure of available information to offer profitable opportunities for informed trade. This, in turn, could then attract large institutional investors who will be helpful in pricing the issue.

6.2.3 Firms Attracting Analyst Coverage

Attracting informed investors is important for firms even after an IPO. In particular, firms that are young or small often find it very difficult or expensive to

\(^\text{12}\)Here, economic benefits from incorporating this information into price include the avoidance of legal action.

\(^\text{13}\)Clearly, pre-announcements depend on the news realization and often come as a surprise to the market. However, the economic rationale for pre-announcements seems entirely consistent with our story.
attract analyst coverage. In this case, investor-specific information is important also because it increases the credibility of the stock price and provides valuable monitoring. After recent high profile events, firms have to obey stricter ethics standards for firms who want to attract analyst following. Furthermore, regulation Fair Disclosure makes it impossible for the firm to lower the cost of information acquisition by making selective disclosures to analysts. In contrast, delaying the disclosure of information offers the firm an ethical and legal way of rewarding security analysis with profitable trading opportunities, enabling analysts to cover their expenses.

6.3 Empirical Predictions

Unfortunately, it is hard to relate the parameters of our model to observable firm characteristics. To a large extent, this is a shortcoming of the whole literature. Nevertheless, we still give some cross-sectional predictions for the timing of corporate disclosures. We believe that there will always be an overlap between investors’ signals and the manager’s information (for example, analyst forecasts do provide significant forecast power for realized earnings), so that we can focus on information that is specific either to management or investors.

Our model predicts that firms with more firm-specific information will announce this information earlier. Asymmetric information is typically more prevalent in young firms or growth firms (high market to book ratio), so that we expect these firms to disclose information earlier. An alternative approach of measuring the relative amount of firm-specific information is by means of analyst forecast errors. If analysts are typically able to forecast the firm’s earnings with a good degree of confidence, we conclude that firm-specific information is relatively unimportant. In this case, we believe that the firm should disclose its information early.

On the other hand, we predict that firms will announce information later if investors contribute significant information. Here, we are typically thinking of valuation information, such as discount rates or current market conditions. Other examples would be information about demand for a specific industry or even a specific product. The manager may be more familiar with the technical specifications, but may be uncertain about its market appeal or future profitability. This type of information should be more pronounced in newly developing and high-growth industries, or in otherwise volatile industries. Similarly, during volatile markets
or regime shifts, sophisticated investors can contribute more valuable valuation information. In these cases, we predict the company will announce its information later.

In our model, more risk (higher $\sigma$) of holding arbitrage positions in the asset leads to later disclosure. We thus predict that riskier firms will announce information later. Admittedly, we would need a multi-asset model to fully address these issues, as we will have to distinguish between priced and non-priced risk.

7 Conclusion

We study how introducing a dynamic financial market changes the economics of disclosure. As Verrecchia writes in his 2001 JAE survey on disclosure theory: “Assessing the effects of disclosure in the context of a single period model of trade risks comingling a host of factors that may obfuscate or obscure disclosure’s role.” We believe that we have disentangled some of these factors, and thus cast new light on the role of disclosure. In a world in which price informativeness is desirable because it leads to more efficient investment decision, we show how a firm would always want to disclose its information in a static setting. But reality is dynamic, and the result does not carry over into a dynamic world. It is still the case that some firms will disclose information as soon as it becomes available to them. In some situations, however, prices and investment decisions are more efficient when the firm delays disclosure. In contrast to current S.E.C. regulatory action, we propose to give firms discretion as to the timing of their disclosures.

We revisit the basic trade-offs of Diamond (1985) and offers some important new insights. Specifically, we show how more information asymmetry can actually occur together with more price informativeness. We do not explore other aspects on disclosure studies such as the relation to insider trading (e.g. Fishman and Hagerty 1995), and proprietary information (Admati and Pfleiderer 2000). Boot and Thakor (2001) analyze disclosure regulation from a financial innovation perspective, while Gennotte and Trueman (1996) explore the differential intra-day timing of disclosure depending on the whether the information is positive or negative. Finally, Dow and Gorton (1997) also focus on the differential signals of managers and shareholders, but exogenously assume a unidirectional flow of information at each point in time. While we want to keep our focused, we believe that
our intuition affects all the research mentioned above.

In our paper, we deliberately stress results on the optimal timing of corporate disclosures. Other important implications can be obtained from the same model. These include new insights on the behavior of stock prices and trading volume around earnings announcement dates, and a better understanding of the limitations of applying an event study methodology for analyzing the effect of earnings announcements. These results are left for future work. Promising extensions, such as a multiple asset setting, are likewise left for future research.

References

- Aboody, David, 1996, “Recognition versus Disclosure in the Oil and Gas Industry”, *Journal of Accounting Research* 34, 21-32


8 Appendix - Proofs

Parts of the appendix are still rough and/or missing.

8.1 Updating for Informed Agents

When the manager discloses his private signal, the informed agents can combine their own signal $s_i = \theta + \delta$ with the manager’s signal $s_m = \eta + \theta$. This is done by running a regression of

$$ v_1 = \eta + \theta + \delta \quad (22) $$
onumber

on

$$ v_2 = (s_i, s_m) \quad (23) $$

Thus the new belief, $\beta$, is given as

$$ \beta = \text{Cov}(v_1, v_2) \cdot \text{Var}(v_2) \cdot v_2' \quad (24) $$

i.e.

$$ \beta = \frac{\sigma^2 \sigma^2 + \sigma^2 \sigma^2}{\sigma^2 + \sigma^2} \cdot s_i + \frac{\sigma^2 \sigma^2 + \sigma^2 \sigma^2}{\sigma^2 + \sigma^2} \cdot s_m \quad (25) $$

This yields

$$ \beta = \frac{\sigma^2 \sigma^2 + \sigma^2 \sigma^2}{\sigma^2 + \sigma^2} \cdot s_i + \frac{\sigma^2 \sigma^2 + \sigma^2 \sigma^2}{\sigma^2 + \sigma^2} \cdot s_m \quad (26) $$

The remaining variance of $F$ conditional on observing both $s_i$ and $s_m$ is found to be

$$ V = \text{Var}(F|s_i, s_m) = \text{Var}(F|\beta) = \frac{\text{Var}(F)\text{Var}(\beta) - \text{Cov}(F, \beta)^2}{\text{Var}(\beta)} = \sigma^2 + \frac{\sigma^2 \sigma^2 \sigma^2 \sigma^2}{\sigma^2 + \sigma^2} \quad (27) $$

8.2 Static Model without Managerial Disclosure

This is simply an application of the model of Grossman and Stiglitz, where the informed know $\theta + \delta$, whereas $\eta + \epsilon$ are not known to any market participants. The derivations for the results below follow exactly the steps in their original paper. For brevity, we omit the intermediate steps, and just provide the results here (they are simpler than the steps for the model with managerial disclosure presented below).

1. Holding the number of informed agents, $M$, fixed, the equilibrium is given by

$$ a_i = \frac{M^2(\sigma^2 + \sigma^2)}{M^2(\sigma^2 + \sigma^2) + R^2(\sigma^2 + \sigma^2)^2\sigma^2} \quad (28) $$

and

$$ b = \frac{R(\sigma^2 + \sigma^2)^2}{M} \cdot a = \frac{MR(\sigma^2 + \sigma^2)^2(\sigma^2 + \sigma^2)}{M^2(\sigma^2 + \sigma^2) + R^2(\sigma^2 + \sigma^2)^2\sigma^2} \quad (29) $$

The utility of informed agents is given by

$$ -\left(\frac{M^2(\sigma^2 + \sigma^2) + R^2(\sigma^2 + \sigma^2)^2(\sigma^2 + \sigma^2 + \sigma^2 + \sigma^2)}{M^2(\sigma^2 + \sigma^2) + R^2(\sigma^2 + \sigma^2)^2\sigma^2}\right)^{-1/2} \quad (30) $$

2. Now assume that the number of informed agents in not exogenously given, but endogenously determined in the model. Specifically, assume that investors can choose to collect information at a cost $c$. Agents will not acquire private information $M = 0$ whenever

$$ \log \left[\frac{\sqrt{\sigma^2 + \sigma^2 + \sigma^2}}{\sqrt{\sigma^2 + \sigma^2}}\right] = \log \left[\frac{\sigma^2}{R}\right] \quad (31) $$
Otherwise, some agents will choose to collect information, and their number will be given by

\[ M = \frac{R\sigma_\eta \sqrt{\sigma_\eta^2 + \sigma_m^2}}{\sqrt{e^{-2Rc} - 1} \sqrt{\sigma_\theta^2 + \sigma_\eta^2}} \]  \hspace{1cm} (32)

The coefficients thus become

\[ a_i = \frac{(\sigma_\theta^2 + \sigma_\eta^2) + (1 - e^{-2Rc}) (\sigma_\theta^2 + \sigma_\eta^2)}{(\sigma_\theta^2 + \sigma_\eta^2)} \]  \hspace{1cm} (33)

and

\[ b = \frac{(e^{-2Rc} - 1) \sqrt{\sigma_\theta^2 + \sigma_\eta^2} \sqrt{\sigma_\theta^2 + \sigma_\eta^2} - e^{-2Rc} (\sigma_\theta^2 + \sigma_\eta^2 + \sigma_\theta + \sigma_\eta)}{\sqrt{1 - e^{-2Rc}} \sqrt{\sigma_\theta^2 + \sigma_\eta^2} \sigma_\theta^2} \]  \hspace{1cm} (34)

Finally, the conditional variance of the payoff is found to be

\[ \text{Var}(\eta + \theta + \delta + \epsilon P) = e^{-2Rc} \cdot (\sigma_\theta^2 + \sigma_\eta^2) \]  \hspace{1cm} (35)

### 8.3 Static Model with Managerial Disclosure

If the manager discloses his signal \( s_m \), the informed agents first update their beliefs about asset payoff as outlined above. The updated belief is denoted \( \beta \), and the variance of asset payoff conditional on \( \beta \) is denoted \( V \). Standard mean variance arguments yield that the informed agents demand schedule is given by

\[ x_1(P) = \frac{E(F) - P}{R \text{Var}(F | P)} = \beta - \frac{P}{RV} \]  \hspace{1cm} (36)

The market maker thus observes two pieces of information, \( s_m \) and net order flow \( Mx_1 + z_1 \). Thus, price is found by regressing asset payoff \( F \) on the \( s_m \) and \( Mx_1 + z_1 \). The latter is given as

\[ M \left( \sigma_\eta^2 \sigma_\theta^2 (-P1 + s_1 + s_m) + (1 \cdot \left( P1 \left( \sigma_\theta^2 + \sigma_\eta^2 \right) \right) + \sigma_\theta^2 s_1 + \sigma_\eta^2 s_m \right) \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\eta^2} + z_1 \]  \hspace{1cm} (37)

which is observationally equivalent to (since both \( P \) and \( s_m \) are known)

\[ \tau = \frac{M \sigma_\eta^2 s_1 \left( \sigma_\theta^2 + \sigma_\eta^2 \right)}{R \sigma_\theta^2 \sigma_\eta^2 \sigma_\theta^2 + R \left( \sigma_\theta^2 \sigma_\eta^2 + \sigma_\theta^2 \sigma_\eta^2 \right) \sigma_\theta^2} + z_1 \]  \hspace{1cm} (38)

Thus, price is found as

\[ P = \text{Cov}(F, (s_m, \tau)) \cdot \text{Var}(s_m, \tau) \cdot (s_m, \tau)^t \]  \hspace{1cm} (39)

which yields the coefficients

\[ a_i = \frac{M^2 \sigma_\theta^2 \left( \sigma_\theta^2 + \sigma_\eta^2 \right)^2}{M^2 \sigma_\theta^2 \left( \sigma_\theta^2 + \sigma_\eta^2 \right) \left( \sigma_\theta^2 \sigma_\eta^2 + \sigma_\theta^2 \sigma_\eta^2 + \sigma_\theta^2 \sigma_\eta^2 + \sigma_\theta^2 \sigma_\eta^2 \right) + R^2 \left( \sigma_\theta^2 \sigma_\eta^2 \sigma_\theta^2 + \sigma_\theta^2 \sigma_\eta^2 + \sigma_\theta^2 \sigma_\eta^2 + \sigma_\theta^2 \sigma_\eta^2 \right) \sigma_\theta^2} \]  \hspace{1cm} (40)

\[ a_m = 1 - \frac{M^2 \sigma_\theta^2 \sigma_\eta^2 \left( \sigma_\theta^2 + \sigma_\eta^2 \right)}{M^2 \sigma_\theta^2 \left( \sigma_\theta^2 + \sigma_\eta^2 \right) \left( \sigma_\theta^2 \sigma_\eta^2 + \sigma_\theta^2 \sigma_\eta^2 + \sigma_\theta^2 \sigma_\eta^2 + \sigma_\theta^2 \sigma_\eta^2 \right) + R^2 \left( \sigma_\theta^2 \sigma_\eta^2 \sigma_\theta^2 + \sigma_\theta^2 \sigma_\eta^2 + \sigma_\theta^2 \sigma_\eta^2 + \sigma_\theta^2 \sigma_\eta^2 \right) \sigma_\theta^2} \]  \hspace{1cm} (41)

and

\[ b = \frac{M \sigma_\theta^2 \left( \sigma_\theta^2 + \sigma_\eta^2 \right)}{(R \sigma_\theta^2 \sigma_\eta^2 + R \left( \sigma_\theta^2 \sigma_\eta^2 + \sigma_\theta^2 \sigma_\eta^2 \right) \sigma_\theta^2)} \frac{M^2 \sigma_\theta^2 \left( \sigma_\theta^2 + \sigma_\eta^2 \right) \left( \sigma_\theta^2 \sigma_\eta^2 + \sigma_\theta^2 \sigma_\eta^2 \right) \sigma_\theta^2}{(R \sigma_\theta^2 \sigma_\eta^2 + R \left( \sigma_\theta^2 \sigma_\eta^2 + \sigma_\theta^2 \sigma_\eta^2 \right) \sigma_\theta^2)} \]  \hspace{1cm} (42)
End of period wealth for the informed agents is thus given by

$$W = x_1 (F - P),$$

which is found to be the following quadratic form of the five random variables $\theta, \eta, \delta, \epsilon$ and $z_1$:

$$W = a_i (\delta + \theta) - b_1 z_1 \left( - a_i (\delta + \theta) - a_m (\eta + \theta) + \frac{\sigma^2_x \sigma^2_y + \sigma^2_y \sigma^2_z}{\sigma^2_x + \sigma^2_y + \sigma^2_z} \right) - b_2 z_1$$

$$R \left( \sigma^2 + \frac{\sigma^2_x \sigma^2_y}{\sigma^2_x + \sigma^2_y + \sigma^2_z} \right)$$

Expected Utility can now be found by rewriting the quadratic form for wealth in matrix-form $W = (1/2) J(\theta, \eta, \delta, \epsilon, z_1)$, where

$$J = \begin{pmatrix} 2j_1 & j_2 & j_3 & j_4 & j_5 \\ j_2 & 2j_6 & j_7 & j_8 & j_9 \\ j_3 & j_7 & 2j_{10} & j_{11} & j_{12} \\ j_4 & j_8 & j_{11} & 2j_{13} & j_{14} \\ j_5 & j_9 & j_{12} & j_{14} & 2j_{15} \end{pmatrix}$$

where the entries are given as:

$$j_1 = \frac{(-1 + a_i + a_m) \left( (-2 + a_i + a_m) \sigma^2_x \sigma^2_y + (-1 + a_i + a_m) \left( \sigma^2_x + \sigma^2_y \right) \sigma^2_z \right)}{R \sigma^2_x \sigma^2_y + R \left( \sigma^2_x \sigma^2_y + \sigma^2_z \left( \sigma^2_x + \sigma^2_y \right) \right) \sigma^2_z}$$

$$j_2 = \frac{(-1 + a_i) (-3 + 2a_i + 2a_m) \sigma^2_x \sigma^2_y + (-1 + a_i + a_m) \left( 2 (-1 + a_i) \sigma^2_x + (-1 + 2a) \sigma^2_y \right) \sigma^2_z}{R \left( \sigma^2_x \sigma^2_y + \sigma^2_z \left( \sigma^2_x + \sigma^2_y \right) \right) \sigma^2_z}$$

$$j_3 = \frac{(-1 + a_m) (-3 + 2a_i + 2a_m) \sigma^2_x \sigma^2_y + (-1 + a_i + a_m) \left( (-1 + 2a_m) \sigma^2_x + 2 (-1 + a_m) \sigma^2_y \right) \sigma^2_z}{R \left( \sigma^2_x \sigma^2_y + \sigma^2_z \left( \sigma^2_x + \sigma^2_y \right) \right) \sigma^2_z}$$

$$j_4 = - \left( \frac{(-2 + a_i + a_m) \sigma^2_x \sigma^2_y + (-1 + a_i + a_m) \left( \sigma^2_x + \sigma^2_y \right) \sigma^2_z}{R \sigma^2_x \sigma^2_y + R \left( \sigma^2_x \sigma^2_y + \sigma^2_z \left( \sigma^2_x + \sigma^2_y \right) \right) \sigma^2_z} \right)$$

$$j_5 = \frac{(-3 + 2a_i + 2a_m) \sigma^2_x \sigma^2_y + 2 (-1 + a_i + a_m) b \left( \sigma^2_x + \sigma^2_y \right) \sigma^2_z}{R \sigma^2_x \sigma^2_y + R \left( \sigma^2_x \sigma^2_y + \sigma^2_z \left( \sigma^2_x + \sigma^2_y \right) \right) \sigma^2_z}$$

$$j_6 = \frac{(-1 + a_i) \left( a_i \sigma^2_x \sigma^2_y + (-1 + a_i) \sigma^2_y \right) \sigma^2_z}{R \sigma^2_x \sigma^2_y + R \left( \sigma^2_x \sigma^2_y + \sigma^2_z \left( \sigma^2_x + \sigma^2_y \right) \right) \sigma^2_z}$$

$$j_7 = \frac{(-1 + 2a_i) (-1 + a_m) \sigma^2_x \sigma^2_y + (-1 + a_i) \sigma^2_x \left( 2 (-1 + a_m) \sigma^2_y + (-1 + 2a_m) \sigma^2_z \right)}{R \sigma^2_x \sigma^2_y + R \left( \sigma^2_x \sigma^2_y + \sigma^2_z \left( \sigma^2_x + \sigma^2_y \right) \right) \sigma^2_z}$$

$$j_8 = - \left( \frac{a_i \sigma^2_x \sigma^2_y + (-1 + a_i) \sigma^2_x \left( \sigma^2_y + \sigma^2_z \right)}{R \left( \sigma^2_x \sigma^2_y + \sigma^2_z \left( \sigma^2_x + \sigma^2_y \right) \right) \sigma^2_z} \right)$$

$$j_9 = \frac{b \left( (-1 + 2a_i) \sigma^2_x \sigma^2_y + 2 (-1 + a_i) \sigma^2_x \left( \sigma^2_y + \sigma^2_z \right) \right)}{R \left( \sigma^2_x \sigma^2_y + \sigma^2_z \left( \sigma^2_x + \sigma^2_y \right) \right) \sigma^2_z}$$
The equilibrium coefficients then become

\[ j_{10} = \frac{(-1 + a_m)((-1 + a_m) \sigma_a^2 \sigma_b^2 + \sigma_a^2 ((-1 + a_m) \sigma_a^2 + a_m \sigma_b^2))}{R \sigma_b^2 \sigma_a^2 + R (\sigma_a^2 \sigma_b^2 + \sigma_a^2 (\sigma_a^2 + \sigma_b^2)) \sigma_b^2} \]

\[ j_{11} = -\frac{((-1 + a_m) \sigma_a^2 \sigma_b^2 + \sigma_a^2 ((-1 + a_m) \sigma_a^2 + a_m \sigma_b^2))}{R \sigma_b^2 \sigma_a^2 + R (\sigma_a^2 \sigma_b^2 + \sigma_a^2 (\sigma_a^2 + \sigma_b^2)) \sigma_b^2} \]

\[ j_{12} = \frac{2(-1 + a_m) b \sigma_a^2 \sigma_b^2 + b ((-1 + 2a_m) \sigma_a^2 + 2(-1 + a_m) \sigma_a^2 \sigma_b^2)}{R \sigma_b^2 \sigma_a^2 + R (\sigma_a^2 \sigma_b^2 + \sigma_a^2 (\sigma_a^2 + \sigma_b^2)) \sigma_b^2} \]

\[ j_{13} = 0 \]

\[ j_{14} = -\left( \frac{b}{R \left( \sigma_a^2 + \frac{\sigma_a^2 \sigma_b^2}{\sigma_a^2 \sigma_b^2 + \sigma_a^2 (\sigma_a^2 + \sigma_b^2)} \right)} \right) \]

\[ j_{15} = \frac{b^2}{R \left( \frac{\sigma_a^2 \sigma_b^2}{\sigma_a^2 \sigma_b^2 + \sigma_a^2 (\sigma_a^2 + \sigma_b^2)} \right)} \]

Calculating the determinant yields an expected utility of

\[ \left( \sigma_a^2 \sigma_b^2 + \sigma_a^2 (\sigma_a^2 + \sigma_b^2) \right) \left( M^2 \sigma_a^2 (\sigma_a^2 + \sigma_b^2) + R^2 \left( \sigma_a^2 + \sigma_b^2 \right) \left( \sigma_a^2 \sigma_b^2 + \sigma_a^2 (\sigma_a^2 + \sigma_b^2) \right) \sigma_b^2 \right) \]

\[ \left( \frac{\sigma_a^2 \sigma_b^2}{\sigma_a^2 \sigma_b^2 + \sigma_a^2 (\sigma_a^2 + \sigma_b^2)} \right) \left( M^2 \sigma_a^2 (\sigma_a^2 + \sigma_b^2) + R^2 \left( \sigma_a^2 + \sigma_b^2 \right) \left( \sigma_a^2 \sigma_b^2 + \sigma_a^2 (\sigma_a^2 + \sigma_b^2) \right) \sigma_b^2 \right) \]

Next we allow for endogenous information acquisition. Specifically, assume that investors can choose to collect information at a cost \( c \). No agents will acquire information if the cost is too high, i.e. whenever

\[ \log \left( \frac{\sqrt{(\sigma_a^2 + \sigma_b^2) \sigma_a^2 \sigma_b^2 + \sigma_a^2 (\sigma_a^2 + \sigma_b^2)}}{\sigma_a^2 \sigma_b^2 + \sigma_a^2 (\sigma_a^2 + \sigma_b^2)} \right) \]

\[ \log \left( \frac{\sqrt{(\sigma_a^2 + \sigma_b^2) \sigma_a^2 \sigma_b^2 + \sigma_a^2 (\sigma_a^2 + \sigma_b^2)}}{\sigma_a^2 \sigma_b^2 + \sigma_a^2 (\sigma_a^2 + \sigma_b^2)} \right) \]

Otherwise, some agents will find it optimal to acquire information, and their number \( M \) is given by

\[ R \sigma_a \sqrt{\left( \sigma_a^2 \sigma_b^2 + \sigma_a^2 \left( \sigma_a^2 \sigma_b^2 + \sigma_a^2 (\sigma_a^2 + \sigma_b^2) \right) \right) \left( (1 - e^{2Re}) \sigma_a^2 \sigma_b^2 \sigma_a^2 + \sigma_a^2 (\sigma_a^2 + \sigma_b^2) \right)} - \left( e^{2Re} - 1 \right) \sigma_a^2 \left( \sigma_a^2 \sigma_b^2 + \sigma_a^2 (\sigma_a^2 + \sigma_b^2) \right) \]

\[ \sigma_a \sqrt{(e^{2Re} - 1) \left( \sigma_a^2 + \sigma_b^2 \right) \left( \sigma_a^2 \sigma_b^2 + \sigma_a^2 (\sigma_a^2 + \sigma_b^2) \right)} \]

The equilibrium coefficients then become

\[ a_i = \frac{\sigma_a^2 + \sigma_a^2 - e^{2Re} \left( \sigma_a^2 \sigma_b^2 + \sigma_a^2 (\sigma_a^2 + \sigma_b^2) \right)}{\sigma_a^2 \sigma_b^2 + \sigma_a^2 (\sigma_a^2 + \sigma_b^2)} \]

\[ a_m = \frac{(-1 + e^{2Re}) \sigma_a^2 \sigma_b^2 + \sigma_a^2 \sigma_b^2 (\sigma_a^2 + \sigma_b^2) + \sigma_a^2 \sigma_b^2 (\sigma_a^2 + e^{2Re} \sigma_a^2 \sigma_b^2 + (e^{2Re} - 1) \sigma_a^2 (\sigma_a^2 + \sigma_b^2))}{\sigma_a^2 \left( \sigma_a^2 + \sigma_b^2 \right) \left( \sigma_a^2 \sigma_b^2 + \sigma_a^2 (\sigma_a^2 + \sigma_b^2) \right)} \]

and

\[ b = \frac{R \sqrt{\left( \sigma_a^2 \sigma_b^2 + \sigma_a^2 \left( \sigma_a^2 \sigma_b^2 + \sigma_a^2 (\sigma_a^2 + \sigma_b^2) \right) \right) \left( (1 - e^{2Re}) \sigma_a^2 \sigma_b^2 \sigma_a^2 + \sigma_a^2 (\sigma_a^2 + \sigma_b^2) \right) - \left( e^{2Re} - 1 \right) \sigma_a^2 \left( \sigma_a^2 \sigma_b^2 + \sigma_a^2 (\sigma_a^2 + \sigma_b^2) \right)}}{\sigma_a \sqrt{\left( e^{2Re} - 1 \right) \left( \sigma_a^2 + \sigma_b^2 \right) \left( \sigma_a^2 \sigma_b^2 + \sigma_a^2 (\sigma_a^2 + \sigma_b^2) \right)}} \]

Finally, the variance of the payoff conditional on price is given by

\[ \text{Var}(\eta + \theta + \delta + \epsilon|P) = e^{2Re} \sigma_a^2 + \frac{e^{2Re} \sigma_a^2 \sigma_b^2}{\sigma_a^2 \sigma_b^2 + \sigma_a^2 (\sigma_a^2 + \sigma_b^2)} \]
8.4 $Q(\chi)$-Lemma

For deriving ex-ante utilities and informed agents’ hedge demands, we need the following well-known result about multivariate normal distributions.

**Proposition 5.** Let $Q(.)$ be a quadratic function of the random vector $\chi \sim N(\mu, \Sigma)$:

\[
Q(\chi) = C + Bt\chi - \chi'Ax
\]

Then $E[\exp(Q(\chi))]$ is given by

\[
|\Sigma|^{-1/2} \cdot |2A + \Sigma^{-1}|^{-1/2} \cdot \exp(C + Bt\mu + \mu'Ax + (1/2)(Bt - 2\mu'Ax)(2A + \Sigma^{-1})^{-1}(B - 2A\mu))
\]

8.5 Dynamic Model: Informed Agents’ Demand Functions

At date $t = 2$, agents demands are simply found by mean-variance analysis. Let $\Psi_t$ denote the informed investors’ information set at time $t$. Informed agents demand

\[
x_2(P_2) = \frac{E(F|\Psi_2) - P_2}{R \cdot \text{Var}(F|\Psi_2)}
\]

This implies an expected indirect utility of wealth at time 2 of

\[
E[u_2(W)|\Psi_1] = E[-\exp(-RW)|\Psi_2]
\]

\[
= E\left[-\exp\left(-R\left(x_1(P_2 - P_1) + \frac{(E(F|\Psi_2) - P_2)(F - P_2)}{R \cdot \text{Var}(F|\Psi_2)}\right)\right)\right]
\]

Thus, at time 1, the informed agent maximizes the expectation of period-2 indirect utility. We now proceed to show that first period demand is of the form

\[
x_1(P) = \frac{E(P_2|\Psi_1) - P_1}{RS} + \frac{E(F - P_2|\Psi_1)}{R \cdot \text{Var}(F|\Psi_2)} \cdot \frac{S - T}{S}
\]

We can rewrite the expression for indirect period 2 utility $u_2(W)$ as

\[
u_2(W) = -\exp\left(-\frac{1}{2}h'tAx + h't\chi + l\right)
\]

where

\[
\chi = (P_2 - E(P_2|\Psi_1), E(\beta|\Psi_2) - E(\beta|\Psi_1))^t
\]

and let $\Sigma$ denote the covariance matrix of $\chi$. Furthermore,

\[
h = \left(Rx_1 + \frac{E(P_2|\Psi_1) - E(\beta|\Psi_1)}{\sigma^2}, \frac{E(\beta|\Psi_1) - E(P_2|\Psi_1)}{\sigma^2}\right)^t
\]

\[
A = \left(\Sigma^{-1} + \begin{pmatrix}
\sigma_z^{-2} & -\sigma_z^{-2} \\
-\sigma_z^{-2} & \sigma_z^{-2}
\end{pmatrix}\right)
\]

and $l = Rx_1(E(P_2|\Psi_1) - P_1) + \text{constant}$, where the constant does not depend on $x_1$. Now we can apply the $Q(\chi)$-lemma to find

\[
E[u_2(W)|\Psi_1] = |\Sigma|^{-1/2} \cdot |A|^{-1/2} \exp\left(\frac{1}{2}h'tA^{-1}h - l\right)
\]

Differentiation yields the first-order condition

\[
htS^{-1} \frac{\partial h}{\partial x_1} - \frac{\partial l}{\partial x_1} = 0
\]
which yields after substitution and simplification
\[ x_1(P) = \frac{E(P_2|\Psi_1) - P_1}{RS} + \frac{E(F - P_2|\Psi_1)}{R\text{Var}(F|\Psi_2)} \cdot \frac{S - T}{S} \]  
where
\[ S^{-1} = \begin{pmatrix} S & T \\ T & U \end{pmatrix} \]

In the case of immediate disclosure, the expression simplifies as mentioned in the text. The same calculation can be used, with \( \text{Var}(\beta|\Psi_2) \to 0 \), or otherwise the result can be obtained in the same way with simpler expressions. In that case, we know that \( T = 0 \) and \( S = \left( \frac{1}{\text{Var}(F|\Psi_2)} + \frac{1}{\text{Var}(P_2|\Psi_1)} \right)^{-1} \).

### 8.6 Equilibrium Prices

Before any trading takes place, the manager discloses his signals, and informed agents update their belief to \( \beta \) as discussed earlier. Then, trading takes place. The market maker sets prices as his expectation of asset payoff conditional on his information, which consists of the manager's signal \( s_m \) and total net order flow \( y_1 = Mx_1 + z_1 \). Thus, prices can be found by regressing asset payoff on the vector \( (s_m, y_1) \).

Note that it is observationally equivalent to remove the terms involving \( \text{Var}(\theta|\Psi_2) \).

Then in the next period, the informed agents again submit their demand. Thus, the market maker can use the \( \text{Q}(\chi) \)-lemma, since wealth is a quadratic form. For the case of intermediate disclosure, prices can be found by regressing asset payoff on the vector \( (s_m, y_1, \tau_1) \). Then first period price can be found as a function of second period price coefficients as
\[ P_1 = \text{Cov}(v_1, F) \text{Var}(v_1)^{-1} v_1 \]

which yields
\[ a_m = \frac{2\sigma^2 M^2 (\sigma^2 + \sigma_0^2)}{R M^2} + \frac{\sigma^2 M^2 (\sigma^2 + \sigma_0^2)}{R^2 \sigma_0} + \sigma_2 (\sigma^2 + \sigma_0^2) + \frac{M^2 (\sigma^2 + \sigma_0^2)(\sigma^2 + \sigma_0^2 + \sigma_2) \sigma_2 + f^2 s_m s_\alpha (\sigma^2 + \sigma_0^2)^2}{f^2 \sigma_2^2 (\sigma_0^2 + \sigma_2^2 + \sigma_2^2 + (\sigma^2 + \sigma_0^2) \sigma_2) \sigma_0^2} \]

\[ b = \frac{M \sigma_2^2 (\sigma^2 + \sigma_0^2) \left( \frac{\sigma^2 M^2 (\sigma^2 + \sigma_0^2)}{R^2 \sigma_0} + \sigma_2 + \frac{M^2 (\sigma^2 + \sigma_0^2)(\sigma^2 + \sigma_0^2 + \sigma_2) \sigma_2 + f^2 s_m s_\alpha (\sigma^2 + \sigma_0^2)^2}{f^2 \sigma_2^2 (\sigma_0^2 + \sigma_2^2 + \sigma_2^2 + (\sigma^2 + \sigma_0^2) \sigma_2) \sigma_0^2} \right)}{R \left( \frac{2\sigma^2 M^2 (\sigma^2 + \sigma_0^2)}{R^2 \sigma_0} + \sigma_2 + \frac{M^2 (\sigma^2 + \sigma_0^2)(\sigma^2 + \sigma_0^2 + \sigma_2) \sigma_2 + f^2 s_m s_\alpha (\sigma^2 + \sigma_0^2)^2}{f^2 \sigma_2^2 (\sigma_0^2 + \sigma_2^2 + \sigma_2^2 + (\sigma^2 + \sigma_0^2) \sigma_2) \sigma_0^2} \right)} \]

Then in the next period, the informed agents again submit their demand. Thus, the market maker can now condition on the vector \( v_2 = (s_m, \tau_1, \tau_2) \) where \( \tau_2 \) is obtained from \( y_2 = Mx_2 + z_1 + z_2 \) by removing the \( P_2 \) and \( s_m \) terms. Thus, period 2 price is given by
\[ P_2 = d_s s_1 + d_m s_m + \epsilon z_1 + f z_2 = \text{Cov}(v_2, F) \text{Var}(v_2)^{-1} v_2 \]

This yields a system of four equations in the four unknown price coefficients. This system is highly nonlinear, and thus problematic to solve. However, since the expression for hedge demand simplifies with early disclosure, we have been able to solve the equilibrium analytically. Ex-ante utility can be found by using the \( \text{Q}(\chi) \)-lemma, since wealth is a quadratic form. For the case of intermediate disclosure, prices are found as
\[ P_1 = \frac{\text{Cov}(\tau_1, F)}{\text{Var}(\tau_1)} \tau_1 \]

and
\[ P_2 = \text{Cov}(v_2, F) \text{Var}(v_2)^{-1} v_2 \]

where \( v_2 = (s_m, \tau_1, \tau_2) \) as before.
This figure explains the behavior of the model with disclosure at an intermediate point in time ($t = 2$). The parameters used here are: $R = 100, \sigma_{\eta} = 0.01, \sigma_{\epsilon} = \sqrt{0.1}, \sigma_\theta = 1, \sigma_z^2 = 1$, and $c = 0.048$. 
This figure displays the behavior of the model if the firm were to disclose its information late (at date $t = 3$). It is qualitatively similar to early disclosure $t = 1$.

The parameters used here are: $R = 100, \sigma_\eta = 0.01, \sigma_\epsilon = \sqrt{0.1}, \sigma_\theta = 1, \sigma_\epsilon^2 = 1$, and $c = 0.048$. 
Figure 3: The Effect of Investor-Specific Information

This figure illustrates how the optimal disclosure policy depends on how much information is unique to investors, as measured by $\sigma_\delta$. We see that the higher $\sigma_\delta$, the more profitable it is to delay disclosure, meaning that the manager may disclose at an intermediate point in time despite a high level of firm specific information.

The parameters used here are: $R = 100$, $\sigma_\eta = 0.4$, $\sigma_\epsilon = \sqrt{0.1}$, $\sigma_\theta = 1$, $\sigma_z^2 = 1$, and $c = 0.048$. 
Figure 4: The Effect of Information Overlap

This figure illustrates how the optimal disclosure policy depends on the amount of information overlap between outside investors and the manager. Note the non-monotonic effect.

The parameters used here are: $R = 100$, $\sigma_\eta = 0.4$, $\sigma_\epsilon = \sqrt{0.1}$, $\sigma_\delta = 1$, $\sigma_\kappa^2 = 1$, and $c = 0.048$. 
This figure illustrates how the optimal disclosure policy depends on the specialness of the manager's information, captured by the ratio $\sigma_\eta / \sigma_\theta$.

The parameters used here are: $R = 100, \sigma_\epsilon = \sqrt{0.1}, \sigma_\delta = 1, \sigma_\theta = 1, \sigma_2^z = 1$, and $c = 0.048$. 
This figure illustrates how the optimal disclosure policy depends on the amount of risk that informed agents have to bear when holding the asset until the liquidation date. The parameters used here are: $R = 100$, $\sigma_\eta = 0.4$, $\sigma_\delta = 1$, $\sigma_\theta = 1$, $\sigma_z^2 = 1$, and $c = 0.048$. 