Equilibrium "Anomalies"

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ABSTRACT

Many empirical "anomalies" are actually consistent with the single beta CAPM if the empiricist utilizes an equity-only proxy for the true market portfolio. Equity betas estimated against this particular inefficient proxy will be understated, with the error increasing with the firm's leverage. Thus, firm-specific variables that correlate with leverage (such as book-to-market and size) will appear to explain returns after controlling for proxy beta simply because they capture the missing beta risk. Loadings on portfolios formed on relative leverage and relative distress completely subsume the powers of the Fama and French (1993) SMB and HML factors in explaining cross-sectional returns.

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Assume a world where the single-beta capital asset pricing model (CAPM) prices all assets. Suppose further that an empiricist, wanting to learn about the risk-return relationship in the equity market, calculates equity betas against a market portfolio proxy that ignores the economy's debt claims. If the empiricist then estimates the cross-sectional relationship between return and proxy beta, what results does theory tell us to expect?

We show that positive pricing errors, downwardly biased market risk premiums, and what appear to be cross-sectional anomalies related to corporate leverage are actual implications of the single-beta CAPM in this situation. We demonstrate this by constructing a hypothetical economy where the single-beta CAPM prices all assets and where firms are allowed to have simple capital structures. This framework allows us to compare the true betas of the economy's financial assets with the proxy betas generated from any market proxy. [1]

As it turns out, use of an equity-only proxy leads to firm-specific beta estimation errors that generally escalate with a firm's relative degree of leverage and distress. If observed returns are then regressed onto these erroneous proxy betas in a one-factor cross-sectional model, the expected values of the alpha and slope coefficients will not be equal to the CAPM pricing error and market price of risk. Furthermore, any firm-specific variables that correlate with relative leverage and/or relative distress (such as size, book-to-market, earnings-to-price, and debt-to-equity) will serve as instruments for the uncaptured beta risk and will appear to explain excess return in the cross-section.

The intuition behind this result is relatively simple. Suppose that, in equilibrium, the CAPM holds, and the market portfolio *M* can be divided into two subportfolios: The economy's debt claims (*D*), and the economy's equity claims (*E*). In this equilibrium, which we will detail in Section I, the covariance between firm *i*'s equity claim (S_i) and M is

$$\sigma_{Si,M} = \frac{E}{M}\sigma_{Si,E} + \frac{D}{M}\sigma_{Si,D},$$

where $\sigma_{Si,M}$, $\sigma_{Si,E}$ and $\sigma_{Si,D}$ are the stock's covariances with the asset, equity, and debt markets, respectively. It follows that the true beta of firm *i*'s equity claim can be written

$$\beta_{Si} = \frac{\sigma_{Si,M}}{\sigma_M^2} = \frac{E}{M}\frac{\sigma_{Si,E}}{\sigma_M^2} + \frac{D}{M}\frac{\sigma_{Si,D}}{\sigma_M^2},$$

where σ_M^2 , σ_E^2 , and σ_D^2 are the return variances of the asset, equity, and debt markets, respectively. If we ignore the economy's debt claims in the construction of our market proxy, the *proxy* beta of firm *i*'s equity will be

$$\hat{\beta}_{Si}^E = \frac{\sigma_{Si,E}}{\sigma_E^2},$$

where the superscript *E* is appended to denote the proxy with respect to the equity market. We can hence write the proxy equity beta as a transformation of the true equity beta:

$$\hat{\beta}_{Si}^E = \Phi^{-1}\left[\beta_{Si} - \Omega\hat{\beta}_{Si}^D\right], \tag{1}$$

where $\Phi = \frac{E}{M}\frac{\sigma_E^2}{\sigma_M^2}$, $\Omega = \frac{D}{M}\frac{\sigma_D^2}{\sigma_M^2}$, and $\hat{\beta}_{Si}^D$ is the beta of the equity calculated against the economy's debt claims only (i.e., the assets omitted from the market proxy).

Equation (1) shows that the error in the proxy equity beta $\left(\hat{\beta}_{Si}^E\right)$ has two components: a scaling error (Φ^{-1}) that is common across all equities, and a firm-specific error ($-\Omega\,\hat{\beta}_{Si}^D$) that reflects the stock's covariance with the (omitted) debt claims. The scaling term implies that proxy betas are generally too low, but it is inconsequential in many applications since all proxy betas in the cross-section will be affected to the same degree. The firm-specific term, however, is critical for the empiricist because it is a function of the firm's financial leverage.

Why? Because a stock's covariance with *any* portfolio is magnified by the firm's leverage, and $\hat{\beta}_{Si}^D$ measures the stock's covariance with the omitted assets. So, in the cross-section, proxy beta errors will be related to an operating characteristic of the firms in the sample (leverage), and metrics that correlate strongly with leverage (such as market value of equity or any scaling of it) will act as instruments for the proxy beta estimation error, and will appear to explain excess returns.[2]

At one level, our results provide a theoretical rationale for the Fama and French (1993) three-factor model. Fama and French (1996) show that loadings on SMB (returns to small minus big market capitalization portfolios) and HML (returns to high minus low book-to-market portfolios) explain a substantial number of well-known anomalies; Fama and French (1993, 1995, 1996, and 1997) suggest that the HML return reflects a priced risk of relative distress.

Why relative distress could be a separately priced risk is not well established, although Fama and French (1996) offer a human capital story. If distress risk is correlated across firms, then workers with specialized human capital in distressed firms will optimally avoid the stocks of all distressed firms--which could lead to an additional risk premium for these firms after market clearing.

An important implication of our result is that complex models that predict an equilibrium premium for relative distress may be unnecessary. The cross-sectional associations among distress, HML, and many anomalies may arise because market betas of distressed firms are underestimated most severely, and loadings on HML provide appropriate corrections.

But our results have a deeper importance. Rather than simply invoke the "ritual argument" that SMB and HML are evidence of a bad market proxy, our model provides a specific roadmap for recovering CAPM expected returns for use in applications of the static model (like performance measurement and estimation of the corporate cost of capital). Our model implies that *if* the single-factor CAPM holds, then factors formed on relative leverage and relative distress should provide the best complements to the equity market index for explaining the cross-section of returns. In other words, a three-factor empirical model that includes factors based on relative leverage and relative distress should outperform the Fama and French (1993) three-factor model in the cross-section.

The economic importance of this argument is an empirical issue, and we provide in Section III preliminary results that are very encouraging. We construct portfolios based on relative leverage (debt-to-equity ratio) and relative distress (Altman's *Z*) through the same method used to generate SMB and HML, and we then investigate whether sensitivities to returns on these portfolios help to explain the cross-section of returns on the Fama-French 25 size- and book-to-market-sorted portfolios.

In standard Fama-MacBeth (1973) cross-sectional regressions, our three-beta model provides more explanatory power than the popular three factor model of Fama and French (1993). Moreover, the parts of SMB and HML that are orthogonal to our leverage and distress portfolios have virtually no explanatory power in the cross-section, while the parts of our leverage and distress portfolios that are orthogonal to SMB and HML provide additional explanatory power when added to a model that includes the equity market index, SMB, and HML.

The paper is organized as follows. Section I presents a simple economy in which the single-beta CAPM prices all assets. Section II derives the main results concerning beta estimation errors when the market proxy ignores the economy's debt claims. Section III provides the empirical investigation mentioned above. Section IV concludes and offers implications of our model for a wide array of puzzles.

I. The Model

We generate a simple continuous-time economy in which the single-beta CAPM prices all real assets. Firms are allowed to finance their real assets with simple capital structures; with the proper assumptions, equity claims will be priced as European calls on the underlying real assets.[3] The model provides the explicit mapping between the covariance structure of asset returns and the covariance structure of equity returns, and therefore allows us to determine explicitly the beta estimation errors that will arise via use of the inefficient equity-only proxy.

A. Equilibrium

We assume that expected returns on real assets are determined by the intertemporal CAPM constructed in Merton (1975):

$$r_i - r_F = \frac{\sigma_{i,M}}{\sigma_M^2}\left(r_M - r_F\right) + \frac{\sigma_{i,O}}{\sigma_O^2}\left(r_O - r_F\right),$$

where the subscript i indicates asset i, the subscript M indicates the market portfolio, and the subscript O indicates the portfolio that best hedges against changes in the state variable. We further restrict the equilibrium by imposing the condition that all investors have logarithmic utility. With log utility, no investor cares to hedge against changes in the investment opportunity set, so the equilibrium return-generating process reduces to the continuous-time equivalent of the single-beta CAPM:

$$r_i - r_F = \frac{\sigma_{i,M}}{\sigma_M^2}\left(r_M - r_F\right) = \beta_i\left(r_M - r_F\right) \quad .$$

For a complete derivation of the equilibrium, including the structural and technical assumptions, see Ingersoll (1987, Ch. 13).

B. Financial Claims

Next, assume that firms in the economy raise capital through some combination of debt and equity claims, with debt restricted to pure discount bonds maturing at T with face value F_i. No dividends are paid before T; at time T, all firms are liquidated, debts are paid, and equity is a residual claimant. Asset values are lognormally distributed at the end of any finite time period, and asset variances per unit time are constant. There are no costs of bankruptcy or other imperfections, and the securities are infinitely divisible and trade in a continuous market.

In this continuous-time economy, the financial assets are redundant securities and can therefore be priced with arbitrage arguments. Specifically, equity claims are priced using the

Black and Scholes (1973) model for European calls. Note that since the financial assets are redundant, their introduction does not alter the equilibrium in the primary assets. Hence, the CAPM will price the financial assets trivially as portfolios of the primary and risk-free assets.[4]

The initial equity value of firm *i* will be

$$S_i = V_i N(d_1)_i - F_i e^{-r_F T} N(d_2)_i , \tag{2}$$

where V_i is the market value of firm *i*'s asset, r_F is the risk-free rate of return, and $N(\cdot)$ is the cumulative density of a standard normal random variable:

$$(d_1)_i \equiv \frac{\ln\left(V_i \big/ F_i \right) + \left(r_F + .5\sigma_i^2 \right) T}{\sigma_i \sqrt{T}} \quad , \tag{3}$$

where σ_i^2 is the variance in the return on asset *i*, and

$$(d_2)_i \equiv (d_1)_i - \sigma_i \sqrt{T} \ . \tag{4}$$

The initial value of firm *i*'s debt claim will be $B_i = V_i - S_i = V_i N(-d_1)_i + F_i e^{-r_F T} N(d_2)_i$.

C. Variances and Covariances

The variance of the equity claim on firm *i* is established by

$$\sigma_{Si}^2 \equiv E\left\{ \left(r_{Si} - \bar{r}_{Si} \right)^2 \right\} = E\left\{ \left(\frac{\partial S_i}{\partial V_i} \frac{V_i}{S_i} \right)^2 \left(r_i - \bar{r}_i \right)^2 \right\} = \left(N(d_1) \frac{V_i}{S_i} \right)^2 \sigma_i^2 \equiv \eta_{Si}^2 \sigma_i^2 \tag{5}$$

where the subscript S_i refers to firm *i*'s equity claim and $\eta_{Si} \equiv N(d_1) \dfrac{V_i}{S_i}$ is defined as the elasticity of firm *i*'s equity claim.[5] By a similar manipulation, the standard deviation of firm *i*'s debt claim is established as $\sigma_{Bi} = \eta_{Bi} \sigma_i$, where $\eta_{Bi} \equiv N(-d_1) \dfrac{V_i}{B_i}$.

The covariance between firm i's debt claim and its equity claim is $\sigma_{Bi,Si} = \eta_{Bi}\eta_{Si}\sigma_i^2$; the covariance between the equity claim of firm i and the equity claim of firm j is $\sigma_{Si,Sj} = \eta_{Si}\eta_{Sj}\sigma_{ij}$; the covariance between the debt claim of firm i and the debt claim of j is $\sigma_{Bi,Bj} = \eta_{Bi}\eta_{Bj}\sigma_{ij}$; and the covariance between the debt claim of firm i and the equity claim of firm j is $\sigma_{Bi,Sj} = \eta_{Bi}\eta_{Sj}\sigma_{ij}$.

Note that all financial asset covariances are directly tied to real asset covariances through the elasticities of the financial claims, and that the elasticities are themselves determined by the moneyness of the financial claims. Moreover, $\eta_{Si} \geq 1$ for all firms i, and equality holds only when the firm is unlevered ($F_i = 0$). Holding V_i fixed, η_{Si} is increasing and convex with leverage – a point of primary importance in our results.

D. Portfolios

It is easy to demonstrate that a stock's covariance with the true market portfolio is identical to its covariance with the global portfolio of financial claims: $\sigma_{Si,FA} = \sigma_{Si,M}$. Since the global portfolio of financial claims can be partitioned into debt claims and equity claims (with aggregate values D and E), we can say that

$$\sigma_{Si,M} = \frac{E}{M}\sigma_{Si,E} + \frac{D}{M}\sigma_{Si,D} \ . \tag{6}$$

It follows that the *true* beta of firm i's equity can be written as

$$\beta_{Si} = \frac{\sigma_{Si,M}}{\sigma_M^2} = \frac{E}{M}\frac{\sigma_{Si,E}}{\sigma_M^2} + \frac{D}{M}\frac{\sigma_{Si,D}}{\sigma_M^2} = \Phi\hat{\beta}_{Si}^E + \Omega\hat{\beta}_{Si}^D \ , \tag{7}$$

where Φ, $\hat{\beta}_{Si}^E$, Ω, and $\hat{\beta}_{Si}^D$ are as defined, and $M = D + E$.

Equation (7) provides the foundation of our analysis of proxy beta errors. When the proxy portfolio consists only of the economy's equity claims, the proxy beta error for an individual equity will be driven by the last term, which is the equity's covariance with the economy's debt claims (the omitted assets). Using the covariance relationships established above, it is easy to demonstrate that $\hat{\beta}_{Si}^{D} = \eta_{Si}\hat{\beta}_{i}^{D}$. In other words, the amount of beta risk not captured by the inefficient proxy is related to the firm's leverage. We formalize this intuition in Section II.

II. Proxy Betas and Errors

A. Proxy Betas for Individual Firms

We begin our analysis with a preliminary result (needed for our main proposition), which has some importance in its own right. In our equilibrium, the asset beta of firm $i\left(\beta_i\right)$ is invariant to the firm's leverage choice. The lemma below, however, tells us that under a very loose restriction, the *proxy* for the firm's asset beta (i.e., its asset beta calculated against the inefficient equity-only portfolio) *declines* with the firm's leverage.

LEMMA: *If the proxy beta of the firm's equity is positive (that is, $\hat{\beta}_{Si}^{E} > 0$), and if*

$$\frac{2\rho_{Si,E}^{2} - 1}{\sigma_E \rho_{Si,E}\sqrt{T}} < \frac{N\left(d_2\right)_i}{Z\left(d_2\right)_i}, \tag{MR}$$

then $\dfrac{\partial \hat{\beta}_i^E}{\partial F_i} < 0$. That is, the proxy beta of the firm's assets declines with the firm's leverage.

Proof: See Appendix.

The right-hand side of (MR) in the lemma is the Mills Ratio for $-d_2$.[6] The Mills Ratio is always greater than zero, so a sufficient condition for (MR) will be $\rho_{Si,E} < \sqrt{.5} \cong .71$ (which guarantees that the left hand side of MR is negative).[7] The quantity $\rho_{Si,E}$ is empirically observable, and indeed it is uncommon to find an equity whose return correlation with the CRSP index is greater than .70. But this is merely a sufficient condition, and (MR) will hold unconditionally as long as $\frac{N(d_2)_i}{Z(d_2)_i}$ is large enough. Since $\frac{N(d_2)_i}{Z(d_2)_i}$ increases with d_2, (MR) will hold as long as the firm is not severely distressed.[8]

In short, for stocks with positive proxy betas, the only time the proxy asset beta does not decline with the firm's leverage is when the stock's correlation with the equity market is extremely high (greater than about .71), *and* the firm is severely distressed.[9]

We are primarily interested in the estimation error of the proxy *equity* beta. In Proposition 1, we examine the degree of the estimation error in $\hat{\beta}_{Si}^{E}$ by focusing on the ratio of the true equity beta to its proxy $\left(\frac{\beta_{Si}}{\hat{\beta}_{Si}^{E}}\right)$. If both are positive, then this ratio rises with firm *i*'s leverage as long as (MR) holds. In other words, as firm *i* increases its leverage, the true beta of its equity increases faster than the proxy for its equity beta.[10]

PROPOSITION 1: *If the true and proxy equity betas of firm i are both positive (that is,* $\beta_{Si} > 0$ *and* $\hat{\beta}_{Si}^{E} > 0$*), and if (MR) holds, then the ratio* $\frac{\beta_{Si}}{\hat{\beta}_{Si}^{E}}$ *increases with firm i's leverage.*

Proof:

$$\beta_{Si} = \sum_j \frac{V_i}{M}\sigma_{Si,M} = \eta_{Si}\sum_j \frac{V_i}{M}\sigma_{i,M} = \eta_{Si}\beta_i$$

and

$$\hat{\beta}_{Si}^{E} = \sum_j \frac{S_i}{E}\sigma_{Si,Sj} = \eta_{Si}\sum_j \frac{S_i}{E}\sigma_{i,Sj} = \eta_{Si}\hat{\beta}_i^{E}$$

so

$$\frac{\beta_{Si}}{\hat{\beta}_{Si}^E} = \frac{\beta_i}{\hat{\beta}_i^E}.$$

That is, the ratio of the true beta of an equity to the proxy beta of the same equity is equal to the ratio of the true beta of the underlying assets to the proxy beta of those assets. So:

$$\frac{\partial\left(\frac{\beta_{Si}}{\hat{\beta}_{Si}^E}\right)}{\partial F_i} = \frac{\partial\left(\frac{\beta_i}{\hat{\beta}_i^E}\right)}{\partial F_i}.$$

It is easier to take the derivative of the term on the right-hand side of the equation

$$\frac{\partial\left(\frac{\beta_i}{\hat{\beta}_i^E}\right)}{\partial F_i} = \frac{\hat{\beta}_i^E \frac{\partial \beta_i}{\partial F_i} - \beta_i \frac{\partial \hat{\beta}_i^E}{\partial F_i}}{\left(\hat{\beta}_i^E\right)^2} = \frac{-\beta_i \frac{\partial \hat{\beta}_i^E}{\partial F_i}}{\left(\hat{\beta}_i^E\right)^2},$$

because a firm's true asset beta does not change with leverage. From the lemma $\frac{\partial \hat{\beta}_i^E}{\partial F_i} < 0$, and $\beta_i > 0$ (by assumption the true equity beta is positive); so

$$\frac{\partial\left(\frac{\beta_{Si}}{\hat{\beta}_{Si}^E}\right)}{\partial F_i} = \frac{-\beta_i \frac{\partial \hat{\beta}_i^E}{\partial F_i}}{\left(\hat{\beta}_i^E\right)^2} > 0.$$

Q.E.D.

We can now make the intuition from the introductory section more precise by recalling equation (1):

$$\hat{\beta}_{Si}^E = \Phi^{-1}\left[\beta_{Si} - \Omega\hat{\beta}_{Si}^D\right].$$

If the firm's assets are uncorrelated with the debt market (the assets omitted from the proxy), then $\eta_{Si}\sigma_{i,D} = \sigma_{Si,D} = 0$, so $\hat{\beta}_{Si}^D = 0$, and the relationship between the proxy and true equity beta is simply a scaling by the economywide constant Φ^{-1}. But when the firm's asset (or equity) returns covary positively with the claims omitted from the inefficient proxy (the debt claims), then $\Omega\hat{\beta}_{Si}^{D}$ is not zero. The result of Proposition 1 is driven by the fact that $\Omega\hat{\beta}_{Si}^{D}$ increases with the firm's leverage.[11] Since $\hat{\beta}_{Si}^{D} = \eta_{Si}\hat{\beta}_{i}^{D}$, and since η_{Si} is greater than one and convex with the firm's leverage, we can say that proxy beta errors will be most serious for firms that are more highly levered and relatively more distressed.[12]

The implication is that *in the cross-section* the equity beta estimation errors will not be random. Rather, they will be systematically related to the relative leverage and relative distress of each firm in the sample.

B. Cross-Sectional Implications of the Inefficient Proxy

We next derive the cross-sectional results that should be expected when the inefficient equity-composite portfolio is used to proxy for the market portfolio of assets. The decomposition of the firm's true equity beta in equation (7) implies the equilibrium model of excess returns:

$$r_{Si} - r_F = \left[r_M - r_F\right]\beta_{Si} = \Phi\left[r_M - r_F\right]\hat{\beta}_{Si}^{E} + \Omega\left[r_M - r_F\right]\hat{\beta}_{Si}^{D}. \tag{8}$$

If equation (8) is the equilibrium model of returns, then the cross-sectional market model regression

$$r_{Si} - r_F = \gamma_0^{*} + \gamma_{ME}^{*}\hat{\beta}_{Si}^{E} + \varepsilon_{Si} \tag{9}$$

will be misspecified. Proposition 2 establishes the exact theoretical values of γ_0^{*} and γ_{ME}^{*}.

PROPOSITION 2*: If equation (8) is the equilibrium model of returns, the theoretical value of the pricing error γ_0^{*} and the market risk premium γ_{ME}^{*} in equation (9) will be*

$$\gamma_0^{*} = \left[\bar{\hat{\beta}}_{S}^{D} - \Delta\bar{\hat{\beta}}_{S}^{E}\right]\Omega\left(r_M - r_F\right)$$

$$\gamma_{ME}^{*} = \left[\Phi + \Omega\Delta\right]\left(r_M - r_F\right)$$

where overbars represent cross-sectional means; the i subscripts are suppressed in cross-sectional operations (means, variances, and covariances); and $\Delta = \dfrac{\operatorname{cov}\left(\hat{\beta}_S^D, \hat{\beta}_S^E\right)}{\operatorname{var}\left(\hat{\beta}_S^E\right)}$.

Proof: Using equation (8):

$$\gamma_{ME}^* = \frac{\operatorname{cov}\left(\hat{\beta}_S^E, r_S - r_F\right)}{\operatorname{var}\left(\hat{\beta}_S^E\right)} = \frac{\operatorname{cov}\left(\hat{\beta}_{S,}^E \Phi\left(r_M - r_F\right)\hat{\beta}_S^E + \Omega\left(r_M - r_F\right)\hat{\beta}_S^D\right)}{\operatorname{var}\left(\hat{\beta}_S^E\right)}$$

$$= \Phi\left(r_M - r_F\right)\frac{\operatorname{cov}\left(\hat{\beta}_S^E, \hat{\beta}_S^E\right)}{\operatorname{var}\left(\hat{\beta}_S^E\right)} + \Omega\left(r_M - r_F\right)\frac{\operatorname{cov}\left(\hat{\beta}_S^D, \hat{\beta}_S^E\right)}{\operatorname{var}\left(\hat{\beta}_S^E\right)}$$

$$\gamma_{ME}^* = \left[\Phi + \Omega\Delta\right]\left(r_M - r_F\right) \quad ; \tag{10}$$

$$\gamma_0^* = \overline{r_S - r_F} - \gamma_{ME}^* \bar{\hat{\beta}}_S^E$$

$$= \overline{\Phi\left(r_M - r_F\right)\hat{\beta}_S^E + \Omega\left(r_M - r_F\right)\hat{\beta}_S^D} - \left[\Phi + \Omega\Delta\right]\left(r_M - r_F\right)\bar{\hat{\beta}}_S^E$$

$$= \left(r_M - r_F\right)\left[\Phi\bar{\hat{\beta}}_S^E + \Omega\bar{\hat{\beta}}_S^D - \Phi\bar{\hat{\beta}}_S^E - \Omega\Delta\bar{\hat{\beta}}_S^E\right]$$

$$\gamma_0^* = \left[\bar{\hat{\beta}}_S^D - \Delta\bar{\hat{\beta}}_S^E\right]\Omega\left(r_M - r_F\right). \tag{11}$$

Q.E.D.

Proposition 2 shows that the very common joint findings of a positive alpha and a non-positive market price of beta risk are consistent with a CAPM equilibrium as long as proxy betas are measured using the equity-only proxy. The theoretical pricing error in equation (9) will be zero if and only if $\Delta\bar{\hat{\beta}}_S^E = \bar{\hat{\beta}}_S^D$ in the cross-section, and the market price of risk in equation (9) will be correct if and only if $\Delta = 0$. So the *only* time equation (9) provides the correct pricing error *and* the correct market price of risk in expectation is when $\hat{\beta}_{Si}^E$ is uncorrelated with $\hat{\beta}_{Si}^D$ in the cross-section *and* the average $\hat{\beta}_{Si}^{D}$ is zero. A positive alpha is expected if $\Delta < \frac{\bar{\hat{\beta}}_S^D}{\bar{\hat{\beta}}_S^E}$. A downwardly biased market price of risk is expected if $\Delta < 0$; and both findings are to be expected from equation (9) simultaneously as long as Δ is less than $\min\left[\frac{\bar{\hat{\beta}}_S^D}{\bar{\hat{\beta}}_S^E}, 0\right]$.[13]

The expression for γ_0^* in equation (11) can be rewritten as $\gamma_0^* = p_0\Omega\left(r_M - r_F\right)$, where p_0 is the intercept of the regression $\hat{\beta}_{Si}^{D} = p_0 + p_1\hat{\beta}_{Si}^{E} + v_{Si}$. In other words, the expected pricing error in equation (9) is the expected excess return on a positive-beta portfolio that is orthogonal to the equity-only proxy. This is a portfolio that *appears* to have zero beta risk when, in fact, it does not. The implication, then, is that there are portfolios that generate positive average excess returns with what appears to be zero beta risk. These are the equilibrium "anomalies".

Anomaly strategies obtain in the equilibrium in our economy as long as *any* inefficient proxy is used. When the proxy portfolio includes only the economy's equity claims, the anomaly strategies can be exploited by finding variables that, in the cross-section, are correlated with the part of $\hat{\beta}^{D}$ that is orthogonal to $\hat{\beta}^{E}$.

Our model allows us to say something about where to look for these strategies. As we show in Proposition 1, a stock's covariance with the portfolio of omitted debt claims (and hence its missing common risk) increases with the firm's leverage. Furthermore, the effect is nonlinear. A stock's elasticity increases with leverage at an increasing rate, so the effect should be more pronounced for more distressed firms.

Prime candidates for anomaly status are therefore firm-specific variables that are strongly related to leverage and distress in the cross-section. Previous research suggests that size (market value of equity) and book-to-market equity are strong candidates. Fama and French (1992) argue that book-to-market equity is a ratio of market and book leverage measures, and they find that size and book-to-market subsume the effects of leverage (as well as the effects of the earnings-to-price ratio and market returns).

Chan and Chen (1991) provide convincing evidence that market value of equity (ME) is strongly related to a firm's financial health. They document that two-thirds of firms in the bottom ME quintile fell there from higher quintiles, and that only 14 percent of the firms in the bottom ME quintile were originally listed there in the previous ten years. Bottom quintile firms have poorer operating results and higher interest expense than their higher-quintile same-industry counterparts; over half of the firms that cut dividends by 50 percent or more in the last year appear in the bottom quintile.

The relationship between book-to-market and financial distress has generated considerable attention. Fama and French (1995) document that high book-to-market firms show deteriorating profitability for five years before ranking, while low book-to-market firms show improving profitability over the same period. Moreover, high book-to-market firms are less profitable than low book-to-market firms for both the five years prior to ranking and the four years after. Similarly, Loughran (1997) finds that the book-to-market effect is predominantly attributed to firms with poor operating performance in the portfolio formation period, and that the firms that populate the extreme (high) book-to-market portfolio tend to be the firms that experience distressed delistings. Griffin and Lemmon (2002) present a measure of financial distress that is strongly correlated with how individual stocks load on the Fama and French (1993) HML factor. Gutierrez (2001) shows that bond ratings decline with firm book-to-market ratios. Results in Fama and French (1997) indicate that distressed industries load heavily on HML, and that the changes in these loadings over time correspond to changes in industry health.

Our model predicts that firm leverage and financial distress will capture the convex beta estimation errors induced by the use of an equity-only market proxy. The empirical research suggests that size and book-to-market may provide the same service. Whether using direct estimates of firm leverage and distress provides an economically meaningful description of returns (relative to size and book-to-market) is an empirical issue.

III. Empirical Analysis

It is well known that the unconditional CAPM performs poorly in explaining the cross-section of average returns in every stock market studied; there appears to be little relationship in the cross-section between return and proxy beta, but there is common variation in return associated with size and book-to-market. The shared feature of all the analyses is use of an equity-composite proxy for the market portfolio. Our theoretical model suggests that, at least to the extent that size and book-to-market (or loadings on SMB and HML) are instruments for leverage, these joint findings are consistent with a single-factor CAPM equilibrium.

Hence our empirical question is whether there is common variation in return associated with leverage and financial distress after controlling for proxy beta risk, and, if so, whether this accounts for a meaningful portion of the size and book-to-market effects. The central question therefore concerns variation in return *across* assets, so a cross-sectional analysis is appropriate for testing the model's predictions.

The most direct way to examine the issues raised in our theoretical section would be to expand the equity market index to include debt claims. Stambaugh (1982) does exactly this, by mixing (in four different weighting schemes) the returns on several asset classes (NYSE stocks, high-grade corporate bonds, U.S. government bonds, Treasury bills, residential real estate, house furnishings, and automobiles). His well-known result is that the four different market proxies lead to identical statistical conclusions. No matter what proxy is used, a zero-beta excess return of zero is rejected, but positivity and linearity of the risk premium are not. An important finding is that the zero-beta coefficient estimate is not affected much by the choice of proxy. The results in Stambaugh (1982) certainly cast some doubt on our theoretical predictions.

Finding a comprehensive index of returns to debt claims is tricky, however. While a long time series of returns on high-grade corporate debt is readily available, return observations for low-grade bonds are difficult to come by. Moreover, returns for bank debt (which are predominantly low-grade) are generally unobservable.[14] As documented by Fama and French (1995), there appears to be substantial common variation between low-grade debt and equity returns. Investment-grade bonds, on the other hand, show little common variation with the equity market when term structure measures are included. Thus, adding only investment-grade debt to the equity market index is unlikely to provide much more information about the beta errors.

For this reason, we take a different tack. Our approach is to create portfolios based on relative leverage and relative distress, and then examine whether sensitivities to returns on these portfolios help to explain the cross-section of excess returns in the standard two-pass approach of Fama and Macbeth (1973). The test assets will be the Fama-French 25 size- and book-to-market-sorted portfolios.[15]

Empirical specification of our theoretical model requires more than a simple single measure of leverage such as a firm's debt-to-equity ratio. We care as much about relative distress as about absolute leverage ratios, because in our theoretical model the amount of beta risk missed by using the wrong proxy is a function of the elasticity of a firm's equity (which is convex with the firm's leverage). It is possible that a firm's likelihood of distress could be much higher or lower than its relative debt-to-equity ratio might suggest.

For example, a firm with substantial cash flow and few growth opportunities might find high debt levels attractive and could appear in the highest debt-to-equity categories without risking bankruptcy. Conversely, a firm in the middle leverage portfolios but with highly volatile cash flow could face substantial risk of distress. To capture this, we construct two portfolios to mimic the part of common return associated with relative leverage (based on the ratio of debt-to-equity) and the part of return associated with relative distress (based on Altman's *Z*).[16]

We generate our leverage and distress return time series $R^{D/E}$ and R^{Z} the same way that Fama and French (1993) create the SMB and HML factors. In June of each year *t*, firms are assigned to one of three book debt-to-market equity (BD/ME) portfolios based on the one-third and two-third percentile cuts determined only from the NYSE firms in the sample.

Independently and simultaneously, firms are assigned to one of two Altman's *Z* portfolios: $Z \leq 2.675$ and $Z > 2.675$.[17] Altman recommends the cutoff level of 2.675 to minimize misclassification (total type I plus type II) errors. Firms with $Z > 2.675$ are predicted to be in the healthy group, while firms with $Z \leq 2.675$ are predicted to be in the distressed group.

To be included in portfolio construction, a firm must have CRSP market equity for December of (t - 1), and data available for all the relevant COMPUSTAT items for *Z* in (t - 1). Only firms with ordinary common equity, as defined by CRSP, are used to form the leverage and distress portfolios.

One small problem arises due to COMPUSTAT data limitations. COMPUSTAT did not systematically record individual equity accounts (like paid-in capital or retained earnings) until 1963. Since construction of a *Z* portfolio in year *t* requires COMPUSTAT data in year t-1, there are very few firms that meet our requirements for inclusion in the D/E and Z portfolios in June 1963. Hence, we lose one year of data (1963); our return series begins in June 1964.

The intersection of the two sorts above results in six debt-to-equity/*Z* portfolios as of June 30 of each year. For July of *t* through June of (t + 1), the return on each portfolio is calculated as the value-weighted average return of the stocks in the portfolio (where value is the market value of equity at the beginning of the return month). In the end, we have six return series that cover the 438 months from July 1964 through December 2000.

In each month *t*, $R_t^{D/E}$ is calculated as the simple average return of the two *Z* portfolios within *D/E* portfolio three (the highly levered firms) minus the simple average return of the two *Z* portfolios within *D/E* portfolio one (the least levered firms). Similarly, R_t^{Z} is the simple average return of the three *D/E* portfolios within *Z* portfolio two (high-*Z* firms) minus the simple average return of the three *D/E* portfolios within *Z* portfolio one (the low-*Z* firms).

The two-way cutting procedure moderates the fact that *D/E* appears in the calculation of *Z*. Cutting on *D/E* reduces the cross-sectional dispersion in this variable within each *D/E* class; within each *D/E* class, the balance sheet variables become magnified in importance in determining the *Z* classification.

For the 438-month period from July 1964 to December 2000, the average returns on $R^{D/E}$ and R^{Z} are 10.6 basis points per month (t = 0.55) and 13.1 basis points per month (t = 1.03), respectively. For comparison purposes, the average market excess return is 49.4 basis points per month (t = 2.31), while the average returns on the Fama-French SMB and HML factors are 18.2 (t = 1.16) and 38.9 (t = 2.78) basis points per month, respectively. While the average monthly returns on the leverage and distress portfolios are low, it turns out that the loadings and return per unit loading on these portfolios are high.

Although it is somewhat surprising that the average return on R^{Z} is positive, this is not a new result. Fama and French (1992) provide strong evidence that cross-sectional returns are positively related to a market-based leverage measure (log of book assets to market equity) but *negatively* related to book leverage (log of book assets to book equity).[18] They interpret the book-to-market result as capturing the difference between market-imposed leverage and book leverage, since ln(*BE*/*ME*) = ln(*A*/*ME*) – ln(*A*/*BE*). Opler and Titman (1994) find a similar result. Stocks of firms with high book leverage experience very poor returns. Altman and Brenner (1981) demonstrate that low-*Z* firms perform poorly over extended periods; for a sample of December fiscal yearend firms, those with $Z < 2.675$ had significantly negative CARs for the 12- and 18-month periods starting the following March. Altman (1968) and Beaver (1966) show that bankrupt firms exhibit considerably negative market-adjusted rates of return right up to exit.

Be that as it may, the loadings on R^Z provide risk premiums consistent with the size and book-to-market effects. It turns out that high book-to-market firms and small firms load positively on R^Z (providing an additional premium over and above proxy beta risk), while low book-to-market firms and large firms load negatively on it (reducing the predicted return from proxy beta). The *Z* factor is determined predominantly by book measures of leverage, and while there is no available explanation for this negative relationship between book leverage and return, we believe that R^Z is capturing the book leverage effect described in Altman and Brenner (1981), Fama and French (1992), and Opler and Titman (1994).

Table I shows some characteristics of the six *D/E-Z* portfolios. Within the high-*Z* class, the typical negative size effect and positive book-to-market effect can be seen. The same *cannot* be said for the low-*Z* (distressed) class, however, as the high-leverage (and high book-to-market) firms in that class underperform the low-leverage (and low book-to-market) firms. While the average number of firms in the low D/E-low *Z* portfolio appears to be small (162), this portfolio is actually larger than the Fama-French large size-high book-to-market portfolio (which over the same period averages 136 firms).

[Insert Table I here]

Table II documents the time series relationships between our leverage portfolios and market excess return (Panel A), SMB (Panel B), and HML (Panel C). The time series variation in $R^{D/E}$ and R^Z explains over 50 percent of the time series variation in HML, but only seven percent of the time series variation in SMB.

[Insert Table II here]

As we document below, the power of the Fama-French (1993) three-factor model in the cross-section is provided predominantly by HML. It is quite possible that HML is a priced factor (or an instrument for a state variable) and that our $R^{D/E}$ and R^Z series are nothing more than good proxies for it. Of course, it is also conceivable that what is captured by HML in the cross-section could be the effect predicted in our theoretical model.

It is interesting to note that R^Z is negatively related to HML but positively related to SMB (after controlling for $R^{D/E}$). That is, in months when distressed (low-Z) firms outperform healthy ones, one would predict that high book-to-market firms would outperform low book-to-market companies, but at the same time that large firms would outperform small ones. Note that SMB and HML exhibit a strong negative correlation $(\rho = -0.29)$ over this 438-month time period, however.[19] Finally, R^Z is unrelated to the part of SMB that is orthogonal to HML.[20]

As we have noted, we use the familiar two-pass methodology of Fama and MacBeth (1973). The first pass consists of 25 multivariate time series regressions (one for each of the Fama-French 25 portfolios). The slope coefficient estimates from the first pass are then used as the explanatory variables in a series of 438 cross-sectional regressions (the second pass) that take the average excess return of size/book-to-market portfolio i ($i = 1, \ldots, 25$) in month t ($t = 1, \ldots, 438$) as the independent variable. The time series averages of the estimated monthly intercept and slope coefficients become the intercept and slope estimates for the overall cross-sectional model, and the standard errors of the overall coefficient estimates are calculated from the time series standard deviation of the monthly estimates.

Cochrane (2001) shows that the resulting *t*-statistics are corrected for cross-sectional correlation in the error terms but not for time series correlation in the residuals, since the dependent variables in the second pass are not fixed but rather are generated in the first pass regression. Shanken (1992) provides a correction for the standard errors, which involves a multiplicative term (which is generally very small) and an additive term (which can be large, depending on the factor variance). We present the traditional Fama-MacBeth *t*-statistics and *p*-values, along with the Shanken (1992) corrected *t*-statistics and *p*-values.[21]

Table III presents the standard single-factor model and the Fama-French three-factor model for purposes of comparison with our model, which is in Table IV. Panel A of Table III shows the familiar failure of the single-beta model. For the 438 months from July 1964 through December 2000, the estimated return per unit beta risk is a negative 55 basis points per month. Panel B of Table III demonstrates the dramatic improvement in explanatory power that is gained by adding the SMB and HML factors to the analysis. The estimated return per unit of SMB risk is not significantly different from zero, although the cross-sectional variation in $\hat{\beta}_i^{SMB}$ from the first pass is large (nearly 1.7 units). The variation in $\hat{\beta}_i^{HML}$ in the first pass is also large (1.32 units).

[Insert Table III here]

Panel A of Table IV presents our primary empirical analysis of the model suggested by the theoretical analysis in Section II. In the Fama-MacBeth regression

$$RP_{i,t} - RF_t = \gamma_0 + \gamma_{MKT}\hat{\beta}_i^{MKT} + \gamma_{D/E}\hat{\beta}_i^{D/E} + \gamma_Z\hat{\beta}_i^Z + \varepsilon_{i,t},$$

loadings on the *D/E* and *Z* portfolios are strongly related to average excess return in the cross-section. The estimated return per unit $\beta^{D/E}$ risk is 165 basis points per month, which is over three standard errors away from zero. Furthermore, the cross-sectional dispersion in estimated $\hat{\beta}_i^{D/E}$ from the first pass is large – about .84 beta units – giving a spread in predicted return from sensitivities to this portfolio of 139 basis points per month. The estimated return per unit β^Z risk is similarly large (102 basis points per month) and statistically significant (corrected-*t* = 2.33). With the cross-sectional spread in estimated $\hat{\beta}_i^Z$ of .99, sensitivity to the *Z* (distress) portfolio provides a spread in predicted returns of about 102 basis points per month.

[Insert Table IV here]

In Figure 1, we construct fitted-versus-actual average monthly excess returns (the time series average of the 438 monthly return observations) for the three cross-sectional models. Panel

A illustrates the poor performance of the single-factor model, while Panel B documents the improvement realized by adding the SMB and HML factors. The additional improvement achieved by using sensitivity to the leverage and distress portfolios is exhibited by the tightness of the plotted points to the characteristic line in Panel C.

[Insert Figure 1 here]

Next, we remove the effects of the $R_t^{D/E}$ and R_t^Z from the R_t^{SMB} and R_t^{HML} factors. The portion of the SMB factor orthogonal to the leverage and distress portfolio returns, $R_t^{SMB\perp}$, is the estimated intercept plus the monthly residual from the time-series regression

$$R_t^{SMB} = \alpha_0 + \alpha_1 R_t^{D/E} + \alpha_2 R_t^Z + \varepsilon_t \,,$$

and the portion of the HML factor orthogonal to the leverage and distress portfolio returns, $R_t^{HML\perp}$, is the estimated intercept plus the monthly residual from the time-series regression

$$R_t^{HML} = \alpha_0 + \alpha_1 R_t^{D/E} + \alpha_2 R_t^Z + \varepsilon_t \,.$$

Panel B of Table IV presents the results of an expanded cross-sectional model that includes the market factor, the two leverage portfolios, and the orthogonalized SMB and HML factors. Adding the orthogonalized SMB and HML factors provides little extra benefit. After removing the variation common with the leverage and distress portfolios, the estimated return per unit HML risk is less than one standard error above zero. Furthermore, the return per unit of SMB risk is now *negative* (but statistically insignificant at the 10 percent level). In other words, after removing the common effects of relative leverage and distress, there is a statistically insignificant large-firm premium.

One possibility here is that we've simply stumbled across two portfolios (leverage and distress) that are multifactor efficient in the sense of Fama (1996). If SMB and HML are also multifactor efficient, as suggested by Fama and French (1996), we should expect that the portions of $R_t^{D/E}$ and R_t^Z orthogonal to SMB and HML would provide no additional power when introduced into the Fama-French three-factor model.

Panel C of Table IV indicates that this is not the case. In Panel C of Table IV, $R_t^{D/E\perp}$ is the estimated intercept plus the monthly residual from the regression

$$R_t^{D/E} = \alpha_0 + \alpha_1 R_t^{SMB} + \alpha_2 R_t^{HML} + \varepsilon_t ,$$

and $R_t^{Z\perp}$ is the estimated intercept plus the monthly residual from the regression

$$R_t^{Z} = \alpha_0 + \alpha_1 R_t^{SMB} + \alpha_2 R_t^{HML} + \varepsilon_t .$$

The estimated returns per unit of $R_t^{D/E\perp}$ and $R_t^{Z\perp}$ risk are 184 basis points per month (which is over 2.5 standard errors above zero) and 154 basis points per month (over 3.2 standard errors above zero), respectively. In the first-pass regressions, the estimated $\beta^{D/E\perp}$ range from -0.15 to 0.21 and the estimated $\beta^{Z\perp}$ range from -0.25 to 0.34. In other words, the spread in predicted returns due to sensitivity to the parts of our leverage/distress portfolios that are orthogonal to SMB and HML are 66 basis points per month for the leverage portfolio and 91 basis points per month for the distress portfolio.

Time series tests paint a different picture. Table V presents the intercepts and associated *t*-statistics as well as adjusted R^2s from the first-pass estimations of the models in Table III and Panels A and B of Table IV. Included in Table V are *F*-statistics derived in Gibbons, Ross, and Shanken (1989), which test the null hypothesis that the 25 intercepts are jointly zero.

[Insert Table V here]

The results in Table V show that even though our leverage measures help explain average return in the cross-section, they do not help matters much in the time series. Adding $R_t^{D/E}$ and R_t^{Z} to the single-factor model does not change the magnitude or the significance of the intercepts much. This is not entirely surprising. In the time series model, all factor risk premiums are estimated directly by the factor's time series mean return, and all zero-beta excess returns are predicted to be zero. The very low average returns on our leverage and distress portfolios (about 10 and 13 basis points per month, respectively) will therefore lead to large pricing errors.

The high return on our low-Z, low-leverage portfolio (see Table I) is both problematic and symptomatic. It is out of line with the predictions of our model, and it substantially affects the time series average returns of both the leverage and the distress portfolios.

SMB and HML, on the other hand, are very important in time series estimations. One possibility is that relative distress is indeed a priced factor, and that SMB and HML pick it up better than do our leverage and distress portfolios.

Another possibility is that SMB and HML are capturing the effects of other state variables unrelated to relative leverage and distress. Brennan, Wang, and Xia (2002), for example, develop an intertemporal CAPM with mean-reverting state variables, and show that the prices of the portfolios used to form SMB and HML incorporate information about the changing investment opportunity set, so the loadings on SMB and HML could measure sensitivities to the state variables.

A third possibility, derived in Berk, Green, and Naik (1999), is that a firm's book-to-market ratio conveys information about its changing risk (relative to its asset base), and that its size encapsulates the importance of its growth options relative to its assets in place. Or, as in Jagannathan and Wang (1996), size and book-to-market could summarize the risk of time-varying betas.

IV. Concluding Remarks

Our primary contribution is to provide a theoretical framework for explanatory variables that are helpful in explaining the cross-section of returns. Betas calculated against equity-only proxies will be understated, and since the missing beta risk will be systematically related to relative leverage and relative distress, factor portfolios formed on variables statistically related to relative leverage and relative distress should improve on the explanatory power of the single-factor model in cross-sectional studies of average return. We thus provide a theoretical rationale for the famous Fama and French (1993) three-factor model.

But more important, our theory provides the foundation for a better empirical model. To cite one example, we estimate a three-factor model that incorporates the market return along with the returns on portfolios formed on relative leverage and relative distress. We find that, in the cross-section, this model outperforms the Fama and French (1993) three-factor model in explaining the returns on the 25 size- and book-to-market-sorted portfolios.

Our work provides a consistent potential explanation for a wide array of puzzles. First, in a CAPM world, the cross-sectional dispersion of proxy betas will not reflect the cross-sectional dispersion of true equity betas as long as proxy betas are calculated against a market proxy that neglects the economy's debt claims. Since the understatement of a firm's proxy equity beta is directly related to its leverage, while its true equity beta also increases with leverage, we should expect too little cross-sectional dispersion in proxy betas. This is one of the most common findings in studies of average return.

Second, non-zero pricing errors should be expected, simply because some of the economy's common risk can be measured only through each equity's covariance with the assets not included in the market proxy.

Third, average return should be related to relative leverage and relative distress in addition to proxy beta, as long as the empirical proxy for the market index excludes the economy's debt claims. Complex stories that predict an equilibrium premium for relative distress are unnecessary.

Fourth, variables that correlate strongly with leverage (such as size, book-to-market, and earnings-to-price) should not be considered anomalies, but rather should be expected regularities in a CAPM world, as long as the market proxy does not incorporate the economy's debt claims. In other words, we should expect to see size effects and book-to-market effects in any dataset. Indeed, one of the most notable developments in the literature is the pervasive finding of these effects in virtually every market studied; see Chan, Hamao, and Lakonishok (1991), Fama and French (1998), and Rouwenhorst (1999).

A fifth prediction of our model is that as firms change their capital structures, proxy betas calculated against the equity market index will not completely reflect the leverage-induced change in the risk of the equity. In a study of firms that undertake highly leveraged recapitalizations, Kaplan and Stein (1990) find that average equity betas rise from 1.01 before the recap to only about 1.40 afterward, even though average debt rates rise from 25 percent of total capitalization to over 80 percent.

Similarly, firms that have experienced severe deterioration in the market value of their equities will be highly levered, and their equity betas will understate their true systematic risk. If portfolios are formed on these long-run losers, and returns are compared to those on similarly formed portfolios of long-run winners, superior ex-post performance should be expected in the loser portfolio after controlling for proxy beta risk (simply because the proxy betas of the long-run losers will be severely understated) – which is the exact result found in DeBondt and Thaler (1985).

A final prediction of the model is that abnormal drift should be detected after controlling for proxy beta risk in studies of long-run returns if the event sample firms have different leverage profiles from the control sample firms. Eckbo, Masulis, and Norli (2000) show that the long-run negative drift following seasoned equity offerings can be explained by leverage differences between the SEO sample (with very low leverage) and the control sample. Eckbo and Norli (2002) find the exact same result in post-IPO returns.

Appendix

Proof of Lemma 1: We must find the sign of $\frac{\partial \hat{\beta}_i^E}{\partial F_i}$. This becomes easier if we break the proxy beta of the assets into parts:

$$\hat{\beta}_i^E = \frac{\sigma_{i,E}}{\sigma_E^2} = \frac{1}{\sigma_E^2}\sum_j \frac{S_j}{E}\sigma_{i,Sj} = \frac{1}{E\sigma_E^2}\sum_j S_j \eta_{Sj}\sigma_{i,j} = \frac{1}{E\sigma_E^2}\sum_j V_j N\left(d_1\right)_j \sigma_{i,j}$$

because $S_j \eta_{Sj} = V_j N\left(d_1\right)_j$ by the definition of the elasticity of an equity claim. So

$$\frac{\partial \hat{\beta}_i^E}{\partial F_i} = \frac{E\sigma_E^2 \cdot \left[\partial\left(\sum_j V_j N\left(d_1\right)_j \sigma_{ij} \right) \Big/ \partial F_i \right] - \sum_j V_j N\left(d_1\right)_j \sigma_{ij} \left[E\frac{\partial \sigma_E^2}{\partial F_i} + \sigma_E^2 \frac{\partial E}{\partial F_i} \right]}{\left(E\sigma_E^2\right)^2}.$$

Evaluating the individual pieces:

$$\left[\partial\left(\sum_j V_j N\left(d_1\right)_j \sigma_{ij} \right) \Big/ \partial F_i \right] = V_i N'\left(d_1\right)_i \sigma_i^2 \quad \text{where } N'\left(d_1\right)_i = \frac{-Z\left(d_1\right)_i}{F_i \sigma_i \sqrt{T}} < 0$$

and where $Z(\bullet)$ is the unit normal density, and

$$\frac{\partial E}{\partial F_i} = \frac{\partial S_i}{\partial F_i} = -e^{-rT} N\left(d_2\right)_i .$$

To evaluate $\frac{\partial \sigma_E^2}{\partial F_i}$, we start by noting that

$$\sigma_E^2 = \sum_j \sum_k \frac{S_j}{E}\frac{S_k}{E}\sigma_{Sj,Sk} = \frac{1}{E^2}\sum_j \sum_k S_j S_k \eta_{Sj}\eta_{Sk}\sigma_{j,k} = \frac{1}{E^2}\sum_j \sum_k V_j V_k N\left(d_1\right)_j N\left(d_1\right)_k \sigma_{j,k} .$$

So

$$\frac{\partial \sigma_E^2}{\partial F_i} = \frac{E^2\left(2V_i N'\left(d_1\right)_i \cdot \sum_j V_j N\left(d_1\right)_j \sigma_{i,j} \right) - \sum_j \sum_k V_j V_k N\left(d_1\right)_j N\left(d_1\right)_k \sigma_{j,k} \left(2E\frac{\partial E}{\partial F_i} \right)}{\left(E^2\right)^2}$$

$$=\frac{1}{E}\left(2V_iN'\left(d_1\right)_i\cdot\frac{\sum_j V_jN\left(d_1\right)_j\sigma_{i,j}}{E}\right)-\frac{1}{E^2}\frac{\sum_j\sum_k V_jV_kN\left(d_1\right)_j N\left(d_1\right)_k\sigma_{j,k}}{E^2}\left(-2Ee^{-rT}N\left(d_2\right)_i\right)$$

$$=\frac{1}{E}2V_iN'\left(d_1\right)_i\sigma_{i,E}+\sigma_E^2\frac{1}{E}2e^{-rT}N\left(d_2\right)_i\quad .$$

Therefore,

$$\frac{\partial\hat{\beta}_i^E}{\partial F_i}=\frac{1}{E\sigma_E^2}\left[V_iN'\left(d_1\right)_i\sigma_i^2\right]$$
$$-\frac{1}{E\sigma_E^2}\frac{\sum_j V_jN\left(d_1\right)_j\sigma_{ij}}{E\sigma_E^2}\left[E\left(\frac{1}{E}2V_iN'\left(d_1\right)_i\sigma_{i,E}+\frac{1}{E}\sigma_E^2 2e^{-rT}N\left(d_2\right)_i\right)+\sigma_E^2\left(-e^{-rT}N\left(d_2\right)_i\right)\right]$$

$$=\frac{1}{E\sigma_E^2}V_iN'\left(d_1\right)_i\sigma_i^2-\frac{1}{E\sigma_E^2}\frac{\sigma_{i,E}}{\sigma_E^2}\left[2V_iN'\left(d_1\right)_i\sigma_{i,E}+\sigma_E^2e^{-rT}N\left(d_2\right)_i\right]$$

$$=\frac{1}{E\sigma_E^2}V_iN'\left(d_1\right)_i\sigma_i^2-\frac{1}{E\sigma_E^2}\hat{\beta}_i^E 2V_iN'\left(d_1\right)_i\sigma_{i,E}-\frac{1}{E\sigma_E^2}\hat{\beta}_i^E\sigma_E^2e^{-rT}N\left(d_2\right)_i .$$

(Notice here that if $\hat{\beta}_{Si}^E=0$, the sign of the derivative is strictly negative because the first term is negative and $\hat{\beta}_{Si}^E=0\Rightarrow\eta_{Si}\hat{\beta}_i^E=0\Rightarrow\hat{\beta}_i^E=0$.) Continuing:

$$\frac{\partial\hat{\beta}_i^E}{\partial F_i}=\frac{1}{E\sigma_E^2}\left[V_i\left(\frac{-Z\left(d_1\right)_i}{F_i\sigma_i\sqrt{T}}\right)\sigma_i^2-\hat{\beta}_i^E 2V_i\left(\frac{-Z\left(d_1\right)_i}{F_i\sigma_i\sqrt{T}}\right)\sigma_{i,E}-\hat{\beta}_i^E\sigma_E^2e^{-rT}N\left(d_2\right)_i\right],$$

because $V_iZ\left(d_1\right)_i=F_ie^{-rT}Z\left(d_2\right)_i$ we can write this as:

$$=\frac{1}{E\sigma_E^2}\left[-V_i\left(\frac{Z\left(d_2\right)_ie^{-rT}}{V_i\sigma_i\sqrt{T}}\right)\sigma_i^2+\hat{\beta}_i^E 2V_i\left(\frac{Z\left(d_2\right)_ie^{-rT}}{V_i\sigma_i\sqrt{T}}\right)\sigma_{i,E}-\hat{\beta}_i^E\sigma_E^2e^{-rT}N\left(d_2\right)_i\right]$$

$$=\frac{e^{-rT}}{E\sigma_E^2}\left[-\left(\frac{Z\left(d_2\right)_i}{\sigma_i\sqrt{T}}\right)\sigma_i^2+\hat{\beta}_i^E 2\left(\frac{Z\left(d_2\right)_i}{\sigma_i\sqrt{T}}\right)\sigma_{i,E}-\hat{\beta}_i^E\sigma_E^2N\left(d_2\right)_i\right]$$

$$=\frac{e^{-rT}}{E\sigma_E^2}\left[-\left(\frac{Z(d_2)_i}{\sigma_i\sqrt{T}}\right)\sigma_i^2+\frac{\sigma_{i,E}}{\sigma_E^2}2\left(\frac{Z(d_2)_i}{\sigma_i\sqrt{T}}\right)\sigma_{i,E}-\frac{\sigma_{i,E}}{\sigma_E^2}\sigma_E^2 N(d_2)_i\right]$$

$$=\frac{e^{-rT}}{E\sigma_E^2}\left[-\left(\frac{Z(d_2)_i}{\sigma_i\sqrt{T}}\right)\sigma_i^2+\frac{\sigma_{i,E}^2}{\sigma_E^2}2\left(\frac{Z(d_2)_i}{\sigma_i\sqrt{T}}\right)-\sigma_{i,E}N(d_2)_i\right]$$

$$=\frac{e^{-rT}\sigma_i^2}{E\sigma_E^2}\left[-\left(\frac{Z(d_2)_i}{\sigma_i\sqrt{T}}\right)+\rho_{i,E}^2 2\left(\frac{Z(d_2)_i}{\sigma_i\sqrt{T}}\right)-\frac{\sigma_E\rho_{i,E}}{\sigma_i}N(d_2)_i\right]$$

$$=\frac{e^{-rT}\sigma_i}{E\sigma_E^2\sqrt{T}}\left[-Z(d_2)_i+2\rho_{i,E}^2 Z(d_2)_i-\sigma_E\rho_{i,E}N(d_2)_i\sqrt{T}\right].$$

So the sign of $\frac{\partial\hat{\beta}_i^E}{\partial F_i}$ is determined by the sign of the quantity inside the bracket, which depends on the sign of the proxy beta. By assumption the proxy beta $\hat{\beta}_{Si}^E > 0$.

$\hat{\beta}_{Si}^E > 0 \Rightarrow \hat{\beta}_i^E > 0 \Rightarrow \sigma_{i,E} > 0 \Rightarrow \rho_{i,E} > 0$, so in this case

$$\frac{\partial\hat{\beta}_i^E}{\partial F_i} < 0 \text{ if and only if}$$

$$Z(d_2)_i\left[2\rho_{i,E}^2-1\right] < N(d_2)_i\,\sigma_E\rho_{i,E}\sqrt{T}$$

or

$$\frac{2\rho_{i,E}^2-1}{\sigma_E\rho_{i,E}\sqrt{T}} < \frac{N(d_2)_i}{Z(d_2)_i}.$$

Finally, we note that

$$\rho_{i,E}=\frac{\sigma_{i,E}}{\sigma_i\sigma_E}=\frac{\eta_{Si}\sigma_{i,E}}{\eta_{Si}\sigma_i\sigma_E}=\frac{\sigma_{Si,E}}{\sigma_{Si}\sigma_E}=\rho_{Si,E}.$$

That is, an equity's correlation with the proxy portfolio is equal to the underlying asset correlation with the proxy. This makes the condition

$$\frac{2\rho_{Si,E}^2 - 1}{\sigma_E \rho_{Si,E}\sqrt{T}} < \frac{N(d_2)_i}{Z(d_2)_i}.$$

Q.E.D.

Corollaries to Proposition 1: The proofs are straightforward from the proofs of the lemma and Proposition 1, and hence are not provided here.

COROLLARY 1: *If the equity's proxy beta is positive, but its true beta is negative (that is, $\beta_{Si} < 0$ but $\hat{\beta}_{Si}^E > 0$), then the ratio* $\frac{\beta_{Si}}{\hat{\beta}_{Si}^E}$

- increases *with the firm's leverage if* $\rho_{i,E} > \sqrt{.5}$ *and the firm is severely distressed;*
- decreases *with the firm's leverage if either*
  A. $\rho_{i,E} < \sqrt{.5}$ *or*
  B. $\rho_{i,E} > \sqrt{.5}$ *but the firm is not severely distressed.*

COROLLARY 2: *If the equity's proxy and true betas are both negative (that is,* $\beta_{Si} < 0$ *and* $\hat{\beta}_{Si}^E < 0$*), then the ratio* $\frac{\beta_{Si}}{\hat{\beta}_{Si}^E}$

- increases *with the firm's leverage if either*
  A. $\rho_{i,E} < -\sqrt{.5}$ *or*
  B. $-\sqrt{.5} < \rho_{i,E} < 0$ *but the firm is not severely distressed;*
- decreases *with the firm's leverage if* $-\sqrt{.5} < \rho_{i,E} < 0$ *and the firm is severely distressed.*

COROLLARY 3: *If the equity's proxy beta is negative, but its true beta is positive, (that is,* $\hat{\beta}_{Si}^E < 0$ *and* $\beta_{Si} > 0$*), then the ratio* $\frac{\beta_{Si}}{\hat{\beta}_{Si}^E}$

- increases *with the firm's leverage if* $-\sqrt{.5} < \rho_{i,E} < 0$ *and the firm is severely distressed.*
- decreases *with the firm's leverage if either*
  A. $\rho_{i,E} < -\sqrt{.5}$ *or*
  B. $-\sqrt{.5} < \rho_{i,E} < 0$ *but the firm is not severely distressed.*

PROPOSITION 3: *If* $\sigma_{Si,D} > 0$*, then* $\frac{\partial(\Omega\hat{\beta}_{Si}^{D})}{\partial F_i} > 0.$

Proof:

$$\Omega\hat{\beta}_{Si,D}^{D} = \frac{D\sigma_D^2}{M\sigma_M^2}\frac{\sigma_{Si,D}}{\sigma_D^2} = \frac{D}{M\sigma_M^2}\eta_{Si}\sum_{j=1}^{N}\frac{B_j}{D}\sigma_{i,Bj}$$

$$= \frac{D\eta_{Si}}{M\sigma_M^2}\sum_{j=1}^{N}\frac{B_j}{D}\eta_{Bj}\sigma_{i,j}$$

$$= \frac{\eta_{Si}}{\sigma_M^2}\sum_{j=1}^{N}\frac{B_j}{M}\frac{V_j}{B_j}N(-d_1)_j\sigma_{i,j}$$

$$= \frac{\eta_{Si}}{\sigma_M^2}\sum_{j=1}^{N}\frac{V_j}{M}N(-d_1)_j\sigma_{i,j}$$

So $$\frac{\partial(\Omega\hat{\beta}_{Si}^{D})}{\partial F_i} = \frac{1}{\sigma_M^2}\left[\eta_{Si}\left(\frac{V_i}{M}\sigma_i^2 Z(-d_1)_i\frac{\partial(-d_1)_i}{dF_i}\right) + \left(\sum_{j=1}^{N}\frac{V_j}{M}N(-d_1)_j\sigma_{i,j}\right)\frac{\partial\eta_{Si}}{\partial F_i}\right],$$

where $Z(\bullet)$ is the unit normal probability density. We know that $\sigma_M^2 > 0$, $\eta_{Si} > 0$, $\frac{V_i}{M} > 0$, $\sigma_i^2 > 0$, $Z(\bullet) > 0$, and $\frac{\partial\eta_{Si}}{\partial F_i} > 0$.[22] Further:

$$(-d_1)_i = \frac{-\ln V_i + \ln F_i - (r_F + .5\sigma_i^2)T}{\sigma_i\sqrt{T}} \text{ so } \frac{\partial(-d_1)_i}{\partial F_i} = \frac{1}{F_i\sigma_i\sqrt{T}} > 0.$$

Finally:

$$\sum_{j=1}^{N}\frac{V_j}{M}N(-d_1)_j\sigma_{i,j} = \sum_{j=1}^{N}\frac{D}{M}\frac{B_j}{D}\frac{V_j}{B_j}N(-d_1)_j\sigma_{i,j}$$

$$= \frac{D}{M}\sum_{j=1}^{N}\frac{B_j}{D}\eta_{Bj}\sigma_{i,j}$$

$$= \frac{D}{M}\sum_{j=1}^{N}\frac{B_j}{D}\sigma_{i,Bj} = \frac{D}{M}\sigma_{i,D} = \frac{D}{M}\frac{1}{\eta_{Si}}\sigma_{Si,D}$$

and thus takes the sign of $\sigma_{Si,D}$.

Q.E.D.

PROPOSITION 4: $\dfrac{\partial(\Omega\hat{\beta}_i^D)}{\partial F_i} > 0.$

Proof: Using the same derivation as in Proposition 3:

$$\Omega\hat{\beta}_i^D = \frac{1}{\sigma_M^2}\sum_{j=1}^{N}\frac{V_j}{M}N\left(-d_1\right)_j \sigma_{i,j}$$

So

$$\frac{\partial(\Omega\hat{\beta}_i^D)}{\partial F_i} = \frac{1}{\sigma_M^2}\left[\frac{V_i}{M}\sigma_i^2 Z\left(-d_1\right)_i \frac{\partial\left(-d_1\right)_i}{dF_i}\right] > 0.$$

Q.E.D.

REFERENCES

Altman, Edward I., 1968, Financial ratios, discriminant analysis and the prediction of corporate bankruptcy, *Journal of Finance* 23, 589–609.

Altman, Edward I., and Menachem Brenner, 1981, Information effects and stock market response to signs of firm deterioration, *Journal of Financial and Quantitative Analysis* 16, 35–51.

Beaver, William H., 1966, Financial ratios as predictors of failure, *Journal of Accounting Research* 4, 71–111.

Berk, Jonathan B., Richard C. Green, and Vasant Naik, 1999, Optimal investment, growth options and security returns, *Journal of Finance* 54, 1553-1607.

Black, Fischer, and Myron Scholes, 1973, The pricing of options and corporate liabilities, *Journal of Political Economy* 81, 637-659.

Brennan, Michael, J., Ashley W. Wang, and Yihong Xia, 2002, Intertemporal capital asset pricing and the Fama-French three-factor model, University of California–Los Angeles, Working paper.

Chan, K. C., and Nai-Fu Chen, 1991, Structural and return characteristics of small and large firms, *Journal of Finance* 46, 1467-1484.

Chan, Louis K., Yasushi Hamao, and Josef Lakonishok, 1991, Fundamentals and stock returns in Japan, *Journal of Finance* 43, 309-325.

Cochrane, John H., 2001, *Asset Pricing* (Princeton University Press, Princeton, NJ).

DeBondt, Werner F.M., and Richard Thaler, 1985, Does the stock market overreact? *Journal of Finance* 40, 793-805.

Dybvig, Phillip H., and Jonathan E. Ingersoll, Jr., 1982, Mean-variance theory in complete markets, *Journal of Business* 55, 233-251.

Eckbo, B. Espen, Ronald W. Masulis, and Øyvind Norli, 2000, Seasoned public offerings: resolution of the "new issues" puzzle, *Journal of Financial Economics* 56, 251-291.

Eckbo, B. Espen, and Øyvind Norli, 2002, Liquidity risk, leverage and ong-run IPO returns, Dartmouth College, Working paper.

Fama, Eugene, F., 1996, Multifactor portfolio efficiency and multifactor asset pricing, *Journal of Financial and Quantitative Analysis* 31, 441-465.

Fama, Eugene, F., and Kenneth R. French, 1992, The cross-section of expected stock returns, *Journal of Finance* 67, 427-465.

Fama, Eugene, F., and Kenneth R. French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3-56.

Fama, Eugene, F., and Kenneth R. French, 1995, Size and book-to-market factors in earnings and returns, *Journal of Finance* 50, 131-155.

Fama, Eugene, F., and Kenneth R. French, 1996, Multifactor explanations of asset pricing anomalies, *Journal of Finance* 51, 55-84.

Fama, Eugene, F., and Kenneth R. French, 1997, Industry costs of equity, *Journal of Financial Economics* 43, 153-193.

Fama, Eugene, F., and Kenneth R. French, 1998, Value versus growth: The international evidence, *Journal of Finance* 53, 1975 – 1999.

Fama, Eugene F., and James D. MacBeth, 1973, Return, risk and equilibrium: Empirical tests, *Journal of Political Economy* 81, 607 – 636.

Galai, Dan, and Ronald W. Masulis, 1976, The option pricing model and the risk factor of stocks, *Journal of Financial Economics* 3, 53-81.

Gibbons, Michael R., Stephen A. Ross, and Jay Shanken, 1989, A test of the efficiency of a given portfolio, *Econometrica* 57, 1121-1152.

Gonzalez, Nestor, Robert Litzenberger, and Jacques Rolfo, 1977, On mean variance models of capital structure and the absurdity of their predictions, *Journal of Financial and Quantitative Analysis* 12, 165-179.

Griffin, John M., and Michael L. Lemmon, 2002, Book-to-market equity, distress risk, and stock returns, *Journal of Finance* 57, 2317-2336.

Gutierrez, Roberto C., Jr., 2001, Book-to-market equity and size and the segmentation of the stock and bond markets, Texas A&M University, Working paper.

Harrison, J. Michael, and David M. Kreps, 1979, Martingales and arbitrage in multiperiod securities markets, *Journal of Economic Theory* 20, 381-408.

Ingersoll, Jonathan E., Jr., 1987, *Theory of Financial Decision Making* (Rowman & Littlefield, Savage, MD).

Jagannathan, Ravi, and Zhenyu Wang, 1996, The conditional CAPM and the cross-section of expected stock returns, *Journal of Finance* 51, 3-53.

Kaplan, Steven N., and Jeremy C. Stein, 1990, How risky is the debt in highly leveraged transactions? *Journal of Financial Economics* 27, 215-245.

Loughran, Tim, 1997, Book-to-market across firm size, exchange, and seasonality: Is there an effect? *Journal of Financial and Quantitative Analysis* 32, 249-268.

Mayers, David, 1972, Nonmarketable assets and capital market equilibrium under uncertainty, in Michael C. Jensen, ed.: *Studies in the Theory of Capital Markets* (Praeger Publishers, New York).

Merton, Robert C., 1975, Theory of finance from the perspective of continuous time, *Journal of Financial and Quantitative Analysis* 10, 659-674.

Opler, Tim C., and Sheridan D. Titman, 1994, Financial distress and corporate performance, *Journal of Finance* 49, 1015-1040.

Rouwenhorst, K. Geert, 1999, Local return factors and turnover in emerging stock markets, *Journal of Finance* 54, 1439-1464.

Shanken, Jay, 1992, On the estimation of beta pricing models, *Review of Financial Studies* 5, 1-34.

Stambaugh, Robert F., 1982, On the exclusion of assets from tests of the two-parameter model, *Journal of Financial Economics* 10, 237-268.

Treacy, William F., and Mark Carey, 1998, Credit risk rating at large U.S. banks, *Federal Reserve Bulletin* 84, 897-921.

Table I
Characteristics of Portfolios that Generate Leverage and Distress Factors

In June of each year t, all stocks are assigned to one of three debt-to-equity (D/E) portfolios using breakpoints determined only by NYSE firms in the sample. We define D/E as COMPUSTAT book value of debt for fiscal year ending in t-1 divided by market equity in December of year t - 1. Independently, all stocks are assigned to one of two distress portfolios: Those with Altman's Z above 2.675, and those below. We compute $Z = 1.2WC/TA + 1.4RE/TA + 3.3EBIT/TA + 0.6ME/BD + 1.0S/TA$, where WC is net working capital, TA is total book assets, RE is retained earnings, $EBIT$ is earnings before interest and taxes, ME is market value of equity, BD is book value of total liabilities, and S is sales revenue. We collect WC, TA, RE, $EBIT$, BD, and S from COMPUSTAT at fiscal yearend t-1; ME is from CRSP at the close of December in t-1. Six D/E–Z portfolios are then constructed from the intersections of the three D/E and two Z, and the monthly returns on each of these six portfolios are the value-weighted monthly returns of the firms. Size is the natural log of market capitalization (in thousands) at the end of June of year t; BE/ME is COMPUSTAT book equity at fiscal yearend t - 1 divided by market capitalization at the end of December of t - 1.

Average Monthly Return (%)			Average Number of Firms		
	Altman's Z			Altman's Z	
D/E	Low	High	D/E	Low	High
Low	1.23	1.01	Low	162	1,366
2	1.00	1.28	2	382	497
High	1.06	1.37	High	508	208

Average Value-Weighted D/E			Average Value-Weighted Z		
	Altman's Z			Altman's Z	
D/E	Low	High	D/E	Low	High
Low	0.46	0.25	Low	2.19	8.83
2	1.00	0.84	2	1.87	3.47
High	2.43	2.21	High	1.41	3.40

Average Size			Average Value-Weighted BE/ME		
	Altman's Z			Altman's Z	
D/E	Low	High	D/E	Low	High
Low	10.30	11.37	Low	0.65	0.37
2	11.36	10.97	2	0.86	0.79
High	10.64	10.05	High	1.15	1.21

Table II
Results from Multivariate Regressions of Market Factor, SMB Factor, and HML Factor on the Leverage and Distress Factors
July 1964 – December 2000 (438 Months)

The time series of portfolio returns for the leverage and distress factors, $R_t^{D/E}$ and R_t^{Z}, are constructed each month by calculating the simple average return on the two high-*D/E* portfolios minus the simple average return on the two low-*D/E* portfolios (for $R_t^{D/E}$) and the simple average return on the three high-*Z* portfolios minus the simple average return on the three low-*Z* portfolios (for R_t^{Z}). The D/E and Z portfolios are formed as in Table I. The series R_t^{MKT} is the value-weighted return on all stocks in the Fama-French 25 portfolios for month t, R_t^{SMB} is the return on the Fama-French small minus big portfolio in month t, R_t^{HML} is the return on the Fama-French high-minus-low (book-to-market) portfolio in month t, and RF_t is the risk-free rate of return in month t.

Coefficient	Estimate	t-statistic	p (%)
Panel A: Estimates for the Time Series Model $R_t^{MKT} - RF_t = \alpha_0 + \alpha_{D/E} R_t^{D/E} + \alpha_Z R_t^Z + \varepsilon_t$			
α_0	0.43	2.16	3.2
$\alpha_{D/E}$	-0.32	-5.23	0.0
α_Z	0.73	7.86	0.0
Adjusted R^2 = 12%			
Panel B: Estimates for the Time Series Model $R_t^{SMB} = \alpha_0 + \alpha_{D/E} R_t^{D/E} + \alpha_Z R_t^Z + \varepsilon_t$			
α_0	0.14	0.92	36.1
$\alpha_{D/E}$	-0.01	-0.05	95.7
α_Z	0.34	4.76	0.0
Adjusted R^2 = 7%			
Panel C: Estimates for the Time Series Model $R_t^{HML} = \alpha_0 + \alpha_{D/E} R_t^{D/E} + \alpha_Z R_t^Z + \varepsilon_t$			
α_0	0.42	4.34	0.0
$\alpha_{D/E}$	0.63	21.01	0.0
α_Z	-0.77	-16.87	0.0
Adjusted R^2 = 52%			

Table III
Fama-MacBeth Regression Estimates (7/1964–12/2000)
for the Traditional Single-Factor CAPM and Fama-French Three-Factor Model

The series $RP_{i,t}$ is the return on the Fama-French size/book-to-market portfolio i (i = 1, …, 25) in month t, for the 438-month period July 1964 to December 2000; RF_t is the risk-free return in month t; R_t^{MKT} is the value-weighted return on all stocks in the Fama-French 25 for month t; R_t^{SMB} is the return on the Fama-French small minus big portfolio in month t; and R_t^{HML} is the return on the Fama-French high-minus-low (book-to-market) portfolio in month t. In Panel A, $\hat{\beta}_i^{MKT}$ is the estimated slope coefficient from a first-pass time series regression of ($RP_{i,t}$ - RF_t) on a constant and ($R_t^{MKT} - RF_t$). In Panel B, $\hat{\beta}_i^{MKT}$, $\hat{\beta}_i^{SMB}$, and $\hat{\beta}_i^{HML}$ are the estimated slope coefficients from a first-pass time series regression of ($RP_{i,t}$ - RF_t) on a constant, ($R_t^{MKT} - RF_t$), R_t^{SMB} and R_t^{HML}. The p-values are given in percent (two-sided). Corrected-t and corrected-p values adjust the standard errors using the Shanken (1992) procedure.

	γ_0	γ_{MKT}	γ_{SMB}	γ_{HML}	
Panel A: Results for Traditional Single-Factor CAPM $RP_{i,t} - RF_t = \gamma_0 + \gamma_{MKT}\hat{\beta}_i^{MKT} + \varepsilon_{i,t}$					
Coefficient	1.29	-0.55			
t-value	3.08	-1.17			
p-value	0.22	24.35			R^2 = 15%
Corrected-*t*	3.08	-1.06			
Corrected-*p*	0.22	28.78			
Panel B: Results for Fama-French Three-Factor Model $RP_{i,t} - RF_t = \gamma_0 + \gamma_{MKT}\hat{\beta}_i^{MKT} + \gamma_{SMB}\hat{\beta}_i^{SMB} + \gamma_{HML}\hat{\beta}_i^{HML} + \varepsilon_{i,t}$					
Coefficient	1.10	-0.57	0.13	0.43	
t-value	3.09	-1.37	0.82	2.97	
p-value	0.21	17.28	41.30	0.31	R^2 = 67%
Corrected-*t*	3.09	-1.22	0.58	2.12	
Corrected-*p*	0.21	22.48	55.93	3.42	

Table IV

Fama-MacBeth Regression Estimates (7/1964–12/2000)
For Competing Three-Factor Models Incorporating Marginal Explanatory Power of Size, Book-to-Market, Leverage, and Distress Factors

The time series of returns for the factor portfolios, R_t^{MKT}, RF_t, $R_t^{D/E}$, R_t^{Z}, R_t^{SMB}, and R_t^{HML}, are computed as described in Table II. In Panel A, $\hat{\beta}_i^{MKT}$, $\hat{\beta}_i^{D/E}$, and $\hat{\beta}_i^{Z}$ are the estimated slope coefficients from the first-pass time series regression of ($RP_{i,t}$ - RF_t) on a constant, ($R_t^{MKT} - RF_t$), $R_t^{D/E}$ and R_t^{Z}. In Panel B, $\hat{\beta}_i^{MKT}$, $\hat{\beta}_i^{D/E}$, $\hat{\beta}_i^{Z}$, $\hat{\beta}_i^{SMB\perp}$, and $\hat{\beta}_i^{HML\perp}$ are the estimated slope coefficients from the first-pass time series regression of ($RP_{i,t}$ - RF_t) on a constant, ($R_t^{MKT} - RF_t$), $R_t^{D/E}$, R_t^{Z}, $R_t^{SMB\perp}$, and $R_t^{HML\perp}$. The series $R_t^{SMB\perp}$ is the sum of the intercept plus the month-*t* residual from the regression $R_t^{SMB} = \alpha_0 + \alpha_1 R_t^{D/E} + \alpha_2 R_t^{Z} + \varepsilon_t$, and $R_t^{HML\perp}$ is the sum of the intercept plus the month-t residual from the regression $R_t^{HML} = \alpha_0 + \alpha_1 R_t^{D/E} + \alpha_2 R_t^{Z} + \varepsilon_t$. In Panel C, $\hat{\beta}_i^{MKT}$, $\hat{\beta}_i^{D/E\perp}$, $\hat{\beta}_i^{Z\perp}$, $\hat{\beta}_i^{SMB}$, and $\hat{\beta}_i^{HML}$ are the estimated slope coefficients from the first-pass time series regression of ($RP_{i,t}$ - RF_t) on a constant, ($R_t^{MKT} - RF_t$), $R_t^{D/E\perp}$, $R_t^{Z\perp}$, R_t^{SMl}, and R_t^{HMl}. The series $R_t^{D/E\perp}$ is the sum of the intercept plus the month-*t* residual from the regression $R_t^{D/E} = \alpha_0 + \alpha_1 R_t^{SMB} + \alpha_2 R_t^{HML} + \varepsilon_t$, and $R_t^{Z\perp}$ is the sum of the intercept plus the month-*t* residual from the regression $R_t^{Z} = \alpha_0 + \alpha_1 R_t^{SMB} + \alpha_2 R_t^{HML} + \varepsilon_t$. The *p*-values are given in percent (two-sided). Corrected-*t* and corrected-*p* values adjust the standard errors using the Shanken (1992) procedure.

Panel A: Results for Market, Leverage, and Distress Three-Factor Model

$$RP_{i,t} - RF_t = \gamma_0 + \gamma_{MKT}\hat{\beta}_i^{MKT} + \gamma_{D/E}\hat{\beta}_i^{D/E} + \gamma_Z\hat{\beta}_i^{Z} + \varepsilon_{i,t}$$

	γ_0	γ_{MKT}	$\gamma_{D/E}$	γ_Z	
Coefficient	1.16	-0.67	1.65	1.02	
t-value	3.48	-1.72	3.27	2.44	
p-value	0.05	8.62	0.12	1.52	R^2 = 81%
Corrected-*t*	3.48	-1.51	3.05	2.33	
Corrected-*p*	0.05	13.15	0.24	2.01	

Panel B: Results for Market, Leverage, and Distress Three-Factor Model
with Marginal Contribution of Size and Book-to-Market Factors

$$RP_{i,t} - RF_t = \gamma_0 + \gamma_{MKT}\hat{\beta}_i^{MKT} + \gamma_{D/E}\hat{\beta}_i^{D/E} + \gamma_Z\hat{\beta}_i^{Z} + \gamma_{SMB\perp}\hat{\beta}_i^{SMB\perp} + \gamma_{HML\perp}\hat{\beta}_i^{HML\perp} + \varepsilon_{i,t}$$

	γ_0	γ_{MKT}	$\gamma_{D/E}$	γ_Z	$\gamma_{SMB\perp}$	$\gamma_{HML\perp}$	
Coefficient	0.74	-0.27	2.18	1.53	-0.35	0.17	
t-value	1.97	-0.63	3.11	3.35	-1.65	0.92	
p-value	4.92	53.20	0.20	0.09	10.01	36.03	R^2 = 81%
Corrected-*t*	1.97	-0.56	3.00	3.23	-1.34	0.82	
Corrected-*p*	4.92	57.63	0.29	0.13	18.12	55.33	

Panel C: Results for Fama-French Three-Factor Model
with Marginal Contribution of Leverage and Distress Factors

$$RP_{i,t} - RF_t = \gamma_0 + \gamma_{MKT}\hat{\beta}_i^{MKT} + \gamma_{D/E\perp}\hat{\beta}_i^{D/E\perp} + \gamma_{Z\perp}\hat{\beta}_i^{Z\perp} + \gamma_{SMB}\hat{\beta}_i^{SMB} + \gamma_{HML}\hat{\beta}_i^{HML} + \varepsilon_{i,t}$$

	γ_0	γ_{MKT}	$\gamma_{D/E\perp}$	$\gamma_{Z\perp}$	γ_{SMB}	γ_{HML}	
Coefficient	0.74	-0.27	1.84	1.54	0.16	0.37	
t-value	1.97	-0.63	2.64	3.38	1.04	2.63	
p-value	4.92	53.20	0.86	0.08	30.07	0.88	R^2 = 81%
Corrected-*t*	1.97	-0.56	2.57	3.26	0.73	1.88	
Corrected-*p*	4.92	57.63	1.04	0.12	46.28	6.13	

Table V

Intercepts, R^2s, and *F*-statistics from First-Pass (Time Series) Regressions in Tables III and IV

In June of each year t, firms are assigned to one of three D/E portfolios based on NYSE breakpoints and (independently) one of two Z portfolios (Z < 2.675 and Z > 2.675). The time series of leverage portfolio returns $R_t^{D/E}$ and R_t^Z are constructed each month by calculating the simple average return on the two high-D/E portfolios minus the simple average return on the two low-D/E portfolios (for $R_t^{D/E}$) and the simple average return on the three high-Z portfolios minus the simple average return on the three low-Z portfolios (for R_t^Z). The series $RP_{i,t}$ is the return on the Fama-French size/book-to-market portfolio i (i = 1, …, 25) in month t, for the 438-month period July 1964 to December 2000, and RF_t is the risk-free return in month t. The series R_t^{MKT} is the value-weighted return on all stocks in the Fama-French 25 for month t. $R_t^{SMB\perp}$ and $R_t^{HML\perp}$ are the portions of the Fama-French SMB and HML factors orthogonal to $R_t^{D/E}$ and R_t^Z. The series $R_t^{SMB\perp}$ is the sum of the intercept plus the month-t residual from the regression $R_t^{SMB} = \alpha_0 + \alpha_1 R_t^{D/E} + \alpha_2 R_t^Z + \varepsilon_t$, and $R_t^{HML\perp}$ is the sum of the intercept plus the month-t residual from the regression $R_t^{HML} = \alpha_0 + \alpha_1 R_t^{D/E} + \alpha_2 R_t^Z + \varepsilon_t$. The statistic *GRS F* is the *F*-statistic derived in Gibbons, Ross, and Shanken (1989), which tests the null hypothesis that the estimated γ_0 are jointly zero.

From Traditional Single-Factor CAPM (Table III, Panel A): $\left(RP_{i,t} - RF_t\right) = \gamma_0 + \beta_i^{MKT}\left(R_t^{MKT} - RF_t\right) + \varepsilon_t$

	γ_0						$t(\gamma_0)$						Adjusted R^2				
	Lo	*2*	*3*	*4*	*Hi*		*Lo*	*2*	*3*	*4*	*Hi*		*Lo*	*2*	*3*	*4*	*Hi*
Sm	-0.50	0.17	0.24	0.49	0.54	*Sm*	-2.02	0.77	1.37	2.90	2.98	*Sm*	61.00	60.11	64.81	62.97	59.80
2	-0.31	0.05	0.35	0.44	0.45	*2*	-1.66	0.38	2.64	3.35	2.89	*2*	73.26	75.64	74.54	72.32	68.08
3	-0.24	0.19	0.22	0.39	0.51	*3*	-1.59	1.65	1.94	3.34	3.55	*3*	78.90	82.00	78.00	73.45	68.68
4	-0.05	-0.04	0.23	0.41	0.42	*4*	-0.41	-0.48	2.24	3.79	2.81	*4*	84.30	86.31	81.15	76.49	67.92
Lg	-0.01	-0.01	0.08	0.19	0.26	*Lg*	-0.06	-0.09	0.74	1.67	1.78	*Lg*	86.82	86.04	76.77	69.21	56.85

GRS F = 3.59 (p = 0.00%)

From Fama-French Three-Factor Model (Table III, Panel B): $\left(RP_{i,t} - RF_t\right) = \gamma_0 + \beta_i^{MKT}\left(R_t^{MKT} - RF_t\right) + \beta_i^{SMB} R_t^{SMB} + \beta_i^{HML} R_t^{HML} + \varepsilon_t$

	γ_0						$t(\gamma_0)$						Adjusted R^2				
	Lo	*2*	*3*	*4*	*Hi*		*Lo*	*2*	*3*	*4*	*Hi*		*Lo*	*2*	*3*	*4*	*Hi*
Sm	-0.43	0.03	0.00	0.17	0.11	*Sm*	-4.13	0.41	0.08	2.80	1.62	*Sm*	93.25	94.56	95.71	95.14	94.84
2	-0.17	-0.10	0.08	0.08	-0.01	*2*	-2.07	-1.38	1.15	1.33	-0.15	*2*	95.21	94.61	93.65	93.62	94.67
3	-0.08	0.04	-0.07	0.02	0.04	*3*	-1.03	0.47	-0.93	0.25	0.45	*3*	94.49	90.08	89.57	89.69	90.25
4	0.17	-0.18	-0.03	0.08	-0.05	*4*	2.18	-2.00	-0.40	1.07	-0.51	*4*	93.38	88.32	87.95	88.26	85.38
Lg	0.22	-0.05	-0.04	-0.11	-0.17	*Lg*	3.45	-0.64	-0.45	-1.58	-1.62	*Lg*	92.90	90.03	84.77	88.71	79.80

GRS F = 2.95 (p = 0.00%)

Table V (Continued)

From Market, Leverage, Distress Three-Factor Model (Table IV, Panel A): $\left(RP_{i,t} - RF_t\right) = \gamma_0 + \beta_i^{MKT}\left(R_t^{MKT} - RF_t\right) + \beta_i^{D/E} R_t^{D/E} + \beta_i^{Z} R_t^{Z} + \varepsilon_t$

	γ_0						$t(\gamma_0)$						Adjusted R^2				
	Lo	*2*	*3*	*4*	*Hi*		*Lo*	*2*	*3*	*4*	*Hi*		*Lo*	*2*	*3*	*4*	*Hi*
Sm	-0.50	0.14	0.21	0.45	0.50	*Sm*	-2.06	0.67	1.25	2.85	2.97	*Sm*	61.77	61.67	67.32	67.03	65.67
2	-0.31	0.03	0.32	0.40	0.40	*2*	-1.71	0.23	2.62	3.53	2.99	*2*	74.17	77.13	78.70	79.00	75.56
3	-0.24	0.16	0.19	0.36	0.47	*3*	-1.64	1.53	1.96	3.92	3.85	*3*	80.04	84.11	84.49	84.13	77.47
4	-0.04	-0.06	0.20	0.38	0.38	*4*	-0.36	-0.70	2.39	4.53	3.07	*4*	85.60	88.38	86.89	85.77	77.56
Lg	0.01	-0.01	0.07	0.17	0.23	*Lg*	0.12	-0.12	0.77	1.78	1.82	*Lg*	91.56	86.57	80.10	77.99	67.24

GRS F = 4.05 (p = 0.00%)

From Market, Leverage, Distress Three-Factor Model Incorporating Marginal Explanatory Power of Size and Book-to-Market (Table IV, Panel B):

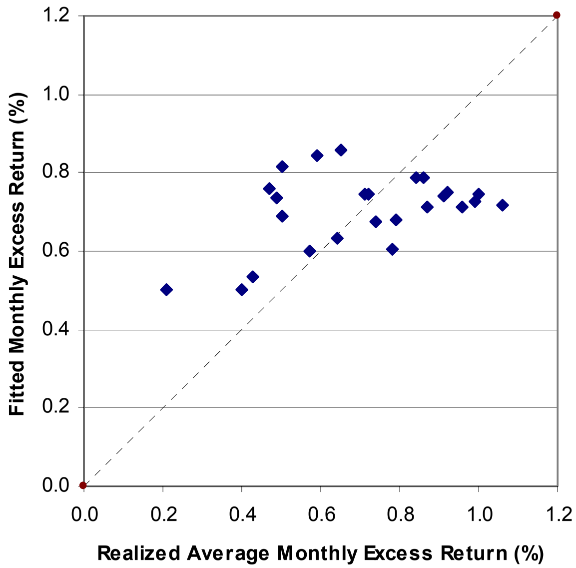
$$\left(RP_{i,t} - RF_t\right) = \gamma_0 + \beta_i^{MKT}\left(R_t^{MKT} - RF_t\right) + \beta_i^{D/E} R_t^{D/E} + \beta_i^{Z} R_t^{Z} + \beta_i^{SMB\perp} R_t^{SMB\perp} + \beta_i^{HML\perp} R_t^{HML\perp} + \varepsilon_t$$

	γ_0						$t(\gamma_0)$						Adjusted R^2				
	Lo	*2*	*3*	*4*	*Hi*		*Lo*	*2*	*3*	*4*	*Hi*		*Lo*	*2*	*3*	*4*	*Hi*
Sm	-0.44	0.02	-0.02	0.15	0.05	*Sm*	-4.23	0.29	-0.25	2.42	0.74	*Sm*	93.38	94.54	95.74	95.19	95.22
2	-0.13	-0.08	0.10	0.10	-0.03	*2*	-1.56	-1.17	1.54	1.57	-0.48	*2*	95.30	94.60	93.74	93.70	94.76
3	-0.05	0.09	-0.01	0.08	0.01	*3*	-0.66	1.01	-0.16	1.13	0.15	*3*	94.52	90.39	90.10	90.29	90.40
4	0.20	-0.09	0.03	0.16	-0.03	*4*	2.60	-1.06	0.41	2.10	-0.30	*4*	93.45	89.07	88.51	89.10	85.36
Lg	0.15	-0.01	0.03	-0.07	-0.17	*Lg*	2.57	0.14	0.33	-1.00	-1.60	*Lg*	94.33	90.18	85.61	88.95	79.71

GRS F = 2.92 (p = 0.00%)

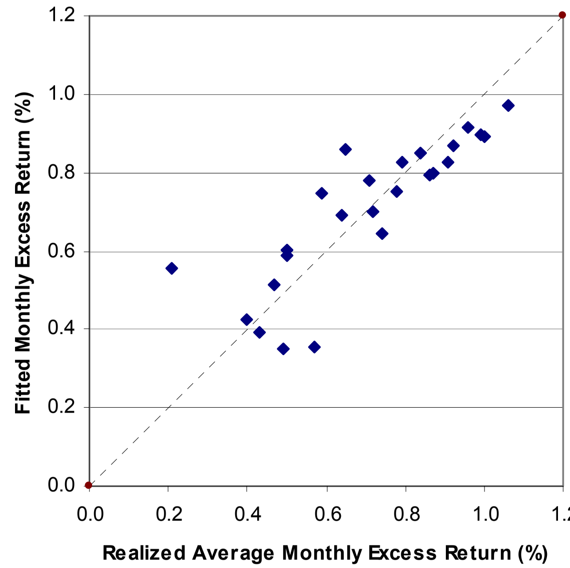
Panel A: Traditional CAPM
Single-Factor Model
(γ and β Estimates From Panel A of Table III)

$$E\{(RP_{i,t} - RF_t)\} = \gamma_0 + \gamma_{MKT}\beta_i^{MKT}$$



Panel B: Fama-French
Three-Factor Model
(γ and β Estimates From Panel B of Table III)

$$E\{(RP_{i,t} - RF_t)\} = \gamma_0 + \gamma_{MKT}\beta_i^{MKT} + \gamma_{SMB}\beta_i^{SMB} + \gamma_{HML}\beta_i^{HML}$$



Panel C: Market, Leverage and Distress
Three-Factor Model
(γ and β Estimates From Panel A of Table IV)

$$E\{(RP_{i,t} - RF_t)\} = \gamma_0 + \gamma_{MKT}\beta_i^{MKT} + \gamma_{D/E}\beta_i^{D/E} + \gamma_Z\beta_i^Z$$

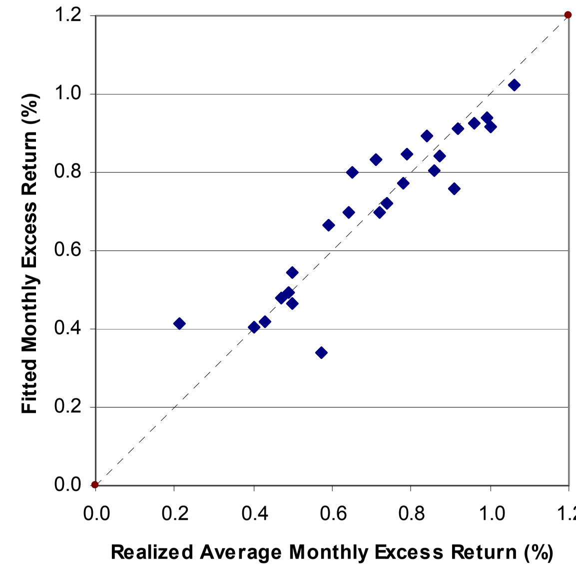


Figure 1

Plots of Fitted versus Actual Returns for 25 Fama-French Portfolios July 1964 – December 2000 (438 months)

The three panels present plots of fitted-versus-actual average monthly returns for the cross-sectional models in Table III and Panel A of Table IV. In Panel A, β_i^{MKT} is the estimated slope coefficient from a time series regression of $(RP_{i,t} - RF_t)$ on a constant and $(R_t^{MKT} - RF_t)$. In Panel B, β_i^{MKT}, β_i^{SMB}, and β_i^{HML} are the estimated slope coefficients from a regression of $(RP_{i,t} - RF_t)$ on a constant, $(R_t^{MKT} - RF_t)$, R_t^{SMB}, and R_t^{HML} where R_t^{SMB} is the return on the Fama-French small minus big portfolio in month t, and R_t^{HML} is the return on the Fama-French high-minus-low (book-to-market) portfolio in month t. In Panel C, β_i^{MKT}, $\beta_i^{D/E}$, and β_i^{Z} are the estimated slope coefficients from a regression of $(RP_{i,t} - RF_t)$ on a constant, $(R_t^{MKT} - RF_t)$, $R_t^{D/E}$, and R_t^{Z}. In all three specifications, R_t^{MKT} is the value-weighted return on all stocks in the Fama-French 25 for month t, and $RP_{i,t}$ is the return on the Fama-French size/book-to-market portfolio i ($i = 1, \ldots, 25$) in month t. In June of each year t, firms are assigned to one of three D/E portfolios based on NYSE breakpoints and (independently) one of two Z portfolios ($Z < 2.675$ and $Z > 2.675$). The time series of leverage and distress portfolio returns $R_t^{D/E}$ and R_t^{Z} are constructed each month by calculating the simple average return on the two high-D/E portfolios minus the simple average return on the two low-D/E portfolios (for $R_t^{D/E}$) and the simple average return on the three high-Z portfolios minus the simple average return on the three low-Z portfolios (for R_t^{Z}).

[1] More specifically, we establish the continuous-time equivalent of the single-beta CAPM so that the non-normality of the derivative returns is not an issue.

[2] Mayers (1972) examines the empirical implications of the single-beta CAPM when non-marketable assets are omitted from the market proxy, and arrives at an equation very similar to equation (1): The true beta of an equity will be a scaling of its beta with respect to the market proxy plus a scaling of its beta with respect to the non-marketable assets omitted from the market proxy. The implications in Mayers (1972) are that proxy betas will be erroneous and that the proxy beta errors will be firm-specific; our contribution is to demonstrate that the firm-specific error is a function of the firm's leverage.

[3] This analysis draws heavily on Galai and Masulis (1976).

[4] It is well known that the one-period CAPM cannot be applied to assets with non-normal returns (such as derivatives or insurance) even in a complete market. The only static model that generates the mean-variance result in a complete market is the quadratic utility specification, but quadratic utility implies the possibility of negative prices (Gonzalez, Litzenberger, and Rolfo (1977); and Dybvig and Ingersoll (1982)). In the continuous-time model, however, derived utility of wealth is always locally quadratic over any instant of time. Thus, as long as we prohibit the doubling strategies outlined in Harrison and Kreps (1979), mean-variance and arbitrage-free models agree on the pricing of the derivatives.

[5] Since the value of an equity claim *S* is a function of the underlying asset value *V* and the time *t*, then by Itô's lemma:

$$\Delta S = \frac{\partial S}{\partial V}\Delta V + \frac{\partial S}{\partial t}\Delta t + \frac{1}{2}\frac{\partial^2 S}{\partial V^2}\left(\Delta V\right)^2 = \frac{\partial S}{\partial V}\Delta V + \frac{\partial S}{\partial t}\Delta t + \frac{1}{2}\frac{\partial^2 S}{\partial V^2}\sigma^2 V^2 \Delta t \ .$$

To get the instantaneous return on the equity of firm *i*, divide both sides by S_i ,and let $\Delta t \to 0$:

$$r_{Si} \equiv \frac{\Delta S_i}{S_i} = \frac{\partial S_i}{\partial V_i}\frac{1}{S_i}\Delta V_i = \frac{\partial S_i}{\partial V_i}\frac{V_i}{S_i}\frac{\Delta V_i}{V_i} = \frac{\partial S_i}{\partial V_i}\frac{V_i}{S_i}r_i = N\left(d_1\right)_i \frac{V_i}{S_i}r_i$$

using the definition $\frac{\Delta V_i}{V_i} \equiv r_i$, which is the instantaneous return on firm *i*'s assets.

[6] The Mills Ratio for x is defined as $e^{.5x^2}\int_x^\infty e^{.5y^2}dy$.

[7] It should be noted that $\rho_{Si,E} = \rho_{i,E}$.

[8] For example, suppose that $\rho_{Si,E}$ = .80, that $\sigma_E = .16$, and that T = 15. The sufficient condition will be $.5648 < \frac{N(d_2)_i}{Z(d_2)_i}$, which holds as long as $d_2 > -1.3$. If we let $r = .05$ and $\sigma_i = .50$, then $d_2 > -1.3$ as long as $\ln V_i - \ln F_i > -1.391$, which is true for $F_i < 4.02 \cdot V_i$. That is, the condition will hold as long as the face value of the debt is anything less than four times the market value of the assets.

[9] Since $\hat{\beta}_{Si}^E = \eta_{Si}\hat{\beta}_i^E$ and $\eta_{Si} \geq 1$, the proxy beta of the stock and its underlying real asset always take the same sign.

[10] The sufficient conditions under which increasing leverage leads to increasing estimation errors for other combinations of $\hat{\beta}_{Si}^E$ and β_{Si} (e.g., $\hat{\beta}_{Si}^E > 0$ and $\beta_{Si} < 0$) are stated as corollaries to Proposition 1 in the Appendix.

[11] A proof that $\sigma_{Si,D} > 0 \Rightarrow \frac{\partial\left(\Omega\hat{\beta}_{Si}^D\right)}{\partial F_i} > 0$ is given as Proposition 3 in the Appendix.

[12] We can also use equation (1) to show why the Mills Ratio condition (MR) is needed. Since $\hat{\beta}_{Si}^D = \eta_{Si}\hat{\beta}_i^D$ and $\beta_{Si} = \eta_{Si}\beta_i$:

$$\frac{\beta_{Si}}{\hat{\beta}_{Si}^E} = \Phi\left[\frac{\beta_{Si}}{\beta_{Si} - \Omega\hat{\beta}_{Si}^D}\right] = \Phi\left[\frac{\beta_i}{\beta_i - \Omega\hat{\beta}_i^D}\right].$$

In Proposition 4 in the Appendix, we show that $\Omega\hat{\beta}_i^D$ strictly increases with leverage. The true beta of asset i is of course fixed, so as long as a change in firm *i*'s leverage does not change the marketwide parameter Φ too much, Proposition 1 holds. While $\Phi = \frac{E}{M}\frac{\sigma_E^2}{\sigma_M^2}$ *usually* increases with any firm's leverage, in extreme cases Φ *falls* as the distressed firm's leverage increases. Hence, the Mills Ratio condition (MR) describes cases where leverage-induced increases in $\Omega\hat{\beta}_i^D$ are not offset by declines in Φ.

[13] We estimate this using the Lehman Brothers Baa index (available monthly starting in July 1975) as a proxy for the debt market, and the Fama-French 25 size- and book-to-market sorted portfolios. For the Fama-French 25, the average $\hat{\beta}_S^D$ is .818, the average $\hat{\beta}_S^E$ (using the Fama-French market portfolio) is 1.009, and the ordinary least squares slope of $\hat{\beta}_S^D$ on $\hat{\beta}_S^E$ is -.0178 (t = -0.208).

[14] According to Treacy and Carey (1998), about 50 percent of the corporate loans held by the 50 largest U.S. banks are below-investment grade (ratings provided by the banks themselves).

[15] We thank Ken French for kindly providing the monthly return data. Details concerning the construction of the portfolios can be found in Fama and French (1993).

[16] Altman (1968) develops a multiple discriminant model for predicting financial distress through the use of balance sheet ratios. Altman's *Z* is defined as follows:

$$Z = 1.2\left(\frac{NWC}{TA}\right) + 1.4\left(\frac{RE}{TA}\right) + 3.3\left(\frac{EBIT}{TA}\right) + 0.6\left(\frac{ME}{BD}\right) + 1.0\left(\frac{S}{TA}\right)$$

where *NWC* is net working capital, *TA* is total book assets, *RE* is retained earnings, and *BD* is book debt (all from balance sheets); *EBIT* is earnings before interest and taxes, and *S* is total sales revenue (both from income statements); and *ME* is market value of equity.

[17] Financial statement measures for calculation of *Z* in year *t* use COMPUSTAT's fiscal year *(t*-1*)* data. The Variable *BD* is either total book liabilities or total book assets minus book equity (as defined by Fama and French (1993)) , in that order; *WC* is net working capital (current assets minus current liabilities); *TA* is total book assets; *RE* is book retained earnings; *EBIT* is earnings before interest and taxes; and *S* is total sales revenue. The variable *ME* is market capitalization at the end of December *(t*-1*)* from CRSP.

[18] Fama and French (1992, Table III). This result is robust to the inclusion of size and E/P variables.

[19] The slope coefficient from a univariate regression of SMB on HML is -.33 with a standard error of .05.

[20] When the part of SMB orthogonal to HML is regressed on $R^{D/E}$ and R^Z , the estimated coefficient on $R^{D/E}$ is 0.20 with a standard error of 0.04, while the estimated coefficient on R^Z is 0.08 with a standard error of 0.07.

[21] See Cochrane (2001) for a derivation of the Shanken (1992) correction.

[22] For a proof that $\frac{\partial \eta_{Si}}{\partial F_i} > 0$, see Galai and Masulis (1976).