To Auction Or To Negotiate

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Abstract

What procedure should a monopolist selling a single object adopt in order to maximize revenue? In settings where potential buyers may derive beneficial externalities from the sale of the object to any one of them, we show that a simple take-it-or-leave-it exclusively to one buyer generates greatest revenue among all “standard selling procedures” - a class of selling procedures that include all typical auctions. In this sense, negotiations dominate auctions!

We discuss the implications of this result for corporate takeover battles and private provision of public goods. An important corollary of our work is the surprising conclusion that a bidder cartel can actually increase the seller’s revenue. (JEL C7, D8, G34)

Keywords: Auctions, Negotiations, Externalities, Standard Selling Procedures, Takeover Battles.

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1 Introduction

When Saint George slayed the dragon\(^1\), as the legend goes, not only did he save the local princess and raise his own chances of becoming the future Patron Saint of England, but he also saved other youngsters of Silene (even ones yet unborn) slated to be dragon-fodder. But saints are a rare breed, which often makes the task of designing the cheapest way for kings to find a slayer non trivial - and while dragons these days rarely venture out from children’s sections in bookstores, solutions to many important economic problems that have emerged lie in finding the right slayer in situations involving a mixture of private and public values.

For instance, an important class of economic phenomena that fits this framework concerns the social design for private provision a public good. In the colorful imagery of Bliss and Nalebuff (1984), whoever undertakes to provide the public good, i.e. chooses to be the “dragon-slayer”, incurs a private cost while everyone benefits from the action. Alternatively, consider the widely discussed issue of how the management of a target should design its sale in takeover battles: When bidders own toeholds in shares of the target, the successful bidder gets private control benefits that are exclusive while all toeholders gain from the improvement in the value of the company from the post-takeover reorganization. Similarly, how should one procure contractual services for building a bridge that would stimulate the local economy? Besides the winning contractor who profits from building the bridge, other construction companies also gain from increased future construction demands. One confronts a similar problem in the choice of a location for parks, recreational facilities, nuclear waste disposal sites, garbage dumps, and other kinds of locally undesirable land use.

In this paper we set about the task of finding a procedure that will find the right slayer at the least cost, i.e. the revenue maximizing selling procedure for environments where exchange generates not just the usual excludable private benefits for the winner (private cost of the “dragon-slayer”, control benefits of successful takeover, profit of firm with winning tender for the bridge), but also a non-excludable externality from which all participants benefit.

We model these environments as an extension to the standard independent private values paradigm of mechanism design by allowing a sale to result in an externality to all the potential buyers. We show that for levels of externalities not too small, the revenue-maximizing auction collapses to an exclusive take-it-or-leave-it offer to a single bidder which targets the lowest type of that bidder. Remarkably, the sufficient condition requires the size of externality to only exceed

\(^1\)The example is inspired by the title of Bliss and Nalebuff (1984).
the range of the excludable private benefits.\textsuperscript{2}

This finding that auctions and many other selling procedures involving multiple potential buyers are dominated under revenue considerations by dealing exclusively with a single buyer (and making a take-it-or-leave-it offer) is in sharp contrast with the received wisdom. There are two reasons for this departure. First, the presence of externalities. Second, the fact that the seller must choose only from a set of selling procedures that we call standard selling procedures. We motivate and formally describe standard selling procedures in a later section. For now, it suffices to point out that it includes all the typically studied auctions.

It is this focus on standard selling procedures that differentiates our work from that of Jehiel, Moldovanu and Stacchetti (1996) (hereafter JMS). (See also Funk (1996) and Krishna (1993).) In fact, with positive externalities, there is a certain “free rider” effect that can lead to a potential loss of revenue through limited participation. One could therefore follow JMS and construct a certain auction-like procedure to eliminate this effect. A key insight from the JMS construction is that in the presence of externalities, the participation constraints are endogenous. By drawing a distinction regarding the seller’s power over actual versus potential buyers, a standard selling procedure limits the seller’s ability to manipulate these participation constraints. Broadly speaking, it is this feature that leads to our result. Our work then exemplifies the rich array of economic insights to be had from alternative approaches to modeling participation constraints. To illustrate, let us discuss in turn, the implications of this result for takeover battles and private provision of public goods.

The management of a takeover target often plays an important role in maximizing the price of the target. A natural question is whether they are better off in a single bidder contest - or should they facilitate the entry of new bidders. Various studies have addressed this issue of single versus multiple bidder contests. In a recent paper, Bulow and Klemperer (1996) show that given a set of bidders, holding a simple English auction with a single additional bidder raises more revenue than an optimally structured negotiating mechanism (without the additional bidder). This suggests that the seller should facilitate entry to raise the sale price. Our result, by modeling takeover battles with toeholds as auctions with externalities, indicates that the best way to sell in such cases is a simple (exclusive) take-it-or-leave-it offer to a buyer who obtains the largest externality - a conclusion that does not depend on the number of bidders. Free entry matters only to the extent of finding the buyer with the largest toehold (greatest externality).

\textsuperscript{2}For instance if the private benefits are distributed between $a$ and $b$, then an externality of $(b - a)$ is sufficient. Clearly, $(b - a)$ could be small compared to even the lowest possible private value $a$.\textsuperscript{2}
The empirical findings on the effect of toeholds lend support to our view. A large number of takeover auctions are single bidder contests, and these typically feature a large toehold. For example, Stulz, Walkling, Song (1990) find larger toeholds in single bidder takeover contests. In a recent paper, Betton and Eckbo (1999) provide evidence on the effect of toeholds on takeovers based on an extensive dataset containing 1353 single and multiple bid takeover contests between 1970 and 1990. They find that 62% of the contests had a single bidder, and larger toeholds by any one bidder implies greater likelihood of single bidder contest. At the same time, multiple bid contests are associated with relatively low toeholds. Stulz, Walkling, Song (1990) report that in such cases the management often facilitates the entry of new bidders. We show that there is an important connection between the existence of toeholds and the optimality of single versus multiple bidder contests. Turning the takeover into a multiple bidder contest can help if common value elements are dominating. Otherwise, in takeover races in which bidders have non-trivial toeholds, negotiating with a single bidder might be the best policy - a result sharply in contrast with the prevalent views in the literature.

Turning to the implications of our result for the provision of public goods that can be supplied by a single individual, our result differs from that of Bliss and Nalebuff (1984) who study the issue in a private values settings, and suggest the “war of attrition” as a solution - the person with the lowest cost is the first to lose patience and set about the job. Unlike us, Bliss and Nalebuff are not concerned with optimizing over different procedures. We are interested in a situation where people (or the king), perhaps ex-ante, set(s) up a scheme that finds the slayer at minimum cost. Our analysis suggests that instead of asking for multiple bids, the best approach is to offer the post to a single person at a take-it-or-leave-it salary.

Three further aspects of our result deserve emphasis. First, the take-it-or-leave-it offer scheme we suggest requires a commitment (similar to that required to post a reserve price) to not sell to any other buyer if the offer is rejected. A priori, this requirement could be a concern. However, it is not an issue here. The fact that the offer in question targets the lowest type of the chosen bidder implies that it is accepted with probability one.

Second, a byproduct of our result suggests that strong cartels that are informationally efficient can enhance the seller’s revenue. Section 4 briefly discusses this issue.

Third, the solution is highly asymmetric: even if all buyers are symmetric, the seller must still choose to deal with only one of them. In a sense, the manner in which comes about bears a similarity in spirit to Bulow, Huang and Klemperer (1996). They show that in common value environments, small private value asymmetries in toeholds can have a dramatic asymmetric effect. Here, small toe-
holds (levels of externalities) can yield an asymmetric solution in a private value environment, provided private control benefits are not very widely dispersed.

The rest of the paper is organized as follows. Section 2 sets up the basic model. Section 2.1 clarifies the intuition behind our result in a simple two buyer setting where the seller is restricted to using reserve price auctions (or make an exclusive take-it-or-leave it offer). Section 2.2 contains the motivation and definition of a standard selling procedure. The optimal selling procedure is characterized in Section 3. Section 4 discusses collusive agreements among buyers. Section 5 concludes.

2 The Model

A monopolist with a certain indivisible object intends to sell to \( n \) potential buyers. The sale of this object results in an externality to all of these buyers. Specifically, utility of buyer \( i \) is

\[
v_i + e_i \quad \text{if buyer } i \text{ wins} \\
e_i \quad \text{if } j \neq i \text{ wins} \\
0 \quad \text{if no sale.}
\]

where \( e_i \) denote the level of the externality generated from the sale of the object and \( v_i \) is the private benefit should \( i \) win the object. The actual value \( v_i \) is private information of bidder \( i \). Others treat \( v_i \) as the realization of a continuous random variable \( V_i \) distributed on \([a, b]\) with a density \( f_i \). Assume that \( f_i(v) \) is strictly positive on its support. The distribution functions are common knowledge.

We also assume that the levels of the externalities (\( e_i \)'s) are common knowledge.

By choosing a selling procedure, be it an auction or a some negotiation scheme, the seller imposes a game among the potential buyers. The revenue generated for the seller is the expected payment she receives in an equilibrium of this game. The objective of the seller is to choose a selling procedure that generates the highest revenue. An important aspect of our analysis is a restriction on the class of selling procedures that the seller can choose from. We shall come to this presently in Section 2.2 but first, the following section presents the intuition behind our result by illustrating how a reserve price auction can be inferior to an exclusive take-it-or-leave-it offer.
2.1 Reserve Price Auctions vs. Negotiations

Assume that there are only two buyers and that they are symmetric. The seller runs sealed bid auction and may set reserve prices. The revenue for the seller can be understood in the following terms. Let \( q_i(v_1, v_2) \) denote the equilibrium probability of buyer \( i \) winning the object when \((v_1, v_2)\) are the realized types. A routine extension of the methods outlined in Myerson (1981) shows that the seller’s revenue is given by

\[
Y = E[q_1(V_1, V_2)h_1(V_1) + q_2(V_1, V_2)h_2(V_2)] - U_1(a) - U_2(a) \\
+ (e_1 + e_2)E[q_1(V_1, V_2) + q_2(V_1, V_2)]
\]

where \( U_i(a) \) is the utility of buyer \( i \) when she is of type \( a \) and \( h_i(v) = v - (1 - F_i(v))/f_i(v) \) is what Myerson (1981) refers to as the “virtual valuation”.

One way to understand how the above expression comes about is as follows. The total surplus is \( v_i + e_1 + e_2 \) if the object is assigned to buyer \( i \) and is equal to zero, if the seller keeps the object. For reasons of incentive compatibility, the seller cannot extract all of this surplus: If the object is assigned to \( i \), she must forgo an informational rent equal to \( (1 - F_i(v_i))/f_i(v_i) \). Taking expectations, we get the expression for \( Y \) above.

We can derive a lower bound for \( U_i(a) \) as follows. An equilibrium of auctions such as these have the property that there is a threshold \( t_i \) (which depends on the equilibrium) such that buyer \( i \) submits a bid above the reserve if her type is above \( t_i \) but not otherwise. Therefore, so long as at least one of the buyers is above her threshold, the object is sold. Fixing her opponent’s strategy then, buyer \( i \)’s payoff from a unilateral deviation of non-participation is \( e_i \) times the probability that the object is sold to her opponent, which in this case is \((1 - F_j(t_j))\).

Equilibrium payoff of a type \( a \) buyer ought to be bounded below by the payoff from this deviation, i.e.

\[
U_i(a) \geq e_i(1 - F_j(t_j)) \quad j \neq i
\]

The arguments of the previous paragraph, present us with upper bound\(^3\), \( R(t_1, t_2) \)

\(^3\)To see this, first note that \( E[q_1(V_1, V_2) + q_2(V_1, V_2)] \), the probability of a sale, is \((1 - F_1(t_1)F_2(t_2))\) - i.e. one minus the probability of no sale. Second, the following are true: (i). \( q_i(v_1, v_2) = 1 \) if \( v_i \geq t_i, v_j \leq t_j \), (ii). \( q_i(v_i, v_j) = 0 \) if \( v_i \leq t_i \), (iii). \( q_1(v_i, v_j) + q_2(v_i, v_j) = 1 \) if \( v_i \geq t_i \), or \( v_j \geq t_j \), (iv). \( \int_{t_i}^{v_j} h_i(v) \, dF_i(v) = (1 - F_i(t_i))t_i \) and (v). \( h_i(v_j) \leq b \). Break up \( E[q_1(V_1, V_2)h_1(V_1) + q_2(V_1, V_2)h_2(V_1, V_2)] \) into expectations over the four different regions of Figure 1 and use (i)-(v), to conclude that it is bounded above by \( \sum_{j \neq i}(1 - F_i(t_j))t_i + (1 - F_1(t_1))(1 - F_2(t_2))b \). These, together with the lower bound for \( U_i(a) \) allows us to conclude that \( R(t_1, t_2) \) is an upper bound for \( Y \).
on the seller’s revenue $Y$:

$$
R(t_1, t_2) = F_2(t_2)(1 - F_1(t_1))(t_1 + e_1) + F_1(t_1)(1 - F_2(t_2))(t_2 + e_2) \\
+ b(1 - F_1(t_1))(1 - F_2(t_2)).
$$

(1)

Note that $R(t_1, t_2)$ can be expressed as the average over the revenues in the different boxes shown in Figure 1:

![Figure 1](image)

Figure 1: $R(t_1, t_2)$ is the average of the payoffs in different boxes.

As is evident from the above figure, the seller is able to recover the externality when exactly one buyer is above her threshold, that is when private values lie in one of the two off diagonal rectangles. It should be obvious that when the externality is large, $R(t_1, t_2)$ is maximized by choosing $t_1, t_2$ such that the total area of these rectangles is maximized. If $e_1 \geq e_2$, this is achieved by setting $t_1 = a$ and $t_2 = b$ with $R(a, b) = a + e_1$ being the maximum. Therefore, the revenue in any reserve price auction is bounded above by $a + e_1$. The seller can achieve this if she sells makes a take-it-or-leave-it offer exclusively to buyer on for the price $a + e_1$.

What is surprising is that the same can be true even when $e$ is smaller than lowest private value, i.e. $a$. For example, the figure below shows the graph of $R(t_1, t_2)$ when $F_i$ is the uniform distribution on $[50, 55]$ and $e = 10$. It is clear that the maxima of $Y$ occur at the extreme points. In fact, our main result shows that whenever the externality $e$ is at least the range of the private benefits $(b - a)$, it
is optimal for the seller to assign the object exclusively to one buyer rather than run an auction with reserve prices. Surprisingly, this result is independent of the number of players and of the precise distributions of players’ types.

Figure 2: \(R(t_1, t_2)\) when \(F_i\) is the uniform distribution on \([50, 55]\) and \(e_i = 10\).

2.2 Standard Selling Procedures

In general, a seller may choose from a variety of instruments in order to extract revenue. Setting reserve prices is one such instrument. Other instruments could be the use of participation fees and/or bidding fees. A key feature of our analysis is the restriction on the procedures that the seller may adopt. To set the stage for motivating our main assumption, let us first see how the seller can increase revenue beyond that in a take-it-or-leave offer scheme of the previous subsection by setting reserve prices and entry fees and making the payments from one player contingent on the participation decisions of her competitor(s). The reserve price \(r\) is a parameter in the following procedure:

**Selling-Procedure:**

- The seller requires an upfront participation fee of \((1 - F_j(r))e_i\) from buyer \(i\).
- Bidders simultaneously choose whether or not to participate.
- The auction proceeds only if both bidders participate. Otherwise, the seller calls off the auction.
– If both bidders participate, they play a second price auction with a reserve price $r$ except for the following caveat: any bid above $r$ requires an additional payment of $F_j(r)e_j$.

One can construct, for the above procedure, an equilibrium in which a buyer participates regardless of her type and bids her true type if and only if it lies above $r$. For an appropriately chosen $r$ this procedure generates more revenue than any other selling procedure. We direct the reader to JMS for details.

To discuss the (un)attractiveness of the above selling procedure, one needs to distinguish between actual and potential participants in the auction. Note that in the above procedure, the seller is able to extract payments from all potential buyers — this includes payments from types who have a zero probability of winning the auction. Such buyers are willing to pay as the seller will otherwise not sell at all, and thus prevent them from receiving the externality benefits resulting from a sale to a competitor. In certain situations this may well be possible, especially if one seller is a public authority. However, it is more typical, and perhaps realistic to assume that the seller has limited powers over potential buyers who do not participate in the auction. For example, one might assume that the seller cannot force payments from a buyer by making a sale to others contingent on her participation decision. Indeed, even a government could hardly make a construction company pay a fee when it had expressed no intention of submitting a tender to build the bridge. It is even less likely for the management of a target to extract payments from all the shareholders in the company before it decides to sell to one of the raiders.

Motivated by the kind of considerations described above, we shall restrict attention to only those procedures with the implication that the seller cannot condition her actions on the non-participation of some of the potential buyers. Our restriction on the choice of mechanisms is along the following lines: suppose that in the scheme that the seller sets up, buyer 1 has a positive probability of winning the object when $(b_1, b_2)$ is the profile of the actions chosen by the two buyers. Therefore buyer 1’s action is adequate to meet the seller’s reservation value. (If the probability that buyer 1 wins differs from one, it is perhaps because $b_2$ is equally attractive.) This would suggest that under any action profile in which buyer 1 chooses $b_1$, the seller faces at least one contender, i.e. buyer 1, who meets her reservation value for the object. Our restriction requires that whenever buyer 1 confronts the seller with the action $b_1$, the object is sold — either to him or perhaps to another buyer whom the seller may find even more attractive.

We call the class of selling procedures satisfying this restriction “standard selling procedures.” Note that this class includes all the well known auction formats
including all-pay auctions. An important aspect of these selling procedures is that the seller cannot extract any payments from a buyer type who has no chance of winning the object in equilibrium: Should the seller seek such a payment, a unilateral deviation from the buyer to not participate and derive externality benefits of a sale to her opponents is profitable.

We shall now formalize the above restriction.

Let \( S \) denote a set of actions and consider a pair of functions \( \{ Q_i, Y_i \} \) for each buyer: if \((s_1, \ldots, s_n) \in S^n\) is the profile of actions chosen the buyers, then \( i \) wins with probability \( Q_i(s_1, \ldots, s_n) \) and is required to pay \( Y_i(s_1, \ldots, s_n) \). Among the actions available to a buyer is her decision not to participate in the seller’s scheme. We shall denote this by \( N \). By definition, the object cannot be assigned to a buyer who does not participate nor can the seller extract payments from such a buyer. Along with feasibility we therefore require

\[
\sum_{i=1}^n Q_i(s_1, \ldots, s_n) \leq 1 \quad \forall (s_1, \ldots, s_n) \\
Q_i(N, s_{-i}) = 0 \quad \forall s_{-i} \\
Y_i(N, s_{-i}) = 0 \quad \forall s_{-i}
\]

We shall refer to a tuple \((S, \{ Q_i, Y_i \}_{i=1}^n)\) which satisfies the restrictions in (2) as a selling procedure.

**Definition 1 (Standard Selling Procedure)** A selling procedure \((S, \{ Q_i, Y_i \}_{i=1}^n)\) is said to be a standard if

\[
Q_i(s_i, s_{-i}) > 0 \implies \sum_{j=1}^n Q_j(s_i, \bar{s}_{-i}) = 1 \quad \forall i, \bar{s}_{-i}
\]

### 3 Optimal Standard Selling Procedures

By choosing a selling procedure, the seller imposes a game among the buyers. In this game, a buyer’s strategy must specify an action as a function of her private information. Let \( \sigma_i : [a, b] \to S \) denote a typical strategy of buyer \( i \). Having specified a strategy profile \( \sigma = (\sigma_1, \ldots, \sigma_n) \), by averaging across types of buyer \( i \)’s opponents, we can easily compute for a type \( v \) buyer \( i \) the following: the probability

\[Q_i(s_i, s_{-i}) > 0 \implies \sum_{j=1}^n Q_j(s_i, \bar{s}_{-i}) = 1 \quad \forall i, \bar{s}_{-i}\]
that she is assigned the object, her expected payment \( y_i(v, \sigma) \). One can also compute the probability \( q_s(v, \sigma) \) that the object is sold to some buyer, given that buyer \( i \) is of type \( v \). The interim expected payoff of such a buyer therefore be written as follows:

\[
U_i(v, \sigma) = v q_i(v, \sigma) - y_i(v, \sigma) + e_i q_s(v, \sigma) \tag{3}
\]

A straightforward modification of the techniques developed in Myerson (1981) shows that if \( \sigma \) is an (Bayesian) equilibrium of the above game, then the ex-ante expected payment\(^5\) from buyer \( i \) in this equilibrium is

\[
y_i(\sigma) = E[q_i(V_i, \sigma) h_i(V_i)] - U_i(a, \sigma) + e_i E[q_s(V_i, \sigma)]. \tag{4}
\]

Recall, \( h_i(v) = v - \frac{1 - F_i(v)}{f_i(v)} \) is the “virtual valuation” of buyer \( i \). The total revenue in this equilibrium is then \( \sum_i y_i(\sigma) \). The following is our main result.

**Theorem** Suppose (i) \( h_i(v) \geq 0 \) for all \( v \in [a, b] \) and (ii) \( \max_i \{e_i\} > (b - a) \). Then,

1. In any equilibrium of a standard selling procedure, the seller’s revenue is bounded above by \( a + \max_i \{e_i\} \). The seller can achieve this upper bound by making an exclusive take-it-or-leave-it offer to the buyer with the highest \( e_i \).

2. Moreover, the above bound on revenue is strict if at least two players have a positive overall probability of winning the object.

The theorem shows that the optimal procedure is to select a single buyer, and make a take-it-or-leave-it offer. Note also that the offer in question targets the lowest type of the buyer, which ensures acceptance with probability 1. This implies that the result does not depend upon any implicit commitment assumption.

We shall present here the essence of the arguments used in the proof of the Theorem with two buyers. Details for the general case and proofs of other formal statements are relegated to the Appendix. The central observation in the proof of the Theorem is Lemma 1 below.

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\(^5\)Integrating the first expression in Eq. (5) (in the appendix) by parts and then using Eq. (3) in the usual way, we note that the ex-ante expected payment from buyer \( i \) in the equilibrium is \( y_i(\sigma) \).
Lemma 1 Any equilibrium of a standard selling procedure specifies a threshold for each buyer such that buyer $i$ does not win the object if her type is below the threshold. Moreover, the object is sold with probability one whenever at least one of the buyers draws a type above her threshold.

Lemma 1 helps us to construct a simple upper bound for the total revenue along the following lines. Suppose that there are two buyers and $(t_1, t_2)$ are the relevant thresholds that result in some equilibrium. Then, $R(t_1, t_2)$ as described in Eq. 1, is an upperbound for total revenue. Now, when $h_i(v) \geq 0$ for all $v$, then $(1 - F_i(t_i))t_i \leq a$. Using this fact and a simple algebraic rearrangement of $R(t_1, t_2)$, gives us $r_i(t_1, t_2)$ below as yet another upper bound for the revenue.

$$r_i(t_1, t_2) = (1 - F_j(t_j))b + F_j(t_j)a$$

$$+ [F_i(t_i)(1 - F_j(t_j)) + (1 - F_i(t_i))F_j(t_j)] e, \quad i, j = 1, 2, i \neq j$$

We will show that $r_i(t_1, t_2)$ must be bounded above by $a + e$, for some $i$. If $t_i = a$, then $r_i(t_1, t_2) = (1 - F_j(t_j))b + F_j(t_j)(a + e)$, which is bounded above by $a + e$. So assume that $F_i(t_i) > 0$ for $i = 1, 2$. Then, we can write

$$r_i(t_1, t_2) = (1 - F_j(t_j))b + F_j(t_j) \left[ a + \left( F_i(t_i) \frac{1 - F_j(t_j)}{F_j(t_j)} + (1 - F_i(t_i)) \right) e \right]$$

Next, suppose that $F_j(t_j) \geq 1/2$. Then, the coefficient of $e$ (within the square brackets) is bounded above by 1. Hence, $r_i(t_1, t_2) \leq (1 - F_j(t_j))b + F_j(t_j)(a + e)$, which again is less than $a + e$. Finally, assume then, $F_i(t_i) \leq 1/2$ for both $i = 1, 2$.

In this case, the expression in square brackets above is larger than $b$. Therefore,

$$r_i(t_i, t_j) \leq a + \left( F_i(t_i) \frac{1 - F_j(t_j)}{F_j(t_j)} + (1 - F_i(t_i)) \right) e \quad i \neq j \quad i, j = 1, 2.$$  

From above, we know that the seller’s revenue is bounded above by $a + H(T)e$ where

$$H(t_1, t_2) = \min \left\{ F_1(t_1) \frac{1 - F_2(t_2)}{F_2(t_2)} + (1 - F_1(t_1)) \right.$$  

$$\left. F_2(t_2) \frac{1 - F_1(t_1)}{F_1(t_1)} + (1 - F_2(t_2)) \right\}$$

whenever $F_i(t_i) \leq 1/2$ for both $i$. Therefore, the proof is complete if we show that in this region, $H(t_1, t_2)$ is bounded above by 1. But this readily follows from the observation that if $(t_1^*, t_2^*)$ were a maximum of $H$, then $F_i(t_i^*) = 1/2$ for at some $i$.  

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4 Bidder Collusion

The academic literature appears to hold the view that cooperative arrangements in auctions harm the seller. Consequently, most studies focus on how the seller should choose the selling methods that minimize losses from the presence of bidding rings. Other studies concentrate on the sustainability and informational efficiency within bidding rings (see Caillaud and Jehiel (1998), Mailath and Zemsky (1991), Graham and Marshall (1990) and Hendricks and Porter (1989)). In a recent paper, Krishna and Morgan (1997) provide some examples of revenue-enhancing collusion under common values. In their model, collusion allows bidders to share information, which reduces the winner’s curse, which could, in turn, enhance revenue. Remarkably, besides these examples, there is very little theoretical work that we are aware of, that directly addresses the interests of the seller when joint bidding is possible. Our results shed some light on this issue under pure private values with externalities.

Consider first the role of a weak cartel. Such a cartel cannot extract payments from all potential bidders, but can select a single bidder to bid for the object. If the seller can directly negotiate with a single bidder, such a collusive arrangement is of no concern to him. Thus collusion cannot hurt. However, in some cases such collusion can strictly benefit the seller. In certain contexts it might be difficult to implement the optimal selling procedure. For example, in procurement auctions where the seller is the government, and the object is a highly visible public project, allowing unrestricted participation might be an objective in itself, as the government wants to appear to be “fair.” In such cases, choosing a single bidder might be problematic, and might even be legally prohibited. One of the reasons is that a selling procedure that chooses a single buyer from a pool of buyers increases the incentives for rent-seeking behavior, and worsens the structure of incentives in the government organization. In such cases, a seller is often restricted to setting a reserve price.

Thus if the seller can choose one bidder and sell to him, she is indifferent between facing a cartel and facing independent buyers, while restrictions on the mechanisms that disallow such a direct choice of a single bidder imply a strict preference for facing a cartel.

In some cases, bidders in an auction might be part of a cartel that exists because the bidders participate in other auctions, or interact repeatedly. In such cases, bidders might collude in an auction even if the cartel mechanism does not

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6This satisfies “symmetry” in the following sense. First, the mechanism allows unrestricted participation and second, any two buyers submitting the same bid have the same probability of winning and the same expected payment.
satisfy individual rationality in terms of the payoffs of the auction. We refer to this as a strong cartel, and such a cartel would only make it easier to derive our results. Consider a strong cartel that can efficiently aggregate the values of all bidders. The value of the cartel is then the highest individual value added to the total externality of the group. Since the seller can now extract the total externality by posting a reserve price of \( a + \sum_i e_i \), he is strictly better off compared to the optimal mechanism with independent bidders.

The Antitrust division has prosecuted and indicted bidders in many auctions for colluding. We show that contrary to popular belief, collusion among bidders can enhance the seller’s revenue, and thus detection of externalities should precede any prosecution on the grounds of collusion in auctions.

5 Discussion

We show that some of our common intuitions about the desirability of auctions over other selling methods as well as the undesirability of collusion in auctions depend crucially on the absence of externalities arising out of the economic transactions involved. However, as we mentioned in the introduction, there is a large class of such transactions that generate externalities - and in such cases the common conclusions we draw from auction theory are often invalid. This suggests that in designing mechanisms we need to pay careful attention to the presence of externalities.

In our main result, we offer a simple sufficient condition that in settings with private values augmented by externalities, the optimal selling procedure collapses to a simple take-it-or-leave-it offer to a single bidder. Thus negotiation is better than all auctions. The sufficient condition that we offer is also remarkable in that it depends only the range of the private value components. Another important feature of the optimal scheme is that the offer targets the lowest type of the chosen bidder, which ensures acceptance with probability 1. Thus the optimal offer requires no special commitment by the seller to not sell if the offer is rejected.

Our effort has been to provide a set of sufficient conditions that cause exclusion to be the optimal mechanism for even low values of the externalities. Our method of proof necessitated the assumption that the virtual valuations are always non-negative. As we mentioned at the very outset (in Section 2.1), it is intuitively clear that when the externalities are large, the result must obtain. Indeed, it is relatively easy show that our main conclusion continues to hold if \( e_i > b \) - even without making additional assumptions regarding the virtual valuations. We omit this result here given as, we feel, the size of the externality required being larger then the private benefit of the highest type is of limited interest.
Finally, we also assume that the externalities received by the players is common knowledge. It is of the utmost interest to find out how our results extend when one relaxes this assumption. Such an exercise is substantially involved and warrants a separate investigation.

Appendix

Proof Of Lemma 1: Despite the presence of externalities, a routine application of Myerson (1981) shows that if $\sigma$ is an equilibrium of the game induced by $(S, \{Q_i,Y_i\}_{i=1}^n)$ then

$$\frac{\partial U_i(v,\sigma)}{\partial v} = q_i(v,\sigma), \quad \frac{\partial q_i(v,\sigma)}{\partial v} \geq 0 \quad (5)$$

Because of the monotonicity of $q_i$ asserted in Eq. (5), there is a threshold $t_i$ that $q_i(v_i) > 0$ if and only if $v_i \geq t_i$. This is true for all selling procedures. Suppose now that buyer $i$ is of type $v_i \geq t_i$. Since $q_i(v_i) > 0$, there is at least one realization of the other players' types, say $v_{-i}$ at which buyer $i$ wins with a positive probability, i.e., $Q_i(\sigma_i(v_i),\sigma_{-i}(v_{-i})) > 0$. From the definition of a standard selling procedure, it follows that $\sum_{j=1}^n Q_j(\sigma(v_i),\sigma_{-i}(\tilde{v}_{-i})) = 1$ for all $\tilde{v}_{-i}$. So the object is sold with probability one, when buyer $i$'s type is $v_i$. As $v_i$ is arbitrarily chosen to be above $t_i$, the result obtains.

Proof Of Main Theorem. Let $e = \max e_i$. Following Lemma 1, let $T = (t_1, \ldots, t_n)$ be a vector of thresholds that correspond to some equilibrium $\sigma$. Also let $A_i(T) = \prod_{j \neq i} F_j(t_j)$, the probability that all players other than $i$ draw a valuation below their threshold. The equilibrium payoff $U_i(a,\sigma)$ must be bounded below by the payoff to the unilateral deviation of non-participation by $i$ when she is of type $i$. On non-participation, buyer $i$ neither wins the object nor is required to pay anything. She however derives the externality benefits should the object be sold to one of her competitors. The object will be sold only if at least one of the players other than $i$ draw a valuation that is above their respective thresholds: the probability of this event is $(1 - A_i(T))$. Therefore,

$$U_i(a_i,\sigma) \geq (1 - A_i(T))e_i \quad (6)$$

Again, from Lemma 1, the overall probability of sale is simply the probability that at least one buyer draws a type above her cutoff. The probability that all the
players draw a type below their cutoffs is \( F_i(t_i)A_i(T) \), which is the probability of no sale. Therefore the probability of a sale is

\[
E[q_s(V_i, \sigma)] = (1 - F_i(t_i)A_i(T)).
\]  

(7)

Use Eq. (7) and the Inq. (6), to conclude that the second line of Eq. (4) is bounded above by \( A_i(T)(1 - F_i(t_i))e \). The first line of Eq. (4) is independent of \( e_i \). Hence, \( A_i(T)(1 - F_i(t_i))e \) is an upper bound for the payment of buyer \( i \) that concerns the externality. Therefore, summing this over all \( i \), gives us the following bound for the part of the total revenue that involves externalities:

\[
\left[ (1 - F_i(t_i))A_i(T) + \sum_{j \neq i} (1 - F_j(t_j))A_j(t_j) \right] e
\]

(8)

Let us now turn to the part of the total revenue that corresponds to the private benefits. Pick an arbitrary player, say \( i \). As long as at least one of the players other than \( i \) draws a valuation above her respective cutoff, (an event that occurs with probability \( (1 - A_i(T)) \)), there will be a sale. In this event gets the virtual valuation of some buyer buyer, which of course is bounded above by \( b \). Now consider the event that all but buyer \( i \) draw a valuation below their cutoffs: the probability of this event is \( A_i(T) \). In this event, the seller either sells it to buyer \( i \), and gets \( h_i(v) \) if \( v \geq t_i \) and zero otherwise. Therefore, total revenue corresponding to the private benefits is bounded above by

\[
(1 - A_i(T))b + A_i(T) \int_{t_i}^{b} h_i(v)dF_i(v) \leq (1 - A_i(T))b + A_i(T)a
\]

(9)

Putting together Eq. 8 and Eq. 9, \( r_i(T) \) below is an upper bound for the seller’s revenue for all \( i \):

\[
r_i(T) = (1 - A_i(T))b + A_i(T)a
\]

\[
+ \left[ (1 - F_i(t_i))A_i(T) + \sum_{j \neq i} (1 - F_j(t_j))A_j(t_j) \right] e
\]

The proof is complete if we show that \( r_i(T) \) must be bounded above by \( (a + e) \) for some \( i \). There are essentially three different cases to consider.

First suppose \( F_i(t_i) = 0 \) for some \( i \). Then, \( A_j(T) = 0 \) for all \( j \neq i \). Therefore,

\[
r_i(T) = (1 - A_i(T))b + A_i(T)(a + e) \leq (a + e)
\]

\footnote{The inequality follows because \( \int_{t_i}^{b} h_i(v)dF_i(v) \) is a decreasing in \( t_i \) if \( h_i(v) \geq 0 \) for all \( v \in [a, b] \). Therefore, it is bounded above by \( \int_{a}^{b} h_i(v)dF_i(v) = a \).}
and the inequality is strict if $A_i(T) < 1$, i.e. at least two players have a positive probability of winning the object. For the remainder of the proof then, assume $F_i(t_i) > 0$ for all $i$. In this case, we case rewrite $r_i(T)$ as follows:

$$r_i(T) = (1 - A_i(T))b + A_i(T)a + A_i(T)[(1 - F_i(t_i)) + F_i(t_i)C_i(T)]e \quad \forall i$$

where $C_i(T) = \sum_{j \neq i} \frac{1 - F_j(t_j)}{F_j(t_j)}$.

If $C_i(T) < 1$ for some $i$, it is immediate that the expression in the square brackets above is bounded above by one, and therefore

$$r_i(T) \leq (1 - A_i(T))b + A_i(T)(a + e) \leq (a + e).$$

Again, the inequality is strict if at least two players win the object with a positive probability. Thus, we are left to only consider the case when $C_i(T) \geq 1$ for all $i$. In this case,

$$r_i(T) = (1 - A_i(T))b + A_i(T)\left[ a + \left( (1 - F_i(t_i)) + F_i(t_i)C_i(T) \right) e \right]$$

$$\leq a + \left( (1 - F_i(t_i)) + F_i(t_i)C_i(T) \right) e$$

As the last expression above is an upper bound for the seller’s revenue $Y(T)$, for all $i$, it follows

$$Y(T) \leq a + H(T)e$$

where

$$H(T) = \min_i \left\{ (1 - F_i(t_i)) + F_i(t_i)C_i(T) \right\}.$$  

The proof is complete if $H(T)$ is bounded above by 1. Consider then the problem of maximizing $H(T)$ with respect to $T$ subject to $C_i(T) \geq 1$ for all $i$. Let $T^*$ be the argmax $^8$ of $H$. (As we are maximizing a continuous function $H$ over a compact domain$^9$, such an argmax must exist.) Note that $t_i \neq 1$ for at least two players. For, if $t_i^* = 1$ for all $j \neq i$, then $C_i(T^*) = 0$, and thereby $T^*$ falls outside the domain.

The fact that $t_i^* \neq 1$ for at least two players implies that $C_i(T^*) = 1$ for some $i$ — if all the constraints were strict, then one can increase the value of $H$ by increasing some $t_i^*$ slightly. Therefore, $H(T^*) = 1$, and this completes the proof.

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$^8$Argmax is well defined as $H$ is continuous and the inequalities $C_i(T) \geq 1$ define a compact set.

$^9$There is a technical issue here that the domain is not actually compact as we require $F_i > 0$. This subtlety can be easily overcome by compactifying the domain by saying that $F_i \geq \varepsilon$, for some arbitrarily small $\varepsilon > 0$. We leave the details to the reader.
References


