On Asset Pricing and the Bid-Ask Spread

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Abstract

Amihud and Mendelson’s (1986) ground-breaking model predicts an increasing and concave relation between expected return and relative spread, designated the clientele effect. In their liquidity-adjusted CAPM, Jacoby et al. (2000) positively relate expected returns to the relative spread, but with a convex relation. Extending Amihud and Mendelson’s model, we demonstrate that the relation is concave and convex for highly liquid and thinly traded assets, respectively. We therefore incorporate the two opposite effects described by the two models. We use NASDAQ data to present evidence supporting the empirical implications. Our results are consistent with Brennan and Subrahmanyam’s (1996) empirical results.

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1 Introduction

In a seminal work, published in 1986, Amihud and Mendelson (AM) present a model of the relation between expected returns and the level of the bid-ask spread relative to the stock price (relative spread). Their model positively relates returns to relative spread, with a decreasing rate (concave relation). They argue that heavily traded securities are likely to be traded more frequently in the future. Therefore, for a given increase in the level of the relative spread ratio, these securities have higher amortized relative spread costs relative to thinly traded securities. Hence, investors demand higher liquidity premiums for a given change in the relative spread, for the more active assets.

Ceteris paribus, given two securities identical in every sense except that one is more liquid than the other, short-term investors are willing to pay a liquidity premium for holding the liquid asset. Long-term investors are indifferent between the two assets, and therefore choose to hold the cheaper illiquid security. AM designate the concave relation between returns and rel-
ative spread the *cliente effect*. They test this theoretical relation using an empirical Capital Asset Pricing Model (CAPM) framework and provide evidence supporting concavity, along with a significant linear relation between excess returns and CAPM beta.

Jacoby, Fowler and Gottesman (JFG) (2000) derive a liquidity-adjusted version of the CAPM. Their model is based on returns net of the relative bid-ask spread. They examine the relation between the expected gross return and the relative bid-ask spread within their CAPM-based model. Like AM, their model positively relates expected returns to the relative spread, but with an increasing rate (convex relation). JFG designate this convex relation the *level effect*.

JFG’s CAPM-based model is a one period model under which all securities, regardless of their level of liquidity, are held for the entire period. This implies that they do not allow highly liquid assets to be traded more often during the underlying period, thereby eliminating AM’s clientele effect and the concave relation. Although this is a shortcoming, their model emphasizes an important issue. JFG’s level effect (convexity) has to hold for securities with high relative spreads. As relative spread approaches 100%, investors expect to pay their entire return as spread costs, and therefore demand an
infinite liquidity premium, in terms of gross return, before entering a long position in such an asset. Thus, the return grows to infinity asymptotically as the relative spread approaches 100%. Therefore, the expected return relative spread relation is convex for high relative spread ratios.

The preceding models give rise to different effects determining the curvature of the function relating gross returns to relative spreads. Empirical evidence presented by Brennan and Subrahmanyam (1996) confirms the validity of both AM’s clientele effect and JFG’s level effect. Brennan and Subrahmanyam decompose the estimated trading cost into variable and fixed components. Their empirical results indicate a concave relation between the variable cost of transacting and the return premium, which is consistent with AM’s clientele effect. They also find the return premium to be convex in the fixed cost component. Brennan and Subrahmanyam note that the fixed cost component is highly correlated with the relative spread (correlation of 0.78), suggesting agreement with JFG’s level effect.

Motivated by the Brennan and Subrahmanyam (1996) empirical evidence, this paper demonstrates that the positive relation between gross return and relative spread is concave for highly liquid assets with low relative spreads and convex for thinly traded assets with higher relative spread ratios. We
incorporate the two opposite effects of the relative spread level on the curvature as described by AM and JFG. The AM model uses riskless assets while the JFG model uses risky assets. We demonstrate below that these theoretical differences do not interfere with the implications of either model with respect to the relation between expected return and relative spread. Altering the assumptions of AM’s model, we demonstrate that the resultant model includes both the clientele effect and the level effect. In Section 2 of this paper, we modify AM’s assumptions to permit JFG’s level effect within AM’s framework. Next, in Section 3, we offer empirical evidence supporting the implications of the altered AM model. Section 4 conclude the paper.

2 Amihud and Mendelson’s (1986) framework and the level effect

AM’s (1986) ground-breaking model describes a concave relation between gross return and relative spread. They describe a world with \( M \) investors indexed by \( i, i = 1, 2, ..., M \). The portfolio is held by a type - \( i \) investor (with initial wealth \( W_i \)) for a random, exponentially distributed, time horizon \( T_i \) with \( E[T_i] = \mu_i^{-1} \). At time \( T_i \), the portfolio is liquidated by the investor to a
market maker for the quoted bid price. There are \( N + 1 \) assets indexed by \( j, j = 0, 1, \ldots, N \), where asset \( j \) pays a \( d_j \) perpetual positive cash flow per unit of time, and has a trading cost reflected by a relative spread of \( S_j \). Asset 0 has zero relative spread with unlimited supply. AM number investor types through increasing expected holding horizons \( \mu_1^{-1} \leq \mu_2^{-1} \leq \cdots \leq \mu_M^{-1} \), and securities through increasing relative spread ratios \( 0 = S_0 \leq S_1 \leq \cdots \leq S_N \). Market makers stand by in the market to provide immediacy services. For each security \( j \) they quote an ask price \( V_j \), and a bid price \( V_j(1 - S_j) \).

AM assume a stationary infinite horizon economy, where assets pay a constant dividend stream (bearing no transaction costs) and asset prices are constant over time. Thus, the net (after spread) return in such an economy does not exhibit JFG’s level effect:

\[
 r_{ij} = \frac{d_j}{V_j} - \mu_i S_j \tag{1}
\]

where

\[
 r_{ij} = \text{expected spread adjusted return of asset } j \text{ to investor } i.
\]

\[
 \frac{d_j}{V_j} = \text{gross (pre-spread) return on security } j.
\]

\[
 \mu_i S_j = \text{expected spread cost on asset } j \text{ for investor } i \text{ (} \mu_i \text{ is the liquidation probability of investor } i \text{)}
\]
We denote the gross (pre-spread) return as \( r_{gj} \equiv \frac{d_j}{V_j} \), and rearrange the above equation to get:

\[
    r_{gj} = r_{ij} + \mu_i S_j
\]  

(2)

Thus, in AM’s economy the gross return is \textbf{linear} in the expected relative spread. We violate this assumption to include JFG’s price level effect. AM assume no short sales, hence the asset equilibrium price in their model is determined by the investor who outbids all other investors (investors are heterogenous with regard to their expected holding periods). This investor agrees to accept the minimum expected net return. Investor \( i \) purchases asset \( j \) which provides him/her with the highest after-spread return:

\[
    r_i^* = \max_{j=0,1,2,...N} \{ r_{ij} \}
\]  

(3)

The gross (pre-spread) return for investor \( i \) on asset \( j \) is given by \( r_i^* + \mu_i S_j \).

Therefore, the equilibrium gross (pre-spread) return on asset \( j \) is the minimal required expected gross return across investors:

\[
    r_g = \min_{i=1,2,...M} \{ r_i^* + \mu_i S_j \}
\]  

(4)
Proposition 2 in AM (1986) states that “in equilibrium, the observed market (gross) return is an increasing and concave piecewise-linear function of the relative spread” (see Fig. 1). They prove this proposition by letting \( f_i(S) = r^*_i + \mu_i S \). By the above minimization, the gross return on an asset with a relative spread \( S \) is \( f(S) = \min_{i=1,2,...,M} f_i(S) \), and proposition 2 follows from the fact that monotonicity is preserved in \( f(S) \) (the minimum function), and a minimum of a finite collection of linear functions is concave piecewise-linear. This proof of a concave piecewise-linear relation between the expected gross return and the relative spread is in the context of the definition of the gross return as a linear function of the expected relative spread.

***Insert Figure 1 here***

We change AM’s economy, such that assets pay no dividends. Instead, issuers distribute cash flows to shareholders through stock repurchase. The dollar amount repurchased for asset \( j \) is \( \omega_j \), a perpetual positive and constant cash flow per unit of time. Investors sell the repurchased shares to the issuer through a market maker for the quoted bid price, so that the periodic after-spread cash flow from stock repurchase to the asset holder is \( \omega_j(1 - S_j) \). In most case, there is an attempt to set the repurchase price such that it
approximates the fair market value of the stock at the time of the repurchase. Usually, the repurchase price is based on the average market price during a given period prior to the relevant purchase. If closing buy and sell trades are equally likely during this period, then it is reasonable to assume that the repurchase price is at the mid price, i.e., the average of the bid and ask prices: \( \omega_j \left(1 - \frac{S_j}{2}\right) \). The following analysis is insensitive to whether the repurchase price is set equal to the mid price or to the bid price (as we assume). In this altered AM economy, asset holders pay transaction costs for the periodic price appreciation. The expected present value of the portfolio held by a type-\( i \) investor is given by:

\[
E_{T_i} \left\{ \int_0^{T_i} e^{-\rho y} \left[ \sum_{j=0}^{N} x_{ij} \omega_j \left(1 - S_j\right) \right] dy \right\} + E_{T_i} \left\{ e^{-\rho T_i} \left[ \sum_{j=0}^{N} x_{ij} V_j \left(1 - S_j\right) \right] \right\}
\]

where \( x_{ij} \) is the number of units of asset \( j \) held by investor \( i \), and \( \rho \) is the riskless return on the zero-spread asset. The above expression is the sum of the expected present value of continuous cash flow from stock repurchase during investor \( i \)'s holding period and the expected present value of the investor’s liquidation revenue. As derived in the appendix, this expression can
be written as:

\[(\mu_i + \rho)^{-1}\sum_{j=0}^{N} x_{ij}[\omega_j(1 - S_j) + \mu_iV_j(1 - S_j)]\]

From the above expression for the expected present value of the portfolio, one may view the firm’s preference of stock repurchase over dividend distribution as a value-eroding strategy for stockholders. Under stock repurchase, even a stockholder with an infinitely long investment horizon that has no plans to liquidate the asset (i.e., \(\mu_i \to 0\)), has to pay bid-ask spread costs based on the amount repurchased periodically. Superficially, this may appear as an unnecessary tax. However, we argue that this expense is necessary, as it ensures the continued provision of liquidity services by market makers. In other words, under this repurchase strategy the firm is ensuring that the market makers are compensated periodically for standing by to provide liquidity, even when the investment horizon of stockholders is infinitely long. Otherwise, market makers will demand a higher bid-ask spread for standing for market making with no trading activity over long durations. This argument corresponds to Amihud and Mendelson’s (2000) argument that companies must engage in strategies designed to increase the liquidity.
of their stocks in order to achieve a higher stock price.

Thus, given asset bid-ask prices, a type-\(i\) investor, maximizing the expected present value of his/her portfolio, solves the following optimization problem:

\[
\max_{x_{ij}} \sum_{j=0}^{N} x_{ij} \left[ \omega_j (1 - S_j) + \mu_i V_j (1 - S_j) \right]
\]

subject to

\[
\sum_{j=0}^{N} x_{ij} V_j \leq W_i \quad \text{and} \quad x_{ij} \geq 0 \quad \text{for all} \quad j = 0, 1, \ldots, N.
\]

The modified net (after spread) expected return in the modified economy, \(r_{ij}^m\), is given by:

\[
r_{ij}^m = \frac{\omega_j (1 - S_j)}{V_j} - \mu_i S_j. \quad (5)
\]

We denote the modified gross (pre-spread) return as \(r_{gj}^m \equiv \frac{\omega_j}{V_j}.\) Thus, the modified expected gross rate of return is given by

\[
r_{gj}^m = \frac{(r_{ij}^m + \mu_i S_j)}{(1 - S_j)}. \quad (6)
\]

This is a strictly increasing and \textbf{convex} function in the relative spread cost.
Thus, if for a given price vector $V$, investor $i$ chooses the asset with

$$r_{i}^{m*} = \max_{j=0,1,2,...,N} \{r_{ij}^{m}\},$$  \hspace{1cm} (7)$$

then the equilibrium gross return on asset $j$ is the minimal required gross return across all investors:

$$\overline{r}_{g}^{m*} = \min_{i=1,2,...,M} \left\{ \frac{(r_{i}^{m*} + \mu_{i}S_{j})}{(1 - S_{j})} \right\}.  \hspace{1cm} (8)$$

Stated differently, the equilibrium price of asset $j$ is determined by the highest bidder:

$$V_{j}^{*} = \max_{i=1,2,...,M} \left\{ \omega_{j}(1 - S_{j}) \right\} \frac{r_{i}^{m*} + \mu_{i}S_{j}}{(1 - S_{j})}.  \hspace{1cm} (9)$$

If in equilibrium the asset is held by investor $i$, then we can rewrite Eq. (9) as:

$$V_{j}^{*} = \frac{\omega_{j}(1 - S_{j})}{r_{i}^{m*}} - \frac{\mu_{i}V_{j}^{*}S_{j}}{r_{i}^{m*}}.  \hspace{1cm} (10)$$

The first term on the right-hand side of Eq. (10) is the present value of perpetual cash flow from stock repurchase. The second term is the expected present value of cash outflows from the cost of transacting.

In the following proposition we demonstrate that AM’s clientele effect
still holds under the modified economy.

**Proposition 1** In equilibrium, higher-spread assets are held by longer-term investors (clientele effect).

**Proof.** Consider assets $j$ and $k$ with $S_k \geq S_j$. Suppose that in equilibrium asset $j$ is held by a type-$i$ investor and asset $k$ is held by investor $i + 1$, where $\mu_i \geq \mu_{i+1}$. This, with maximization (7), implies that $r_{ij}^m \geq r_{ik}^m$ and $r_{i+1,j}^m \geq r_{i+1,k}^m$. Substituting from Eq. (5) we get:

$$\omega_j \left(1 - S_j\right) V_j^* - \mu_i S_j \geq \omega_k \left(1 - S_k\right) V_k^* - \mu_i S_k,$$

and

$$\omega_k \left(1 - S_k\right) V_k^* - \mu_{i+1} S_k \geq \omega_j \left(1 - S_j\right) V_j^* - \mu_{i+1} S_j.$$ 

This implies that $(\mu_i - \mu_{i+1})(S_k - S_j) \geq 0$. Thus, when $\mu_i > \mu_{i+1}$ it must be that $S_k \geq S_j$.

The proof for non-consecutive portfolios immediately follows. Q.E.D.

As in AM’s work, the current model predicts that long-term investors will tend to hold illiquid assets, while short-term investors will hold the more liquid assets. Next, in proposition 2, we modify AM’s proposition regarding the concavity of the gross return-spread relationship.

**Proposition 2** In equilibrium, the gross return-spread relationship is increasing and piecewise-convex (gross return-spread relationship).

**Proof.** Let $h_i(S) = \frac{(r_{ij}^* + \mu_i S_j)}{(1 - S_j)}$. By minimization (8), the market gross return on an asset with a relative spread $S$ is $h(S) = \min_{i=1,2,\ldots,M} h_i(S)$, and propo-
sition 2 follows from the fact that monotonicity is preserved in \( h(S) \) (the minimum function), and a minimum of a finite collection of convex functions is piecewise-convex. Q.E.D. ■

Fig. 2 describes the relation between expected return and spread resulting from minimization Eq. (8). The required expected gross return is increasing and piecewise-convex. This follows, as for every investor \( i \), \( \left( \frac{\sum_{j=1}^{n} \mu_{i} S_{j}}{1 - S_{j}} \right) \) is increasing and convex in \( S_{j} \).

***Insert Figure 2 here***

If we assume that there are more short and intermediate term investors than investors with a very long horizon, minimization (8) produces an approximation for a concave relation between the expected gross return and the relative spread for lower levels of the relative spread. As the relative spread reaches one, the approximated relation becomes convex. The \( M^{th} \) investor, who holds the asset with the highest relative spread, has a convex function growing to infinity asymptotically to the vertical line \( S_{j} = 1 \). This follows, as for an asset with 100% expected relative spread, the investor demands an infinite compensation.
Therefore, allowing asset prices to appreciate instead of paying dividends, AM’s framework permits a function that is roughly concave for low relative spreads and convex for higher relative spread levels (see Fig. 2). This suggests that AM’s clientele effect dominates JFG’s level effect for highly liquid assets, with a higher value of additional amortized spread costs resulting from a given increase in the relative spread. On the other hand, JFG’s level effect dominates AM’s clientele effect for thinly traded assets, as the relative spread approaches 100% and investor’s required gross return approaches infinity.

AM provide important empirical evidence supporting their model. The data used for the empirical test is sampled from the NYSE (New York Stock Exchange) for the years 1960-1979. It is reasonable that a market as liquid as the NYSE is in the concave range, where the clientele effect dominates the level effect. The highest average of the beginning and end-of-year relative spread ratio reported by AM is close 3.2% (or 1.6% relative half spread). Eleswarapu and Reinganum (1993) show that for a sample of NYSE stocks during the period 1961-1990, a positive relation between excess returns and relative bid-ask spreads is significant only for the month of January, with no significant liquidity premium for the non-January months. Eleswarapu (1997) finds a significant positive liquidity premium for NASDAQ stocks over the
1973-1990 period for both the month of January and non-January months. Eleswarapu and Reinganum and Eleswarapu test for a linear relation. Thus, these two studies do not investigate the curvature issue.

To test the extended AM model developed in this study, we consider the relation between excess return and relative spread of stocks traded on the NASDAQ. Since a sample from NASDAQ stocks includes a larger spectrum of relative bid-ask spread ratios relative to a NYSE sample, it is an appropriate laboratory in which to test our conclusions. In the terms of the model derived in this paper, if stocks in the NASDAQ sample used by Eleswarapu (1997) cover both the concave and the convex regions of the relation between excess return and relative spread, then Eleswarapu’s positive relation is expected. We do so in the next section.

3 Empirical evidence

In this section we test the major hypothesis of our analysis through studying the nature of the relation between returns and liquidity. We provide evidence of the existence of a concave relation for liquid stocks and convex relation for illiquid stocks.
The data set consists of all nonfinancial firms trading on NASDAQ between July 1984 and June 1997. The primary data is from the Center for Research and Security Prices (CRSP) database. Monthly return is calculated as the percentage change in the value of one dollar of investment during month $t$. Monthly return is adjusted after distributions so that comparisons can be made on an equivalent basis before and after distributions.

Monthly bid and ask prices represent the closing bid or ask prices on the last trading day of the month. Due to source limitations, CRSP does not report "plain vanilla" bid and asks for all NASDAQ securities in February 1986. On this date, the bid and ask values are proxied by the lowest and highest closing price on the last day of the month, respectively. Monthly price is calculated as the average of the monthly closing bid and ask prices. In other words, price represents the mid price. The half-spread ratio for stock $j$ in month $t$ is calculated as:

$$S_{jt} = \frac{(ask_{jt} - bid_{jt})}{price_{jt}} = \frac{ask_{jt} - bid_{jt}}{ask_{jt} + bid_{jt}}$$

(11)

Our relative spread is consistent with JFG’s definition of relative spread, which is half the conventional definition of relative spread as in AM. The
dollar spread in the current paper is given by \((ask - bid)/2\), not \((ask - bid)\).

Note that the relative half spread is a simple linear transformation of the conventional relative spread. Thus, our choice of relative half spreads rather than full relative spreads has no bearing on the results presented in this section.

The relation is explored using 169 equally weighted portfolios. Portfolio formation takes place as follows. For each year we sort all nonfinancial NASDAQ stocks that have a 36 month history of complete return data, with the last of these 36 months June of the given year. These stocks are placed into one of 13 relative spread portfolio groups based on relative spread ranking. The spread measure used in the ranking is the average of two relative spread observations, where relative spread is specified in equation (11). The two observations are June of the given year and July of the previous year. This method of determining relative spread follows AM and Elsewarapu and Reinganum (1993).

We next calculate beta for each stock in each of the 13 relative spread portfolio group. Beta is calculated as the sum of the slopes in the regression of excess returns on the current and previous month’s excess market returns. Unlike AM, we include a single lag in the calculation, following
Fama and French (1992). Specifically, we estimate the beta coefficient using the following regression model:

\[ R_{jt} = \alpha_j + \beta_{j1} R_{mt} + \beta_{j2} R_{mt-1} + \varepsilon_{jt} \]  

(12)

where \( R_{jt} \) is the month-\( t \) excess returns over the 90-day T-bill rates of stock \( j \). \( R_{mt} \) and \( R_{mt-1} \) are month \( t \) and \( t - 1 \) excess market returns, calculated as the excess return of the value weighted portfolio in the given month. We define the beta for stock \( j \) as \( \beta_j = \beta_{j1} + \beta_{j2} \).

Each of the 13 relative spread portfolio groups are further subdivided into 13 portfolios based on the beta ranking. Hence, we have 169 portfolios for each year. For each portfolio, for each year, we extract the arithmetic average portfolio excess return, \( R_{pt} \), and the arithmetic average portfolio relative spread, \( S_{pt} \) for the following 12 months. Survivorship bias is reduced through including stocks that disappear within a test year. Using \( R_{pt} \), we define portfolio beta as \( \beta^n_p = \beta^n_{p1} + \beta^n_{p2} \) where \( \beta^n_{p1}, \beta^n_{p2} \) are defined by the regression model

\[ R_{pt} = \alpha_p + \beta^n_{p1} R_{mt} + \beta^n_{p2} R_{mt-1} + \varepsilon_{pt} \]  

(13)
Based on the above, each portfolio $p$ for month $n$ is characterized by $(R_{pn}, S_{pn}, \beta^n_p)$. Through calculating $(R_{pn}, S_{pn}, \beta^n_p)$ for the 12 months following the formation of 169 portfolios each year, we have values of $(R_{pn}, S_{pn}, \beta^n_p)$ for each month between July 1987 and June 1997.

Please see Table 1 for descriptive statistics of the three vectors: excess return, relative spread and beta. Table 2 provides the average excess return, relative spread, and beta values for each of the 13 relative spread and beta groupings. Comparison with AM suggests that the relative spreads in our sample encompass a much wider range.\(^1\) Table 2 demonstrates that relative spread ranges from 0.0113 through 0.1725 across the relative spread groupings, while AM reports (half) relative spread ratios ranging from 0.00243 through 0.01604 across the relative spread groupings. Our results are broadly similar to Eleswarapu (1997), who reports relative spreads ranging from 0.010025 through 0.15316 across seven relative spread groups, based on NASDAQ stocks. Figures 3, 4 and 5 plot the excess returns, relative bid-ask spread and beta for each of the 169 groupings, respectively.

\(^1\)In the ensuing discussion, relative spread values from AM and Eleswarapu are adjusted through dividing by two, to permit comparison with our convention.
To test for the predicted relation, we test three cross-sectional regression models.

\[ R_p = \alpha_0 + \alpha_1 \ln(S_p) + \alpha_2 \beta_p + \varepsilon_p \quad (14) \]

\[ R_p = \alpha_0 + \alpha_1 \ln(S_p) + \alpha_2 S^2_p + \alpha_3 \beta_p + \varepsilon_p \quad (15) \]

\[ R_p = \alpha_0 + \alpha_1 \ln(S_p) + \alpha_2 S^2_p + \varepsilon_p \quad (16) \]

The first regression model, Eq. (14), is tested to verify AM’s findings of concavity. The second regression model, Eq. (15), adds convexity to the AM test, to test for both the clientele and level effects incorporated in this paper. The third model, Eq. (16), is tested to determine if we find similar results without the beta variable.
The results of the regressions are presented in Table 3. The results of the first model verify AM’s findings of concavity. The coefficient associated with the ln(relative spread) variable is 0.00537, significant at the 1% level. The results of the second model supports the argument developed in this paper. Both ln(relative spread) and relative spread$^2$ have significant coefficients, at the 10% and 1% levels, respectively. The coefficient associated with ln(relative spread) is 0.00135, while the coefficient associated with relative spread$^2$ is 0.50469. As well, there is a 0.00321 coefficient associated with the beta variable, significant at the 5% level. The adjusted $R$ square of regression model 2 is 0.52, a significant improvement relative to the AM-type concave model with an $R$ square of only 37% of the cross-sectional variation of excess returns.

The results of the third model are broadly similar to the results in model 2. The adjusted $R$ square of regression model 3 indicate that it explains 51% of the cross-sectional variation of excess returns. Since the adjusted $R$ square associated with model 2 is approximately 52%, the inclusion of beta in regression model 2 presents an insignificant improvement in the model’s explanatory power. However, the significance of the beta coefficient in model 2 suggests that when accounting for convexity in the relation between return
and relative spread, we can identify a linear relation. This provides support for JFG, who argue that adjusting the CAPM for liquidity eliminates non-linearity in the model, particularly for thinly traded assets.

***Insert Table 3 here***

The regression results provide evidence that the relation between relative spread and excess return is both concave and convex, confirming the implications of the extended AM model developed in the current paper. The clientele effect dominates the level effect for highly liquid assets with lower relative spread ratios. On the other hand, the level effect dominates the clientele effect for thinly traded assets, as the relative spread approaches 100%. To further illustrate this relation, Fig. 6 presents a scatter plot of the relative spreads and excess returns, as well as the curve defined by regression model 3. Clearly, the relation predicted by the model developed in this paper is supported by the NASDAQ data. The critical (half) relative spread ratio where the curve turns from concave to convex is 3.93%. This suggests that highly liquid NYSE stocks tend to lie in the concave range, where the clientele effect dominates the level effect. Recall that the highest average relative (half) spread ratio reported in AM’s NYSE sample is close to 1.6%. Our
results show that a significant number of less liquid NASDAQ stocks lie in the convex range where the level effect dominates the clientele effect.

***Insert Figure 6 here***

4 Summary and conclusion

In this paper we examine the relation between gross return and relative bid-ask spread. We extend Amihud and Mendelson’s (1986) model, allowing the model to incorporate both Amihud and Mendelson’s clientele effect (concave relation) and Jacoby, et al.’s (2000) level effect (convex relation). Our resultant model predicts that the relation between gross return and relative spread is concave for highly liquid assets with low relative spreads and convex for thinly traded assets with higher relative spread ratios. We use NASDAQ data from CRSP to test the major hypothesis of our analysis through studying the nature of the relation between returns and the relative bid-ask spread. We provide evidence of the existence of a concave relation for liquid stocks and convex relation for illiquid stocks. This empirical evidence is in agreement with the implications of the extended Amihud and Mendelson model. We conclude that highly liquid NYSE stocks tend to lie in the concave range,
where the Amihud and Mendelson’s clientele effect dominates the Jacoby et al.’s level effect. Further, a significant number of less liquid NASDAQ stocks lie in the convex range where the level effect dominates the clientele effect. Finally, after controlling for convexity, we identify a linear relation between returns and beta. This supports Jacoby, et al.’s (2000) argument that such a relation exists once the CAPM is adjusted for liquidity effects.
The expected present value of the portfolio held by a type-\textit{i} investor is given by:

$$E^T_i \left\{ \int_0^{T_i} e^{-\rho y} \left[ \sum_{j=0}^{N} x_{ij} \omega_j (1 - S_j) \right] dy \right\} + E^T_i \left\{ e^{-\rho T_i} \left[ \sum_{j=0}^{N} x_{ij} V_j (1 - S_j) \right] \right\},$$

where \( x_{ij} \) is the number of units of asset \( j \) held by investor \( i \), and \( \rho \) is the riskless return on the zero-spread asset. We rewrite this term as:

$$\left[ \sum_{j=0}^{N} x_{ij} \omega_j (1 - S_j) \right] E^T_i \left\{ \int_0^{T_i} e^{-\rho y} dy \right\} + \left[ \sum_{j=0}^{N} x_{ij} V_j (1 - S_j) \right] E^T_i \left\{ e^{-\rho T_i} \right\}.$$

Since we assume that the investor’s investment horizon, \( T_i \), is exponentially distributed with \( E[T_i] = \mu_i^{-1} \), we write:

$$\left[ \sum_{j=0}^{N} x_{ij} \omega_j (1 - S_j) \right] \int_0^{\infty} \mu_i e^{-\mu_i t} \left( \int_0^{t} e^{-\rho y} dy \right) dt + \left[ \sum_{j=0}^{N} x_{ij} V_j (1 - S_j) \right] \int_0^{\infty} \mu_i e^{-\mu_i t} e^{-\rho t} dt.$$

Integrating the above expression yields:

$$\frac{\mu_i}{\rho} \left[ \sum_{j=0}^{N} x_{ij} \omega_j (1 - S_j) \right] \int_0^{\infty} e^{-\mu_i t} (1 - e^{-\rho t}) dt + \frac{\mu_i}{\mu_i + \rho} \left[ \sum_{j=0}^{N} x_{ij} V_j (1 - S_j) \right].$$
Solving the integral and rearranging, we find:

\[(\mu_i + \rho)^{-1} \sum_{j=0}^{N} x_{ij} [\omega_j (1 - S_j) + \mu_i V_j (1 - S_j)].\]
References


Table 1. Descriptive statistics of the 169 portfolio groupings. This table presents descriptive statistics for the excess return, relative spread and beta variables of the 169 portfolio groupings. We create 169 portfolios groupings of all nonfinancial CRSP-reported firms trading on NASDAQ between July 1984 and June 1997. We created the groupings through ranking the firms based on relative spread, creating 13 groupings. For each of these 13 groupings, we rank the firms based on firm CAPM beta, resulting in a total of 169 groupings. Excess return is firm monthly return in excess of the 90 day monthly T-bill rate. Relative spread is the difference between the ask and bid prices, divided by the sum of the ask and bid prices. Beta is the sum of the nonconstant coefficient estimates in a regression of firm excess return against current period and single lag market excess return.

<table>
<thead>
<tr>
<th></th>
<th>Excess Return</th>
<th>Relative Spread</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0082</td>
<td>0.0508</td>
<td>0.9628</td>
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<tr>
<td>Standard Error</td>
<td>0.0006</td>
<td>0.0036</td>
<td>0.0210</td>
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<tr>
<td>Median</td>
<td>0.0065</td>
<td>0.0345</td>
<td>0.9432</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.0072</td>
<td>0.0472</td>
<td>0.2731</td>
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<tr>
<td>Minimum</td>
<td>-0.0069</td>
<td>0.0089</td>
<td>0.2687</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.0449</td>
<td>0.2010</td>
<td>1.6462</td>
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</table>
Table 2. Average excess return, relative spread, and beta values for each of the 13 relative spread and beta groupings. This table presents the average values of the excess return, relative spread and beta variables for both the 13 relative spread and 13 beta groupings. We created groupings of all nonfinancial CRSP-reported firms trading on NASDAQ between July 1984 and June 1997. We first ranked the firms based on relative spread, creating 13 groupings. For each of these 13 groupings, we rank the firms based on firm CAPM beta, creating 13 groups. The average values reported for the beta groupings represents the average of the first through thirteenth groups of each relative spread group. Excess return is firm monthly return in excess of the 90 day monthly T-bill rate. Relative spread is the difference between the ask and bid prices, divided by the sum of the ask and bid prices. Beta is the sum of the nonconstant coefficient estimates in a regression of firm excess return against current period and single lag market excess return.
<table>
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<tr>
<th>Spread groupings</th>
<th>Beta groupings</th>
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<tr>
<td></td>
<td>Excess return</td>
</tr>
<tr>
<td></td>
<td>Excess return</td>
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<tr>
<td>Lowest</td>
<td>0.0065</td>
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<td></td>
<td>0.0106</td>
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<td></td>
<td>0.0147</td>
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<tr>
<td>Highest</td>
<td>0.0222</td>
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<td></td>
<td>0.0080</td>
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Table 3. Ordinary least square regressions of portfolio excess returns on concavity, convexity, and beta variables. We create 169 portfolios groupings of all nonfinancial CRSP-reported firms trading on NASDAQ between July 1984 and June 1997. We created the groupings through ranking the firms based on relative spread, creating 13 groupings. For each of these 13 groupings, we rank the firms based on firm CAPM beta, resulting in a total of 169 groupings. In all models, the dependent variable is portfolio monthly returns in excess of the 90 day monthly T-bill rate. In model 1, the independent variables are \( \ln(\text{Relative Spread}) \) and Beta. \( \ln(\text{Relative Spread}) \) represents the natural logarithm of the relative spread and beta is calculated as the sum of the slopes in the regression of excess returns on the current and previous months excess market returns. In model 2, the independent variables are \( \ln(\text{Relative Spread}) \), Relative Spread\(^2\), and Beta. Relative Spread\(^2\) represents the square of the relative spread value of each portfolio. In model 3, the independent variables are \( \ln(\text{Relative Spread}) \) and Relative Spread\(^2\). Numbers in parentheses are t-statistics. Results significantly different from zero at 10, 5, and 1 percent level (2-tailed), are represented by *, ** and *** respectively.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Model (1)</th>
<th>Model (2)</th>
<th>Model (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln(Relative Spread)</td>
<td>0.00537 (9.94291)***</td>
<td>0.00135 (1.86473)*</td>
<td>0.00145 (1.98310)**</td>
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<tr>
<td>Relative Spread$^2$</td>
<td>0.50469 (7.29144)***</td>
<td>0.46920 (6.90124)***</td>
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<tr>
<td>Beta</td>
<td>0.00065 (0.39203)</td>
<td>0.00321 (2.16409)**</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.02550 (11.50670)***</td>
<td>0.00720 (2.27438)**</td>
<td>0.01079 (3.95719)***</td>
</tr>
<tr>
<td>N</td>
<td>169</td>
<td>169</td>
<td>169</td>
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<tr>
<td>Adj. R$^2$</td>
<td>0.37090</td>
<td>0.52132</td>
<td>0.51070</td>
</tr>
</tbody>
</table>

*, **, *** Significantly different from zero at 10, 5, and 1 percent level (2-tailed), respectively.
Figure 1. Amihud and Mendelson’s (1986) equilibrium. The gross return, $r_g$, is an increasing and concave piecewise-linear function of the relative spread, $S_j$. 

\[ \bar{r}_g^{m} \]

\[ S_j \]
Figure 2. Amihud and Mendelson’s framework with the clientele effect and Jacoby, Fowler, and Gottesman’s (2000) level effect. The gross return, $\bar{r}_g^m$, is a piecewise-convex function of the relative spread, $S_j$. The relation is approximately increasing and concave for lower levels of the relative spread, and becomes convex as the relative spread reaches 1.
Figure 3. Excess returns of the 169 groupings. We create 169 portfolios groupings of all nonfinancial CRSP-reported firms trading on NASDAQ between July 1984 and June 1997. We created the groupings through ranking the firms based on relative spread, creating 13 groupings. For each of these 13 groupings, we rank the firms based on firm CAPM beta, resulting in a total of 169 groupings. Starting from the left, the first 13 values are in the lowest relative spread grouping, the next 13 in the next-to-lowest relative spread grouping, etc. For each set of 13 values, the left-most value is the lowest beta grouping, the next-to-left is in the next-to-lowest beta groupings, etc.
Figure 4. Relative spreads of the 169 Groupings. We create 169 portfolios groupings of all nonfinancial CRSP-reported firms trading on NASDAQ between July 1984 and June 1997. We created the groupings through ranking the firms based on relative spread, creating 13 groupings. For each of these 13 groupings, we rank the firms based on firm CAPM beta, resulting in a total of 169 groupings. Starting from the left, the first 13 values are in the lowest relative spread grouping, the next 13 in the next-to-lowest relative spread grouping, etc. For each set of 13 values, the left-most value is the lowest beta grouping, the next-to-left is in the next-to-lowest beta groupings, etc.
Figure 5. Beta of the 169 groupings. We create 169 portfolios groupings of all nonfinancial CRSP-reported firms trading on NASDAQ between July 1984 and June 1997. We created the groupings through ranking the firms based on relative spread, creating 13 groupings. For each of these 13 groupings, we rank the firms based on firm CAPM beta, resulting in a total of 169 groupings. Starting from the left, the first 13 values are in the lowest relative spread grouping, the next 13 in the next-to-lowest relative spread grouping, etc. For each set of 13 values, the left-most value is the lowest beta grouping, the next-to-left is in the next-to-lowest beta groupings, etc.
Figure 6. Scatter plot of relative spreads and fitted curve. We create 169 portfolios groupings of all nonfinancial CRSP-reported firms trading on NASDAQ between July 1984 and June 1997. We created the groupings through ranking the firms based on relative spread, creating 13 groupings. For each of these 13 groupings, we rank the firms based on firm CAPM beta, resulting in a total of 169 groupings. The average relative spread and average returns in excess of the 90 day monthly T-bill rate are calculated for the 169 portfolios, formed on the basis of relative spread and beta rankings. The vertical axis represents the excess return, and the horizontal axis represents the relative spread.