

Policyholder Surrender Behaviors under Extreme Financial Conditions

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Abstract

We try to model surrender rates with a few explanatory variables such as the difference between reference market rates and product crediting rates with surrender charges, the policy age since the contract was issued (duration), unemployment rates, economy growth rates, and seasonal effects using logit function. We consider the policy holder surrender behaviors of US single premium deferred annuities (SPDA) and Korean interest indexed annuities under extreme financial conditions.

Keywords: Surrender/Lapse Rate Model, Extreme Financial Conditions, Surrender Rate Changes

1 Introduction

Modeling appropriate interest rate sensitive surrender/lapse rates is essential in managing assets and liabilities of insurance companies. Even though there are a few research papers on the interest sensitivity of the cash flows, the analysis is focused usually on asset sides. For example, in Pesando's (1974) paper, the cash flow analysis considers the prepayment rate impacts only. But we have to mention that the interest sensitivity of cash flows through surrender rate fluctuations is a kind of "dual problem" to that through prepayment rate fluctuations. So it is important to consider surrender rate impacts on cash flow analysis with proper surrender rate models.

There are many factors affecting surrender/lapse rates such as the difference between reference market rate and policy crediting rate, seasonal effect, age and gender of clients, economy growth rate, foreign exchange rate, inflation rate, policy age since

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issue date, and unemployment rate, etc. Kim (2004a) presents surrender rate models with explanatory variables such as the difference between reference rates and crediting rates, policy age since issue, financial crises, unemployment rates, economy growth rates, seasonal effects and so on. He uses the logit function and the complementary log-log function in modeling surrender rates and shows that the logit model and the complementary log-log model are generally better than the existing surrender rate models such as arctangent model. He also shows that the surrender rate models are different according to insurance policy types and finds proper surrender rate models for the four insurance groups: protection plans, education plans, endowment, and annuities.

The surrender rate level has great influences on the cash flows of assets and liabilities. To reflect the exact impacts of surrender rate in asset/liability management (ALM) framework, it is inevitable to consider and devise a proper surrender/lapse rate model. Kim (2004b) investigates the surrender rate impacts on the value, the duration, and the convexity of interest indexed annuities.

In this paper we try to define the extreme economic conditions to be considered and quantify their impact on policyholder surrender behaviors. First we gather data in order to understand and quantify the causes of lapse behavior under extreme conditions. Sources of this data include large US insurance writers and Korean data (to examine economic stress). We consider surrender rate models reflecting the complicated policyholder surrender behaviors with endogenous and exogenous multi-variables. We use Logit model to describe the surrender rate experiences of Korean interest indexed annuities and US single premium deferred annuities. We try to model surrender rates with a few explanatory variables and develop better estimates of policyholder surrender/lapse behavior under extreme conditions, where the extreme condition is defined as more than 2 standard deviations from the expected level (which may vary by duration), under varying economic conditions, and in combination with different policy characteristics.

2 The Structure of Single Premium Deferred Annuities

Many insurance companies are selling single premium deferred annuities (SPDA). But SPDA are sold with the primary focus on accumulation. Only a few of the policy holders purchase SPDA for the purpose of annuitization. In Korea, the annuity market is still young and growing slowly¹ compared to that of the United States

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¹ The volume of in-force and new contracts of annuities in Korea is not really large compared to that in

(US). The SPDA crediting interest rates are declared each month/year by the issuing companies. Although that is the predominant structure in Korea, other variants such as multiple-year guarantees and interest-indexed annuities (IIA) are also popular.

The distinctive features of SPDA are the surrender options and annuitization options. The purchasers of SPDA can surrender at any time before annuitization if the new money rates (or competitive market rates) move to their advantage with reasonable surrender charges. Kim (2004c) discusses the valuation of the surrender options in interest-indexed annuities (IIA). At the date of annuitization, they may also select one type of annuity out of four choices: lump sum of their account value, whole life annuity, fixed term annuity, or inheritance annuity. They might terminate the contract with the lump sum withdrawal of their account value. Selecting whole life annuity, the annuitant receives annuities as long as he/she is alive with ten year fixed annuity guarantees. The annuitant of an inheritance annuity receives only the interest of the account value each year while he/she is alive, and the principal account value at the time of annuitization will be given to the heir/heirress when the annuitant dies.

For Korean IIA, we consider 7 year interest indexed annuities. The death benefits are the account value plus 10% of premium, and another 10% of premium in the case of accidental death.

For US SPDA, we consider multiple annuity products with different surrender charge schedules. An example of the products is the 7 year fixed annuity SPDA, and its interest rate may be reset each year at end of each anniversary. After the first policy year, the policy-owner may surrender up to 10% of total account value each year without a surrender penalty, with excess over 10% subject to surrender charges. On full surrenders, the first 10% is penalty-free. Upon confinement in nursing home/hospital for at least 60 days, some or all of fund value may be withdrawn, provided it is within 90 days after end of confinement. The death benefits are the full fund value. Annuitizations are permitted starting in the first policy year, with no surrender charges provided the pay-out is for at least 5 years.

For various characteristics and valuation of SPDA, we may refer Society of Actuaries (1991), Cox, Laporte, Linney, and Lombardi (1992), and Asay, Bouyoucos, and Marciano (1993).

the United States of America. According to data from American Council of Life Insurers, the reserve value for annuity contracts in USA is about \$1,585,008 million. But, from the Korea Life Insurance Association data, it is about 44,927,906 million Korean won (US \$37,440 million with exchange rate of 1,200 Korean won for US \$1) in Korea in year 2001. The number of annuity contracts in force is about 6,406,000 in Korea (it is 66,548,000 in USA) and the number of newly issued annuities is about 822,000 in Korea (it is 7,641,000 in USA) in year 2001.

2.1 Crediting Interest Rates

Crediting interest rates may be reset each year at end of each anniversary for the fixed annuity SPDA. Many contracts guarantee a minimum interest rate below which the renewal crediting interest rates will not fall. For Korean IIA, the crediting interest rates are announced every month based on current market interest rates, current investment gain rates, and the expected future portfolio income gain rates. The main factor of the crediting rates is the market interest rates and this is why they call the products the interest-indexed annuities.

The majority of contracts guarantee interest for one-year periods; however, longer guarantees are available, with 5-years being the most popular. After the initial 5-year guarantee, the contract might (a) automatically roll into another 5-year guarantee at current rates, (b) automatically switch to annual guarantees, or (c) give a choice between the two. The longer guarantees have gained increasing popularity as some purchasers and salesmen have gotten uncomfortable with “trust me” annual interest declarations.

2.2 Surrender Charges

Many contracts credit the full premium to the account value and assess surrender charges when the policy holder surrenders. The amount of surrender charges are usually from 7% to 10% of the account value and decreased to zero over a 6-10 year period. The range of surrender charges of different companies may be higher or lower and the penalty periods may run for shorter or longer. For Korean IIA, we consider surrender charges from 7% of the account value and decreased to zero over a 6 year period. For US SPDA, we consider multiple annuity products with different surrender charge schedules. An example of the surrender charge schedule is 7%, 7%, 7%, 7%, 6%, 4%, 2% of the account value in years 1-7, 0% thereafter.

Usually the maximum initial surrender charge on an SPDA is about 10% and decreased by 1% annually. Surrender charges are generally waived for certain withdrawals, which are called Free Partial Withdrawals. On full surrenders, the first portion of the account value, for example 10%, is penalty-free.

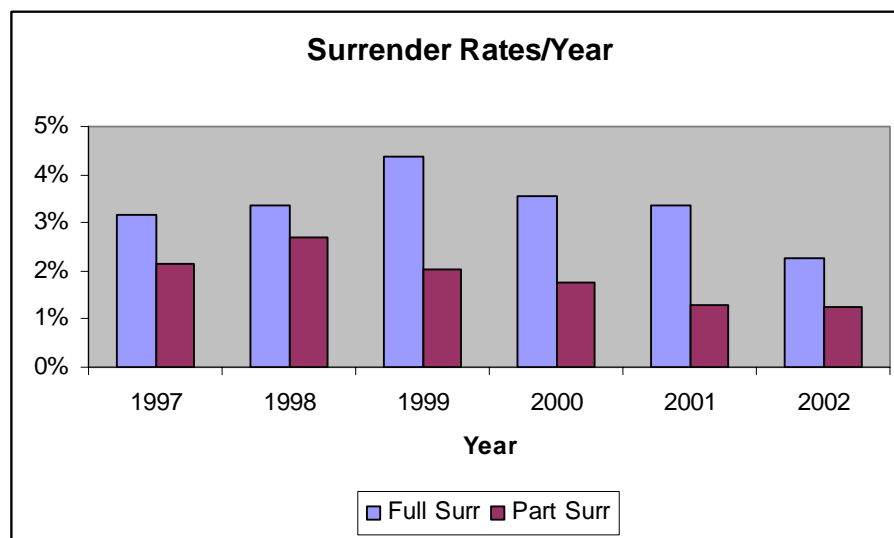
2.3 Free Partial Withdrawals

A portion of the account value can be withdrawn at any time without surrender charges to provide liquidity to the contract owner. The maximum level is 90% of the account value at the time of partial withdrawal, but a few companies might limit the maximum level much lower than 90% of the account value. For example, after the first policy year, the policy-owner may surrender up to 10% of total account value each year without a surrender penalty, with excess over 10% subject to surrender charges. Often the policy holders can take advantage of this partial withdrawal option several times a year. For example, when the stock markets show signs of an upward jump, the policy holders can draw out their savings from the account without any surrender charges and invest this amount of money in the stock markets. After enjoying the profits from the stock market, they can return to their insurance contracts paying relatively low interest. So this characteristic of high maximum level of partial withdrawal without surrender charges is a source that one might overuse the partial withdrawal option. For some contracts, upon confinement in nursing home/hospital for at least 60 days, some or all of fund value may be withdrawn, provided it is within 90 days after end of confinement.

Figure 1 shows the full surrender rates and partial surrender rates of US insurance companies from year 1997 to year 2002. The average partial surrender rate is about 1.9% each year, relatively high compared to the average of the full surrender rate, 3.4% each year.

Moreover the death benefit amount is still guaranteed during the partial withdrawal period.

Figure 1. Full and Partial Surrender Rates of US-SPDA/Year



2.4 Death Benefits

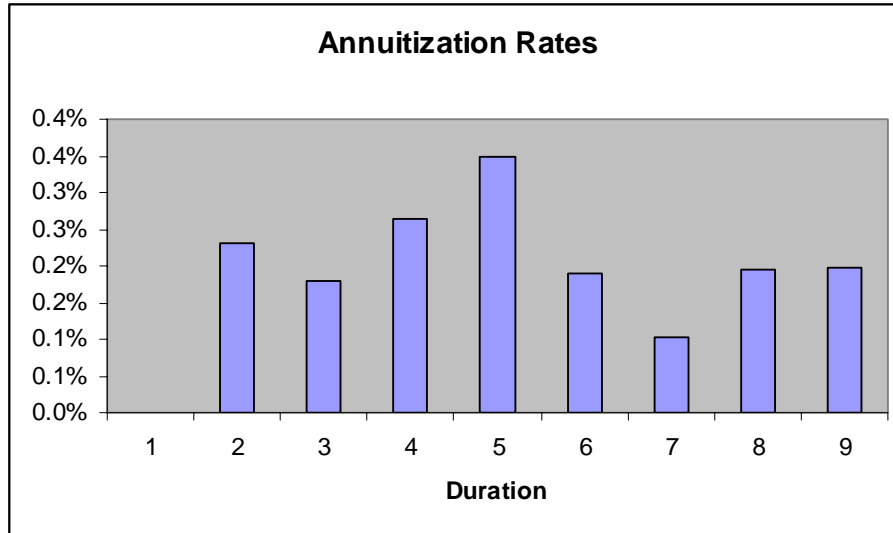
Usually the death benefit is the account value. A few variations of death benefits are considered according to the companies, for example, the account value plus 10% of premium, and another 10% of premium in the case of accidental death. Some contracts allow the spouse to take over ownership of the contract at the time of death of the owner if the spouse was a beneficiary.

2.5 Annuitization

The policy holder can choose the initial annuitization date. The owner may change it before the chosen initial annuitization date. Annuitizations are permitted starting in first policy year, with no surrender charge provided the payout is for at least 5 years for some US SPDA. For Korean IIA, the range of the initial annuitization date is from age 45 to age 70 and usually 10 years after issue. Guaranteed annuitization rates may be announced by the company, but these rates are really conservative. The crediting rates reflect the current market rates and portfolio income gain rates with minimum guaranteed rate of 3%. But the guaranteed annuitization rates may be based on the minimum guaranteed rate of 3% plus very conservative bonus. Some policy holders prefer minimum rate of return guaranteed products. The mortality may be mildly conservative reflecting annual improvement factors, in recognition of anticipated future mortality reductions.

Approximately less than 2% of deferred annuity values are annuitized each year in both Korea and US. There are several factors for this low annuitization ratio. The main reason is that much of the business is still young and could be considered too early for annuitization. Many purchasers want to pass their annuity accumulation values to their heirs at death. The other reason is that many purchasers do not want to give up control of their investment and, consequently, prefer to take partial withdrawals in lieu of annuitization. Figure 2 shows the annuitization rates of US SPDA according to the duration.

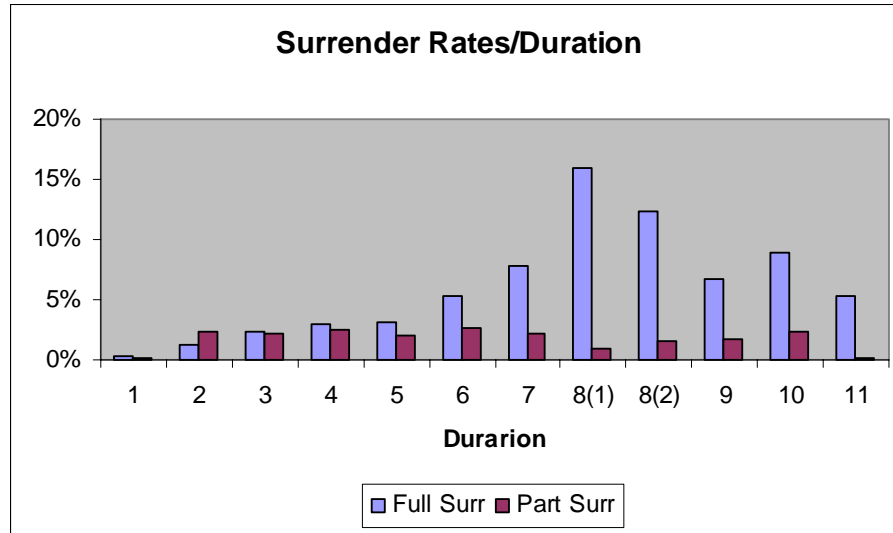
Figure 2. Annuitization Rates of US-SPDA / Duration



3 Modeling Surrender Rates for Korean Interest Indexed Annuities and US SPDA

We have seen that the SPDA/IA product provides the policy holder with a surrender option that he/she may surrender the contract early with specified surrender charges. As market rates rise, we might think that the SPDA/IA owners would surrender their contracts and reinvest the surrender cash value in high yielding alternatives. But the surrender option may not be exercised by every policy holder even though the market rates rise. That is, it is not exercised optimally. As we show below, the surrender option is not a function of interest rate only. It depends on the policy age since the contract was issued. It also reflects the unemployment rate and the economy growth rate. Actually the surrender tendency varies between policy holders. So we have to model the policy holder surrender behavior statistically. The variables considered are (a) the difference between the reference new money rates (or market rates) and the product crediting rates with surrender charges, (b) the policy age since the contract was issued (or the duration), (c) unemployment rates, (d) economy growth rates, and (e) seasonal effects.

Figure 3. Surrender Rates of US-SPDA/ Duration

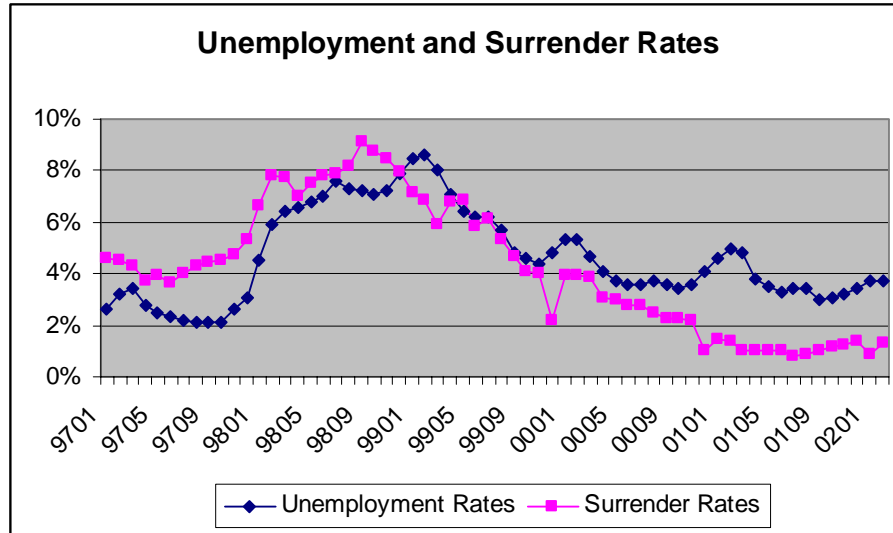


Especially the duration, i.e. the policy age since the contract was issued, is one of the most important factors of surrender rates. Figure 3 shows the surrender rates of US SPDA according to the duration. The policy is seven year fixed SPDA. For the first five years, the surrender rates are increasing slowly. The surrender rates on the 6th and 7th years are relatively high. Duration 8(1) is first 3 months of the 8th contract year, and the surrender rates are almost 16%. Duration 8(2) is months 4-12 of the 8th contract year, and the surrender rates are almost 14%. We can notice that almost 30% of the contracts are withdrawn on the 8th contract year, right after the accumulation period, 7 years.

Figure 4 and Figure 5 show the relationships between the unemployment rates, the market interest rates and the surrender rates of Korean IIA. We can easily notice that the unemployment rates, market interest rates, and the surrender rates soared up rapidly during the financial crises, from December 1997 to December 1998.

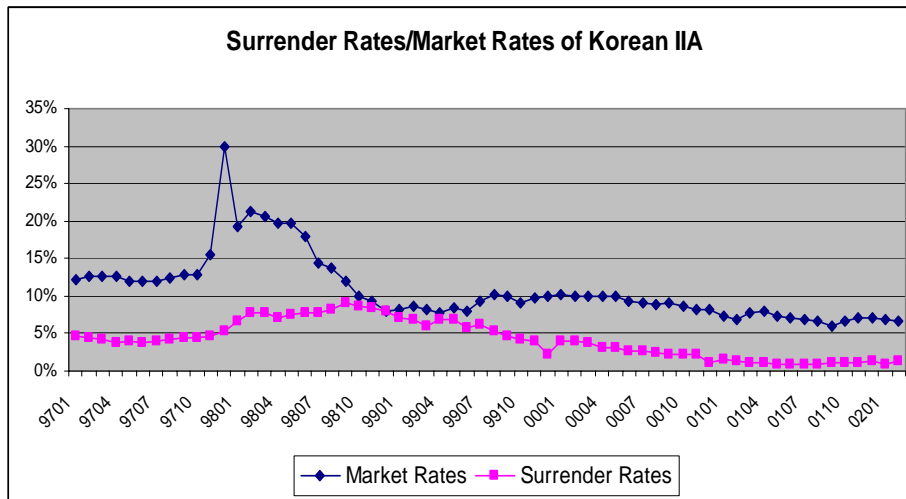
We can conjecture that the surrender rates are dependent not only on interest rates but also on exogenous factors such as unemployment rates, economy growth rates, seasonal effects, and so on.

Figure 4. Unemployment Rates and Surrender Rates of Korean IIA



Source : Unemployment Rate ; Korea National Statistical Office (www.nso.go.kr)

Figure 5. Market Interest Rates and Surrender Rates of Korean IIA



Source: Market Rates; 5 year government bond rates; The Bank of Korea (www.ecos.bok.or.kr)

We use logit link function in modeling the surrender rates of Korean IIA and US SPDA. We can use logit functions for odds and probability functions. There are many

examples in which logit functions are used for financial data analysis. Hall (2000) compares logit analysis of data to the results from his prepayment model. Pinder (1996) demonstrates how multinomial logit models can be used in a decision analysis framework to estimate expected monetary value. Kolari, Glennon, Shin, and Caputo (2002) use the parametric approach of logit analysis to predict large commercial bank failures. We may refer Johnsen and Melicher (1994), and Lo (1986).As modeling programs, we use the Generalized Linear Models², Procedure GENMOD, Logistic Regression Models, and Procedure LOGISTIC, with SAS³.

The Logit function has the following form,

$$\ln\left(\frac{q_s}{1-q_s}\right) = \beta_0 + \beta_1 V_1 + \dots + \beta_n V_n, \quad (1)$$

where q_s is the surrender rate, β_i is the coefficient to be estimated and V_i is the explanatory variable.

For Korean IIA surrender rate models with 3 year duration⁴, we use the Logit Model,

$$\ln\left(\frac{q_s(t)}{1-q_s(t)}\right) = \beta_0 + \sum_{j=0,2,4,6,8,10,12} \beta_j *(i_m(t-j) - i_c(t-j)) + \beta_{UE} * i_{UE}(t) + \beta_{EG} * i_{EG}(t) + \sum_{j=1}^{11} \beta_{month-j} * DV_j, \quad (2)$$

where DV_j is the seasonal effect dummy-variable.

The parameter estimates are shown in Table 1. It is interesting to note that the parameter β_{UE} for the unemployment rates is very large, 50.6348. It means that the surrender rates change very greatly according to the unemployment rate movements. But, considering the unemployment rate change ratio is not so radical as that of the reference market rates (new money rates), it is not strange for us to have a large β_{UE} . It

² We may refer a few books on generalized linear model such as Agresti (1996 and 2002), Harrell (2001), Kutner, Nachtschiem, and Wasserman (1996), McCullagh and Nelder (1989), Firth (1991), and McCulloch and Searle (2000).

³ In programming with SAS, refer Allison (1999), and SAS Institute Inc. (1999).

⁴ We can model surrender rates with the duration (policy age since issue date) as an explanatory variable. For more details see Appendix and Kim(2004a). We notice that almost 30% of the contracts are withdrawn on the 8th contract year, right after the accumulation period, 7 years, as shown in Figure 3. So duration is one of the main factors of the surrender rates. In this paper, we want to investigate the policyholder surrender behaviors under extreme financial conditions. So we just look at the contracts with the same duration; 3 years for Korean IIA and 5 years for US SPDA, this will help us to check the impacts on the surrender rates due to the economic variables.

seems also reasonable that the parameter β_{EG} for the economy growth rates is a negative number, -5.3360 . We can guess that when the economy condition is good the policyholders may not surrender their IIA policies.

Now our final model for the Korean IIA surrender rates, $\{q_s(t)\}$, is given by the following formula,

$$q_s(t) = \frac{1}{1 + \exp(-\alpha)}, \quad (3)$$

where

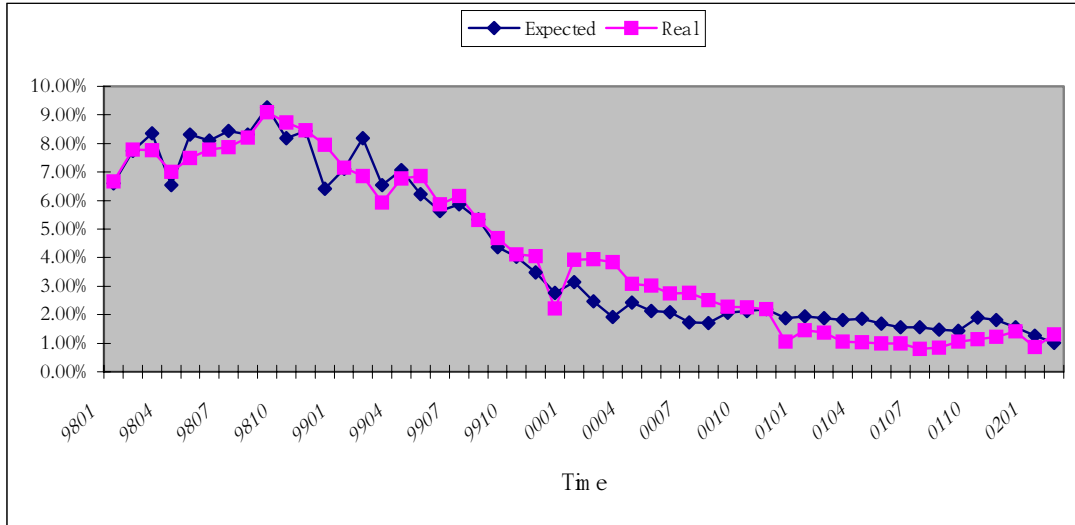
$$\alpha = \beta_0 + \beta_{UE} * i_{UE}(t) + \beta_{EG} * i_{EG}(t) + \sum_{j=0,2,4,6,8,10,12} \beta_j * \{i_m(t-j) - i_c(t-j)\} + \sum_{j=1}^{11} \beta_{month_j} * DV_j. \quad (4)$$

We show the graph of the real and predicted (using Logit model) surrender rates of Korean IIA policies below.

Table 1. Parameter Estimates with Logit Model (IIA)

Analysis of Maximum Likelihood Estimates					
Parameter	DF	Estimate	Standard Error	Chi-Square	Pr > ChiSq
Intercept	1	-6.0132	0.00617	950275.502	<.0001
DIFFLAG0	1	9.3465	0.0563	27551.1981	<.0001
DIFFLAG2	1	0.9728	0.0412	557.6077	<.0001
DIFFLAG4	1	-6.2020	0.0438	20031.9722	<.0001
DIFFLAG6	1	-2.7553	0.0399	4776.8774	<.0001
DIFFLAG8	1	1.4655	0.0390	1410.1121	<.0001
DIFFLAG10	1	0.5252	0.039	182.5160	<.0001
DIFFLAG12	1	-1.8470	0.0468	1557.8107	<.0001
Unemploy	1	50.6348	0.1640	95314.7985	<.0001
Eco GROWTH	1	-5.3360	0.1723	959.5427	<.0001
MONTH1	1	-0.2111	0.00409	2662.3227	<.0001
MONTH2	1	-0.4199	0.00446	8867.3221	<.0001
MONTH3	1	-0.3629	0.00446	6633.6120	<.0001
MONTH4	1	0.1121	0.00415	728.9672	<.0001
MONTH5	1	0.2443	0.00408	3589.7187	<.0001
MONTH6	1	0.2961	0.00424	4879.2107	<.0001
MONTH7	1	0.2111	0.00429	2421.8388	<.0001
MONTH8	1	0.2082	0.00458	2065.2003	<.0001
MONTH9	1	0.4040	0.00452	7970.0766	<.0001
MONTH10	1	0.4919	0.00469	11024.0567	<.0001
MONTH11	1	0.3720	0.00447	6913.5047	<.0001

Figure 6. Real and Predicted Surrender Rates of Korean IIA



For US SPDA surrender rate models with 5 year duration, we also use the Logit Model,

$$\ln\left(\frac{q_s(t)}{1-q_s(t)}\right) = \beta_0 + \beta_M *(i_m(t) - i_c(t)) + \beta_{UE} * i_{UE}(t) + \beta_{EG} * i_{EG}(t), \quad (5)$$

where β_M is the parameter for the difference between current reference market rates and policy crediting rates.

The parameter estimates are shown in Table 2. We can notice that the parameter β_{UE} for the unemployment rates is 24.3694 very smaller than that of the Korean IIA unemployment parameter, 50.6348. It means that the US SPDA surrender rates change less sensitively according to the unemployment rate movements. It seems also reasonable that the parameter β_{EG} for the economy growth rates is a negative number, -2.6450.

For the US SPDA, the surrender rates, $\{q_s(t)\}$, are estimated by the following formula,

$$q_s(t) = \frac{1}{1 + \exp(-\alpha)}, \quad (6)$$

where

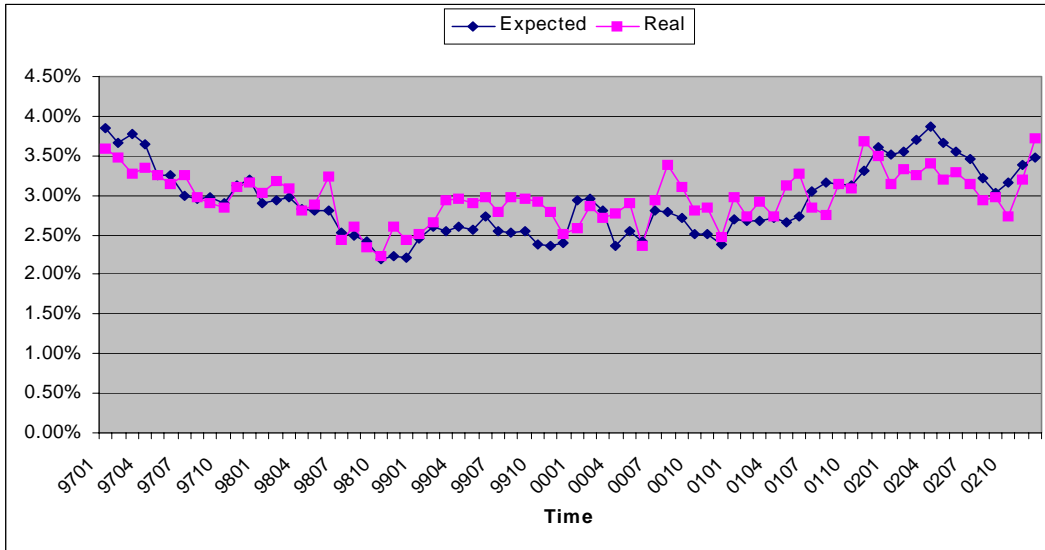
$$\alpha = \beta_0 + \beta_M *(i_m(t) - i_c(t)) + \beta_{UE} * i_{UE}(t) + \beta_{EG} * i_{EG}(t). \quad (7)$$

We show the graph of the real and predicted (using Logit model) surrender rates of US SPDA policies below. The average of the real surrender rates is 2.97% and the average of the expected (predicted) surrender rates using Logit model is 2.92%.

Table 2. Parameter Estimates with Logit Model (US SPDA)

Analysis of Maximum Likelihood Estimates					
Parameter	DF	Estimate	Standard Error	Chi-Square	Pr > ChiSq
Intercept	1	-4.5452	0.00785	89325.4212	<.0001
DIFFLAG	1	12.7525	0.05831	25413.1981	<.0001
Unemploy	1	24.3694	0.27836	59821.6548	<.0001
Eco GROWTH	1	-2.6450	0.86473	4862.2485	<.0001

Figure 7. Real and Predicted Surrender Rates of US SPDA



4 Surrender Rate Changes under Financial Rate Shocks

There are many factors affecting surrender rates such as the difference between reference market rate and policy crediting rate, seasonal effect, age and gender of clients, economy growth rate, foreign exchange rate, inflation rate, policy age since issue date (duration), and unemployment rate, etc. During the stable interest rate period, all of these factors play an important role in determining the surrender rate. But sometimes, if there are any shocks (or sudden changes) on financial rates, such as the unemployment rates, the economy growth rates, or the market interest rates, the surrender rates can be changed much more than expected. For example, during the financial crises in Korea from December 1997 to December 1998, the surrender rates show a sudden peak.

Figure 5 shows the sudden increase in the market interest rates during the financial crises and the surrender rates of Korean IIA, and we can see that interest rate fluctuation is really an important factor in determining the surrender rates. Figure 4 shows the unemployment rates and the surrender rates of Korean IIA. We can easily notice that the unemployment rates and surrender rates soared up during the financial crises. So we can conjecture that the surrender rates are dependent not only on interest rates but also on exogenous factors such as unemployment rates, economy growth rates, seasonal effects, and so on

Now we want to investigate the surrender rate changes under the assumption that there are financial rate shocks (or sudden changes). As an example, we first look at the pattern of the financial rate shocks during the Korean financial crises.

Let us denote $i(t)$ to be a financial rate at time t . We use the following formula for the financial rate at time t ,

$$i(t) = \mu + k(t) \sigma, \quad (8)$$

where μ is the average and σ is the standard deviation of the financial rate during a stable state period.

We define $k(t)$ to be a risk measure of the financial rate $i(t)$,

$$k(t) = \frac{i(t) - \mu}{\sigma}. \quad (9)$$

We define that the financial rate $i(t)$ experiences a financial rate shock at time t if

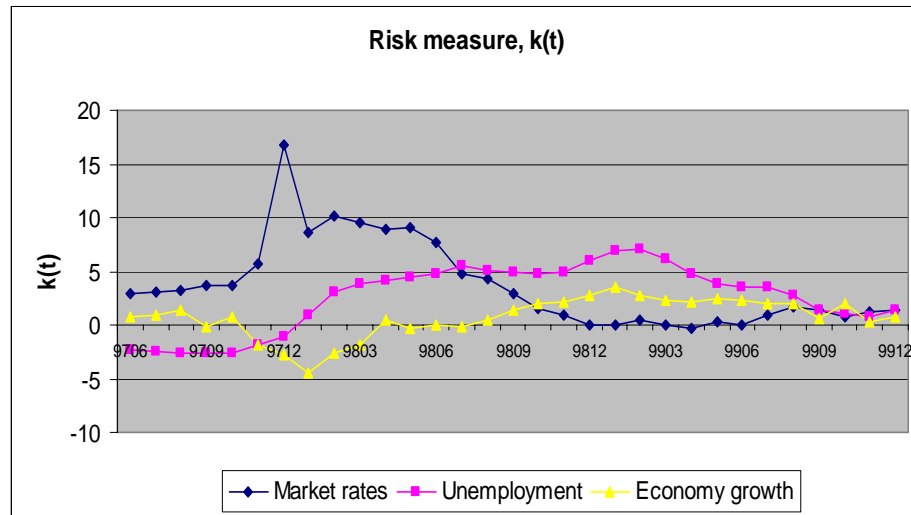
$$|k(t)| \geq 2, \quad (10)$$

and we say that the financial rate is in a stable status at time t if $|k(t)| < 2$. We also say that we are under extreme financial conditions if the financial rates experience financial rate shocks.

Figure 8 shows the risk measure $k(t)$ of the reference market rates (5 year government bond rates), the unemployment rates, and the economy growth rates of

Korea around the financial crises period.

Figure 8. Risk Measure, $k(t)$, of Korean Financial Rates



Source: Market Rates; 5 year government bond rates; The Bank of Korea (www.ecos.bok.or.kr)
 Unemployment Rate ; Korea National Statistical Office (www.nso.go.kr)
 Economy Growth Rates ; Korean Statistical Information System (www.kosis.nso.go.kr)

From Figure 8, we can notice that the market rates experience financial shocks, $k(t) > 2$, for the period from July of 1997 to September of 1998, for 14 months around the financial crises. The unemployment rates experience financial shocks, $k(t) \geq 2$, for the period from February of 1998 to August of 1999, for 19 months around the financial crises. The economy growth rates experience financial shocks, $k(t) \leq -2$, for the period from November of 1997 to March of 1998, for 5 months around the financial crises.

Figure 6 shows the real and expected surrender rates (using Logit model) of Korean IIA considering all of the financial rate shocks. The averages of the real and expected (predicted) surrender rates of Korean IIA are 4.2%.

Now, we want to consider the surrender rate changes of US SPDA under the assumption that US financial rates experience the financial rate shocks. We make two assumptions on the pattern of $k(t)$, the risk measures of the financial rates. Assumption 1 (A1): the pattern of $k(t)$ is same as that of Korean data when the rate experiences financial shocks. When the rate does not experience any financial shocks, the risk measure $k(t)$ is calculated from US data. Assumption 2 (A2): $k(t) = c$, where c is a constant integer such that $|c| \geq 2$ and the financial rate $i(t)$ is changed to $i(t) + c\sigma$.

Figure 9 shows the surrender rate changes of US SPDA under the assumption (A1) that the US market rates (10 year T-bond rates) experience the financial rate shock, $k(t) \geq 2$, as the same pattern of $k(t)$ as that of Korean data.

It shows a very high peak of 12.63% at the beginning of the market rate shock period. The average of the expected (predicted) surrender rates is 3.42% whereas the average of the real surrender rates is 2.97%.

Figure 9. US SPDA Surrender Rate Changes under Market Rate Shock (A1)

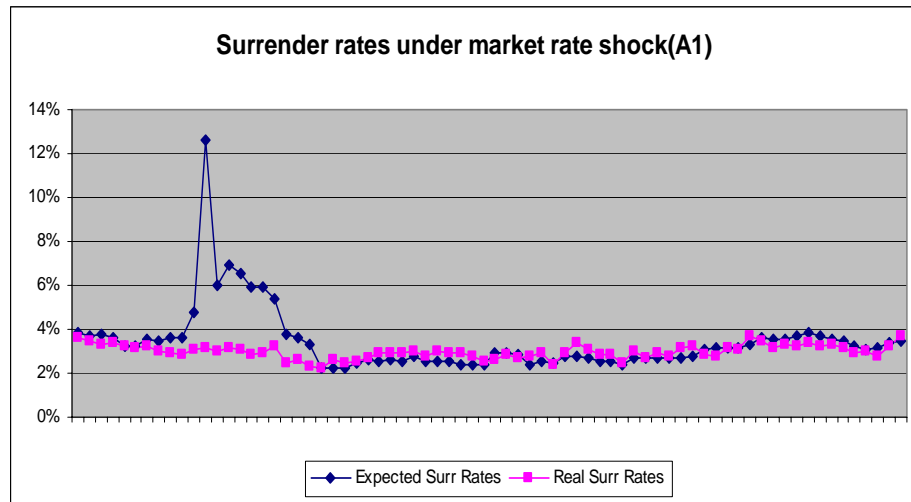


Figure 10. US SPDA Surrender Rate Changes under Market Rate Shock (A2)

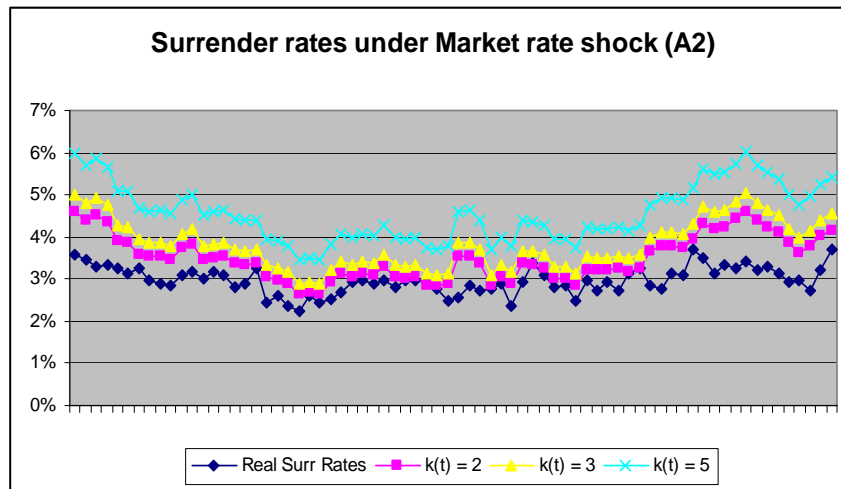


Figure 10 shows the surrender rate changes of US SPDA under the assumption (A2) that the US market rates (10 year T-bond rates) experience the financial rate shock, $k(t) = 2, 3, 5$ over the whole period.

It shows that the surrender rates are increasing as $k(t)$ goes up, i.e. market rates increase. The average of the expected (predicted) surrender rates is 3.49% when $k(t) = 2$, 3.82% when $k(t) = 3$, and 4.56% when $k(t) = 5$, whereas the average of the real surrender rates is 2.97%.

Figure 11 shows the surrender rate changes of US SPDA under the assumption (A1) that the US unemployment rates experience the financial rate shock, $k(t) \geq 2$, as the same pattern of $k(t)$ as that of Korean data.

It shows a very high peak of 7.56% in the middle of the unemployment rate shock period. We also make an interesting notice that the unemployment rate shock period starts later than that of market rate shock, and lasts longer. The average of the expected (predicted) surrender rates is 3.42% whereas the average of the real surrender rates is 2.97%.

Figure 11. US SPDA Surrender Rate Changes under Unemployment Rate Shock (A1)

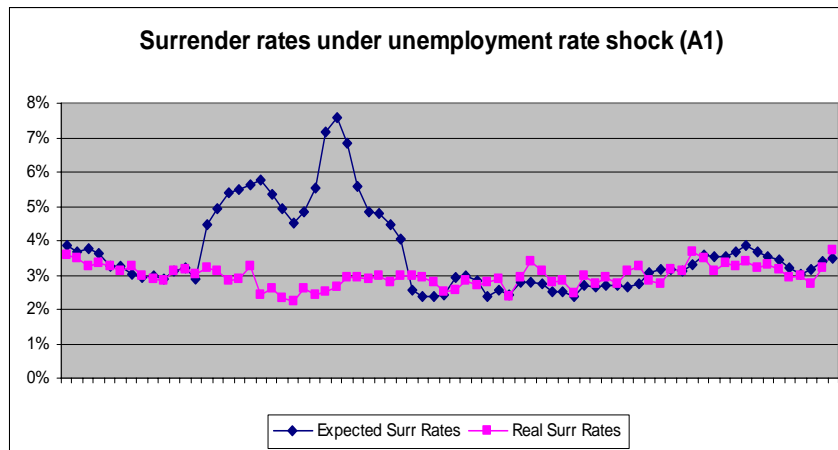


Figure 12 shows the surrender rate changes of US SPDA under the assumption (A2) that the US unemployment rates experience the financial rate shock, $k(t) = 2, 3, 5$ over the whole period.

It shows that the surrender rates are increasing as $k(t)$ goes up, i.e. unemployment rates increase. The average of the expected (predicted) surrender rates is

3.94% when $k(t) = 2$, 4.57% when $k(t) = 3$, and 6.13% when $k(t) = 5$, whereas the average of the real surrender rates is 2.97%.

Figure 12. US SPDA Surrender Rate Changes under Unemployment Rate Shock (A2)

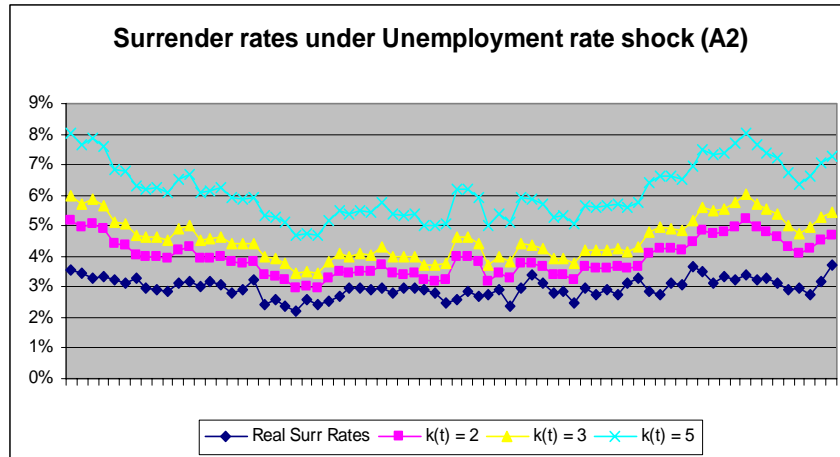


Figure 13 shows the surrender rate changes of US SPDA under the assumption (A1) that the US economy growth (GDP) rates experience the financial rate shock, $k(t) \leq -2$, as the same pattern of $k(t)$ as that of Korean data.

It shows a small peak of 3.89% in the beginning of the shock period. We can notice that the economy growth rate shock period last for short period of 5 months and the impacts of the economy growth rate shock to surrender rates are relatively small. The average of the expected (predicted) surrender rates is 2.98% whereas the average of the real surrender rates is 2.97%.

Figure 13. US SPDA Surrender Rate Changes under Economy Growth Rate Shock (A1)

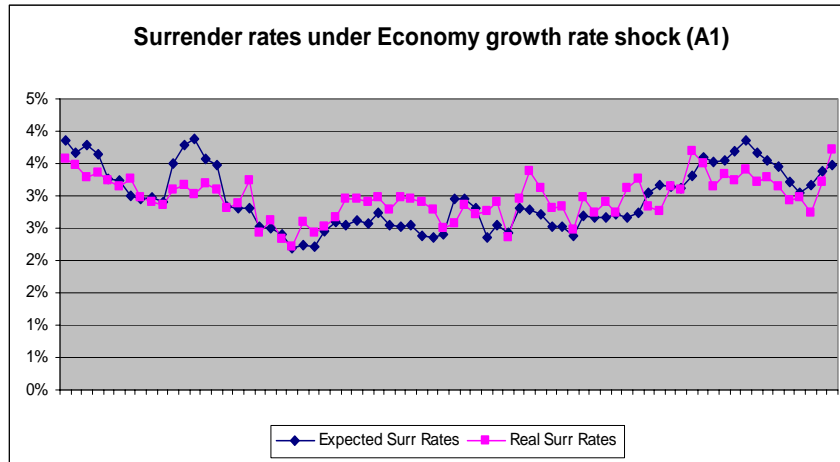


Figure 14 shows the surrender rate changes of US SPDA under the assumption (A2) that the US economy growth (GDP) rates experience the financial rate shock, $k(t) = -2, -3, -5$ over the whole period.

It shows that the surrender rates are increasing as $k(t)$ goes down, i.e. the economy growth rates decrease. The average of the expected (predicted) surrender rates is 3.27% when $k(t) = -2$, 3.46% when $k(t) = -3$, and 3.87% when $k(t) = -5$, whereas the average of the real surrender rates is 2.97%.

Figure 14. US SPDA Surrender Rate Changes under Economy Growth Rate Shock (A2)

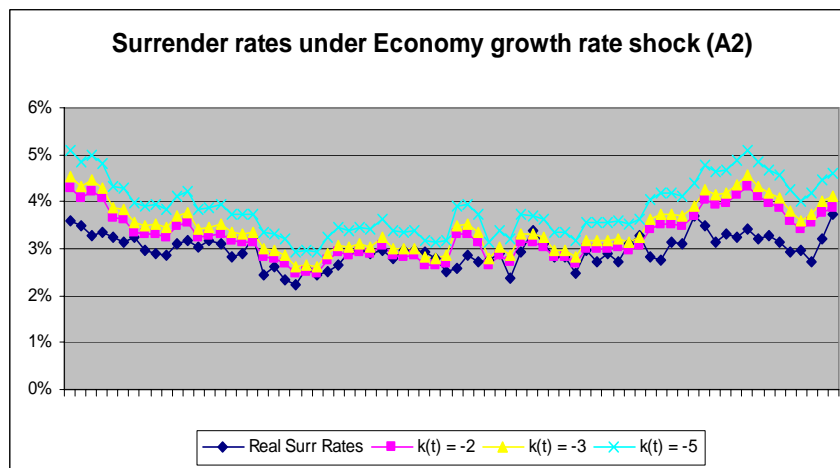


Figure 15. US SPDA Surrender Rate Changes under Total Rate Shock (A1)

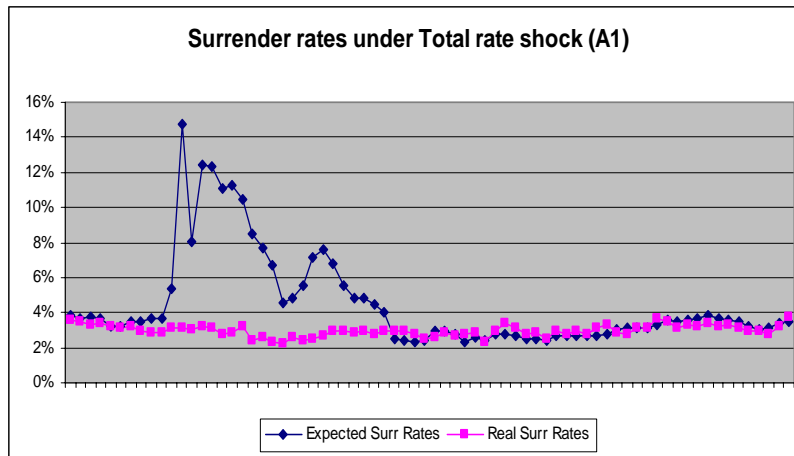


Figure 15 shows the surrender rate changes of US SPDA under the assumption (A1) that the total three US financial rates (market, unemployment, and economy growth rates) experience the financial rate shock, $|k(t)| \geq 2$, at the same time, as the same pattern of $k(t)$ as that of Korean data.

It shows a high peak of 14.71% at the beginning of the shock period. And the surrender rates are quite high with the average of 7.67% during the shock period for almost 2 years. The average of the expected (predicted) surrender rates is 4.48% whereas the average of the real surrender rates is 2.97%.

5 Conclusion

Many insurance companies are selling single premium deferred annuities (SPDA). But SPDA are sold with the primary focus on accumulation. Only a few of the policy holders purchase SPDA for the purpose of annuitization. In Korea, the annuity market is still young and growing slowly compared to that of the United States (US). Interest-indexed annuities (IIA) are one of the most popular SPDA products in Korea. The distinctive features of SPDA are the surrender options and annuitization options. In this paper we consider the surrender behaviors of SPDA /IIA policy holders under extreme economic conditions.

We have considered a model on the policy holder surrender behavior statistically. The variables considered are the difference between reference market rates and product crediting rates with surrender charges, the policy age since the contract was issued, unemployment rates, economy growth rates, and seasonal effects. Especially the duration, i.e. the policy age since the contract was issued, is one of the most important

factors of surrender rates. We use the logit model for the surrender rates.

For extreme events/financial rate shocks, we define $k(t)$, a risk measure of a financial rate $i(t)$,

$$k(t) = \frac{i(t) - \mu}{\sigma}.$$

We define that the financial rate $i(t)$ experiences a financial rate shock at time t if

$$|k(t)| \geq 2,$$

and we say that the financial rate is in a stable status at time t if $|k(t)| < 2$. We also say that we are under extreme financial conditions if the financial rates experience financial rate shocks.

We consider the surrender rate changes of US SPDA under the assumption that US financial rates experience the financial rate shocks. We make two assumptions on the pattern of $k(t)$, the risk measures of the financial rates. Assumption 1 (A1): the pattern of $k(t)$ is same as that of Korean data when the rate experiences financial shocks. When the rate does not experience any financial shocks, the risk measure $k(t)$ is calculated from US data. Assumption 2 (A2): $k(t) = c$, where c is a constant integer such that $|c| \geq 2$ and the financial rate $i(t)$ is changed to $i(t) + c\sigma$. We summarize the analysis results in the following table.

Table 3. Surrender Rate Changes under Extreme Conditions

Assumption	Market rates		Unemployment rates		Economy Growth rates		Total rates	
	max	average	max	average	max	average	max	average
A1	12.63%	3.42%	7.56%	3.66%	3.89%	2.96%	14.71%	4.48%
$ k(t) = 2$	4.62%	3.49%	5.20%	3.94%	4.32%	3.27%	9.23%	4.37%
$ k(t) = 3$	5.04%	3.82%	6.02%	4.57%	4.57%	3.46%	11.57%	6.24%
$ k(t) = 5$	6.01%	4.56%	8.04%	6.13%	5.11%	3.87%	15.03%	10.85%

From the Table 3, we can notice that the surrender rates change very much under extreme conditions. We see a high peak of 14.71% when all of the 3 variables experience financial rate shocks under the assumption 1. And the surrender rates are quite high with the average of 7.67% during the shock period for almost 2 years. The average of the expected (predicted) surrender rates is 4.48% whereas the average of the real surrender rates (without extreme condition assumptions) is 2.97%.

It may be a consideration in risk management of insurance business to predict sudden increase of surrender rates and prepare appropriate hedging strategies.

Appendix Modeling Surrender Rates of Korean Annuities

We want to show a method to model surrender rates with economic variables and durations (policy-age-since issue date) for Korean annuities. We show how to choose the explanatory variables. We also show how to compare the surrender rate models and choose a better model for Korean annuities. This method can be applied to other insurance policies.

A.1 Variables and Assumptions

We summarize the explanatory variables and the assumptions used in modeling the surrender rates of Korean annuities.

Table A1. Explanatory Variables Considered

Variable	Contents	Memo
BASEYM	Year, Month of data	
DIFFLAG0	Difference of rates	=market rate-crediting rate at current time
DIFFLAG2	Difference of rates	=market rate-crediting rate 2 months ago
DIFFLAG4	"	=market rate-crediting rate 4 months ago
DIFFLAG6	"	=market rate-crediting rate 6 months ago
DIFFLAG8	"	=market rate-crediting rate 8 months ago
DIFFLAG10	"	=market rate-crediting rate 10 months ago
DIFFLAG12	"	=market rate-crediting rate 12 months ago
POL-AGE	Policy age	Average policy age since issue
LOST	Unemployment rates	
GROWTH	Economy growth rates	
IMF	Financial crises period under IMF control	Period from 1997.12 to 1998.12 Dummy variable = 1 during the period
MONTH1	January	Dummy variable = 1 on current month
MONTH2	February	"
MONTH3	March	"
MONTH4	April	"
MONTH5	May	"
MONTH6	Jun	"
MONTH7	July	"
MONTH8	August	"
MONTH9	September	"
MONTH10	October	"
MONTH11	November	"
SUR_RATE	Real surrender rate	Dependent variables

For seasonal effects, we investigate the surrender rates from January to November. We consider the financial crises period since the surrender rates

skyrocketed during this period. We use dummy variable 1 during the financial crises period from Dec. 1997 to Dec 1998 and 0 elsewhere.

The dependent variable SUR_RATE denotes the real surrender rates, and it is the face amount of surrendered policies divided by the face amount of initial policies. We consider Korean annuities with more than 1,000,000 policy holders⁵.

A.2 Surrender Rate Models

We use logit link function and complementary log-log (CLL) link function. As modeling programs, we use the Generalized Linear Models, Procedure GENMOD, Logistic Regression Models, and Procedure LOGISTIC, with SAS.

The Logit function has the following form,

$$\ln\left(\frac{q_s}{1-q_s}\right) = \beta_0 + \beta_1 V_1 + \dots + \beta_n V_n, \quad (\text{A.1})$$

and the Complementary Log-Log (CLL) function is of the form,

$$\log(-\log(1-q_s)) = \beta_0 + \beta_1 V_1 + \dots + \beta_n V_n, \quad (\text{A.2})$$

where q_s is the surrender rate, β_i is the coefficient to be estimated and V_i is the explanatory variable.

A.3 Significance of Each Explanatory Variable

We check the significance of each explanatory variable. There are many factors which affect the surrender rate fluctuations such as the difference between reference market rates and crediting rates, policy age since issue, unemployment rates, economy growth rates, financial crises, and seasonal effects. According to each explanatory variable, we analyze the significance as a whole with logistic regression analysis. The specific analysis for the variable selection and the reduced models will be done next.

In Table A2, (*) means the p-value for the test statistic χ^2 is less than 0.0001. Since the p-value is less than 1% or 5%, each variable has its own significance for surrender rates.

The difference between reference market rates and crediting rates are considered important for surrender rate modeling. We summarize the points below.

(i) The estimated parameters are all positive numbers. So the surrender rate goes up as the difference between the reference market rates and crediting rates becomes large.

(ii) From Table A2, we see that each interest rate difference variable has its own

⁵ The data used in modeling surrender rates are from Korean insurance business, so there may be differences on the explanatory variables in modeling them in other countries.

effects on surrender rates. Especially the difference of interest rates 2 months ago is really significant noting the relatively large parameter estimate of 6.9440 (LOGIT) and 6.8517 (CLL).

Table A2. Significance of Explanatory Variables

Variables	LOGIT LINK FUNCTION			CLL LINK FUNCTION		
	Parameter	Std error	Chi-square	parameter	Std error	Chi-square
DIFFLAG0	5.6600	0.0037	2380693(*)	5.5888	0.0036	2391882(*)
DIFFLAG2	6.9440	0.0036	3693589(*)	6.8517	0.0036	3716484(*)
DIFFLAG4	6.6217	0.0037	3221988(*)	6.5291	0.0036	3237336(*)
DIFFLAG6	6.5901	0.0037	3121563(*)	6.4988	0.0037	3136179(*)
DIFFLAG8	5.7086	0.0038	2226700(*)	5.6308	0.0038	2234024(*)
DIFFLAG10	4.3584	0.0040	1205621(*)	4.2986	0.0039	1206910(*)
DIFFLAG12	3.0710	0.0041	552153(*)	3.0297	0.0041	551929(*)
POL-AGE	-0.1076	0.0001	3192912(*)	-0.1066	0.0001	3196557(*)
LOST	13.4398	0.0086	2438932(*)	13.3027	0.0085	2440284(*)
GROWTH	-5.9912	0.0139	186882(*)	-5.9436	0.0137	187356(*)
IMF	0.7662	0.0003	5112065(*)	0.7578	0.0003	5122041(*)
MONTH1	0.1296	0.0006	51236.9(*)	0.1283	0.0006	51292.5(*)
MONTH2	0.1193	0.0006	43069.9(*)	0.1181	0.0006	43112.8(*)
MONTH3	0.1235	0.0006	46335.6(*)	0.1223	0.0006	46383.5(*)
MONTH4	-0.0414	0.0006	4574.70(*)	-0.0410	0.0006	4573.22(*)
MONTH5	-0.0356	0.0006	3386.19(*)	-0.0352	0.0006	3385.25(*)
MONTH6	0.0108	0.0006	326.21(*)	0.0107	0.0006	326.24(*)
MONTH7	-0.0507	0.0006	6811.54(*)	-0.0503	0.0006	6808.86(*)
MONTH8	-0.0667	0.0006	11621.2(*)	-0.0661	0.0006	11615.2(*)
MONTH9	-0.0652	0.0006	11127.1(*)	-0.0646	0.0006	11121.5(*)
MONTH10	-0.1009	0.0006	25883.8(*)	-0.1000	0.0006	25863.9(*)
MONTH11	-0.1191	0.0006	35476.5(*)	-0.1180	0.0006	35444.7(*)

So we may guess that the 2-month-ago interest rates are influencing more on the surrender behaviors of the policy holders. Also the interest rate differences from 2 months ago to 6 months ago are affecting the surrender rate fluctuations. *So the policy holders observe the interest rate movements for 2 – 6 months and decide to surrender their policies.*

The estimated parameter for policy-age since issue is negative. So the surrender rates decrease as the policy age increases.

The positive parameter for unemployment rates indicates that surrender rates go up when the unemployment rates increase. It is natural and it is really significant to take the unemployment rates into account as an explanatory variable in modeling

surrender rates considering the relatively high parameter estimate of 13.4398 (LOGIT) and 13.3027 (CLL).

The parameter for economy growth rates is negative and we may think that the surrender rates go down under good economy conditions.

The positive parameter for the dummy variable, financial crises under IMF control, means that the surrender rates can increase when unexpected economy/finance shock happens.

It is interesting to note that the parameters for January, February, March, and Jun are positive and the others are negative, but all are small. Thus the season has a small effect on surrender behavior.

A.4 Reduced Models

We may not need all of the variables in modeling surrender rates, i.e. the full information model. In this section, we find appropriate reduced models with least number of explanatory variables for Korean annuities. We try to keep the same fit of reduced model as that of full information model. We know that there are a few methods such as forward selection method, backward elimination method, stepwise regression, and all possible regressions, to select the most significant variables.

We follow 3 steps to find the most appropriate reduced models. The first step is to select a few significant explanatory variables with backward elimination method. The second step is to set up reduced models with the selected variables. The third step is to transform the policy age (or duration). The reason we transform the policy-age is that there is a possibility that the fit may become worse if we use the real policy-age without transformation.

Also we compare the three models, arctangent model, Logit model, and CLL model and choose the most appropriate one for Korean annuities⁶.

A.4.1 Step 1. Selecting Explanatory Variables

We want to delete the variables one by one from the least significant one until we get a reduced model. It is a kind of backward elimination method⁷. As criterion for the selection of variables we use $-2 \times \text{Log Likelihood function}$ ($-2 \times \text{Log L}$), Akaike information criterion (AIC), and Bayesian information criterion (BIC). We also show

⁶ For an example of a comparison of models for pricing mortgage-backed securities, we may refer Dunn and McConnell (1981). Hall (2000) compares logit analysis to his modeling results.

⁷ We may refer Weisberg (1985).

Schwartz criterion (SC)⁸.

Comparing the variables in Table A3, we rank the relative contributions of each variable to the model; *interest rate differences, policy age since issue date, unemployment rate (lost), financial crises (IMF), seasonal effects, and economy growth rates.*

And we notice that the financial crises (IMF) and economy growth rates influence very little to the surrender rates. But unemployment rates are really affecting the surrender rate behaviors of the policy holders.

The numbers in Table A3 show the increased model fit statistics (AIC, BIC, -2*Log L) as we delete the variables one by one in each step. The increased amounts indicate the relative significance. That is, the deleted variables make contributions to the fit of the reduced model compared with the full information model as much as the changed amount.

For seasonal effects and interest difference effects, we averaged the increased amount divided by the number of variables. Figure A1 shows the decreased fit according to the reduced model steps. After the reducing step 3, we can notice that the fit is reduced significantly. So we stop at step 3 and decide to delete the first 3 variables which have less significant contributions.

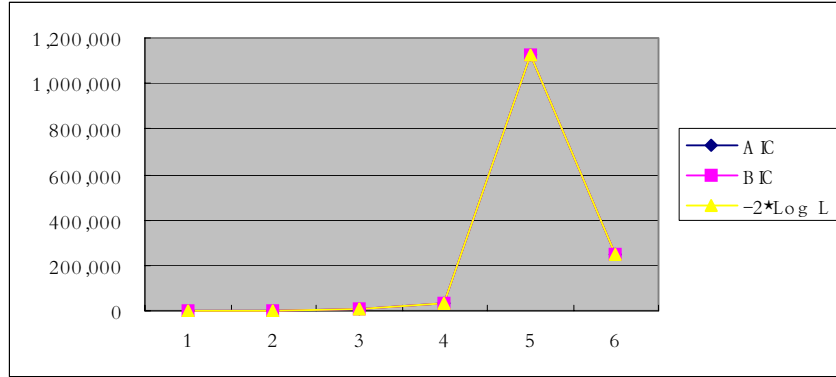
We find out that the interest rate differences and policy-age since issue are the most important factors and the unemployment rates are also important in modeling surrender rates. Modeling with these 3 explanatory variables, we have p-value less than 0.0001 and we conclude that it is reasonable to estimate the 3 parameters.

Table A3. Model Fit Statistics Changed

Reducing step	Reduced variable	AIC	BIC	-2*Log L
1	Economy growth	45	27	47
2	Seasonal	2,039	2,021	2,041
3	IMF	9,015	8,998	9,017
4	Unemployment	29,162	29,144	29,164
5	Policy age	1,127,643	1,127,625	1,127,645
6	Interest diff	251,787	251,769	251,789

⁸ $AIC(r) = -2*\log L + 2*r = \log\{SSE(r)/n\} + (2/n)*(r+1)$, where r is the number of explanatory variables, n is the number of observation, and SSE stands for sum of squared errors. $BIC(r) = \log\{SSE(r)/n\} + \{(\log n)/n\}*(r+1)$. $SC = -2*\log L + r*\log n$. For more explanation, we refer to Hamilton (1994).

Figure A1. Model Fit Statistics Changed



For Korean annuity, we can select policy-age, interest rate differences, and unemployment rates as the explanatory variables.

A.4.2 Step 2. Reduced Models

The second step is to set up reduced models with the selected variables from step 1. We present 3 tables. The first and second tables show the estimated parameters for the selected variables from Logit and CLL model. The third table shows the estimated errors for the three models, arctangent model, Logit model, and CLL model, and also compares the models by the differences of the estimated errors between arctangent model and Logit model and arctangent model and CLL model according to the policy-age since issue⁹. For comparison purposes, we define RMSE and MAPE as follows,

$$RMSE = \sqrt{\frac{\sum (y_i - \hat{y}_i)^2}{n}}, \quad (A.3)$$

and

$$MAPE = \frac{1}{n} \sum \frac{|y_i - \hat{y}_i|}{y_i}, \quad (A.4)$$

where, y_i is the i-th real value, \hat{y}_i is the i-th predicted value, and n is the sample size.

⁹ For mortgage prepayment models, Hall (2000) compares a conventional logit analysis to the results from the model used in the paper.

We define the terminologies used in the third table as follows. RMSE1 is RMSE of Arctangent model, RMSE2 is RMSE of Logit model, and RMSE3 is RMSE of CLL model. MAPE1, MAPE2, and MAPE3 represent the MAPE of Arctangent model, Logit model, and CLL model respectively.

RMSEGAP1 denotes RMSE1-RMSE2, so Logit model is better than Arctangent model if RMSEGAP1 is positive. RMSEGAP2 is RMSE1-RMSE3, so CLL model is better than Arctangent model if RMSEGAP2 is positive. MAPEGAP1 is MAPE1-MAPE2 and Logit model is better than Arctangent model if MAPEGAP1 is positive. MAPEGAP2 is MAPE1-MAPE3 and CLL model is better than Arctangent model if MAPEGAP2 is positive.

We show the parameter estimates from Logit model and CLL model for Korean annuities below. We also show the estimated errors and comparison of models in the following tables.

Table A4. Parameter Estimates with Logit Model

Analysis of Maximum Likelihood Estimates					
Parameter	DF	Estimate	Standard Error	Chi-Square	Pr > ChiSq
Intercept	1	-4.0378	0.00103	15373699.0	<.0001
DIFFLAG0	1	5.0670	0.0151	113287.687	<.0001
DIFFLAG2	1	2.7905	0.0147	36030.6698	<.0001
DIFFLAG4	1	-0.2736	0.0133	425.5740	<.0001
DIFFLAG6	1	2.3560	0.0131	32358.8416	<.0001
DIFFLAG8	1	2.5700	0.0131	38450.9595	<.0001
DIFFLAG10	1	1.7711	0.0136	17042.5285	<.0001
DIFFLAG12	1	2.9900	0.0119	63226.0859	<.0001
POLICY AGE	1	-0.1369	0.000132	1078276.82	<.0001

Table A5. Parameter Estimates with CLL Model

Analysis of Maximum Likelihood Estimates					
Parameter	DF	Estimate	Standard Error	Chi-Square	Pr > ChiSq
Intercept	1	-4.0471	0.00102	15755515.4	<.0001
DIFFLAG0	1	4.9841	0.0147	114220.414	<.0001
DIFFLAG2	1	2.7394	0.0144	36268.0157	<.0001
DIFFLAG4	1	-0.2493	0.0130	367.7235	<.0001
DIFFLAG6	1	2.3170	0.0128	32612.5292	<.0001
DIFFLAG8	1	2.5327	0.0128	38929.2327	<.0001

DIFFLAG10	1	1.7545	0.0133	17367.0213	<.0001
DIFFLAG12	1	2.9481	0.0117	63544.4834	<.0001
POLICY AGE	1	-0.1351	0.000130	1081687.16	<.0001

Table A6. Estimated Errors and Comparison of Models

time	RMSE 1	RMSE 2	RMSE 3	MAPE 1	MAPE 2	MAPE 3	RMSEGA P1	RMSEGA P2	MAPEGA P1	MAPEGA P2
0.5	0.02707	0.02083	0.02078	0.3268	0.29949	0.29887	0.00624	0.00629	0.02731	0.02792
1.5	0.00773	0.00858	0.00861	0.19563	0.22432	0.22502	-0.00084	-0.00088	-0.02869	-0.02939
2.5	0.00569	0.01358	0.01357	0.24269	0.95731	0.95601	-0.00789	-0.00788	-0.71462	-0.71332
3.5	0.00607	0.00903	0.00901	0.30749	0.76591	0.76488	-0.00296	-0.00294	-0.45841	-0.45739
4.5	0.007	0.00477	0.00475	0.24598	0.32145	0.32079	0.00223	0.00225	-0.07547	-0.07481
5.5	0.00734	0.00297	0.00301	0.20046	0.09406	0.09451	0.00437	0.00433	0.1064	0.10595
6.5	0.00869	0.00299	0.00304	0.7049	0.1216	0.1229	0.00569	0.00565	0.5833	0.582
7.5	0.00849	0.00386	0.0039	0.34024	0.20862	0.21083	0.00463	0.00459	0.13162	0.12941
8.5	0.00842	0.0039	0.00393	0.31219	0.20492	0.20738	0.00452	0.00449	0.10727	0.10481
9.5	0.00817	0.00452	0.00454	0.30694	0.20528	0.20597	0.00365	0.00363	0.10166	0.10097

For annuity plan, we may not conclude that Logit or CLL model is better than arctangent model. Even when we add unemployment rates and IMF effects to the Logit and CLL models, we do not have enough evidence that one model is better than the other ones. Also the sign of DIFFLAG4 is negative and it seems to be unexplainable.

A.4.3 Step 3. Transformation of Duration

The third step is to transform the policy-age (duration) since issue. The reason we transform the policy-age is that the surrender rates are dependent on durations and there is possibility that the fit may be decreased if we use the real policy-age without transformation. We try three formulas which are usually used in transformation¹⁰,

¹⁰ For more on transformation of variables, we may refer Kutner, Nachtschiem, and Wasserman (1996).

$$\sqrt[n]{x}, \log x, \text{ and } -\frac{1}{x}. \quad (\text{A.5})$$

The policy age may be transformed to

$$(\text{policy age})^{\frac{1}{n}}, \log(\text{policy age}), \text{ or } -\frac{1}{\text{policy age}} \quad (\text{A.6})$$

We choose the best transformation formula using the model fit statistics -2Log L . We compare arctangent model and Logit model and arctangent model and CLL model, and conclude which model is the best one.

Table A7. Model Fit Statistics According to Transformed Policy Age

$(\text{policy age})^{\frac{1}{n}}$	$-2*\text{Log L}$
n=1	72208277
n=2	71946654
n=3	71863316
n=4	71825035
n=5	71803493
n=6	71789797
n=7	71780359
n=8	71773477
n=9	71768243
n=10	71764133

Formula	$-2*\text{Log L}$
Log(policy age)	71730319
$-1/(\text{policy age})$	71680566

Figure A2. Model Fit Statistics According to Transformed Policy Age

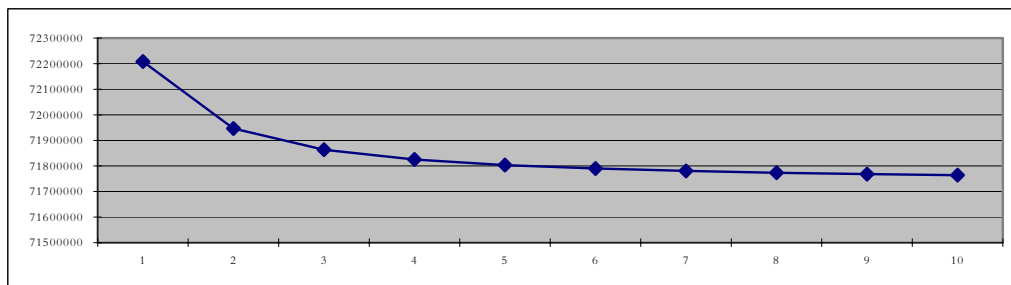


Table A7 and Figure A2 show the model fit statistics (-2*Log L) according to the policy-age. We can notice that the model fits well when we transform the policy age. Comparing the model fit statistics, we conclude that the best transformation formula is

$$-\frac{1}{policy\ age}$$

We show the analysis results in the following tables.

Table A8. Parameter Estimates with Logit Model under Transformation

Analysis of Maximum Likelihood Estimates					
Parameter	DF	Estimate	Standard Error	Chi-Square	Pr > ChiSq
Intercept	1	-5.4203	0.00255	4506516.79	<.0001
DIFFLAG0	1	6.0594	0.0163	137622.131	<.0001
DIFFLAG2	1	1.9039	0.0157	14768.9749	<.0001
DIFFLAG4	1	-1.0608	0.0141	5678.8636	<.0001
DIFFLAG6	1	1.6583	0.0137	14577.7492	<.0001
DIFFLAG8	1	2.3668	0.0131	32620.9627	<.0001
DIFFLAG10	1	1.4915	0.0137	11839.3940	<.0001
DIFFLAG12	1	0.7086	0.0181	1528.7608	<.0001
POLICY AGE	1	-0.6715	0.000478	1971550.62	<.0001
LOST	1	10.5238	0.0617	29044.8354	<.0001

Table A9. Parameter Estimates with CLL Model under Transformation

Analysis of Maximum Likelihood Estimates					
Parameter	DF	Estimate	Standard Error	Chi-Square	Pr > ChiSq
Intercept	1	-5.4089	0.00252	4597008.35	<.0001
DIFFLAG0	1	5.9574	0.0160	138442.495	<.0001
DIFFLAG2	1	1.8506	0.0153	14580.5856	<.0001
DIFFLAG4	1	-1.0303	0.0138	5583.9827	<.0001
DIFFLAG6	1	1.6172	0.0134	14475.6309	<.0001
DIFFLAG8	1	2.3250	0.0128	32949.0219	<.0001
DIFFLAG10	1	1.4808	0.0134	12166.9626	<.0001

DIFFLAG12	1	0.6886	0.0178	1491.4247	<.0001
POLICY AGE	1	-0.6561	0.000465	1991007.04	<.0001
LOST	1	10.4363	0.0610	29316.3498	<.0001

Table A10. Errors and Comparison of Models under Transformation

time	RMSE 1	RMSE 2	RMSE 3	MAPE 1	MAPE 2	MAPE 3	RMSEGA P1	RMSEGA P2	MAPEGA P1	MAPEGA P2
0.5	0.02707	0.01256	0.01274	0.3268	0.19366	0.19315	0.01451	0.01433	0.13314	0.13364
1.5	0.00773	0.00951	0.00954	0.19563	0.26445	0.26563	-0.00177	-0.00181	-0.06882	-0.06999
2.5	0.00569	0.00474	0.00474	0.24269	0.36096	0.36226	0.00095	0.00096	-0.11827	-0.11957
3.5	0.00607	0.00364	0.00368	0.30749	0.34218	0.34641	0.00243	0.00239	-0.03468	-0.03892
4.5	0.007	0.00258	0.00263	0.24598	0.13063	0.13339	0.00443	0.00437	0.11535	0.11259
5.5	0.00734	0.00463	0.00465	0.20046	0.16366	0.16235	0.00272	0.0027	0.0368	0.0381
6.5	0.00869	0.0029	0.00293	0.7049	0.11537	0.11695	0.00579	0.00576	0.58953	0.58795
7.5	0.00849	0.00281	0.00284	0.34024	0.24663	0.25122	0.00568	0.00565	0.09361	0.08902
8.5	0.00842	0.00305	0.00307	0.31219	0.30407	0.30834	0.00537	0.00535	0.00812	0.00385
9.5	0.00817	0.00351	0.00352	0.30694	0.29396	0.29819	0.00466	0.00465	0.01297	0.00875

When we do not transform the policy ages, we do not have enough ground that one model is better than the other ones. *But Logit and CLL models are better than arctangent model on many policy ages after we transform the policy ages.* Below we show the 10 graphs of real and estimated surrender rates for each model according to the policy-age from duration 1 to duration 10 for Korean annuity. Note that the Logit model and the CLL model produce almost the same results and the two graphs are overlapping.

Figure A3. Surrender Rates According to the Policy-age (Duration)

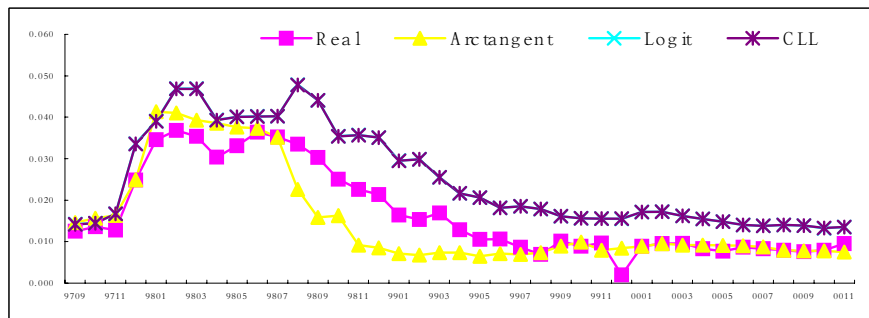
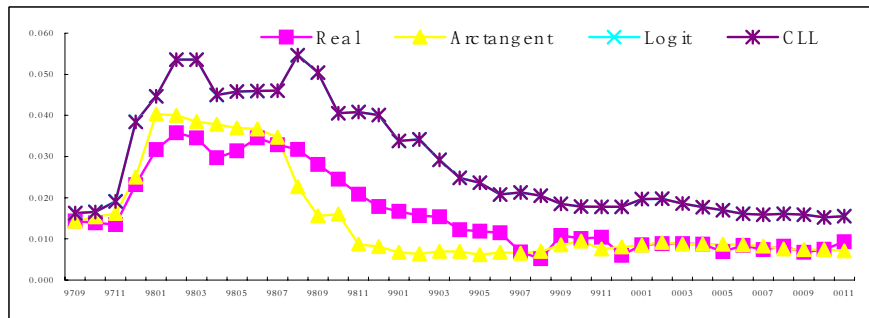
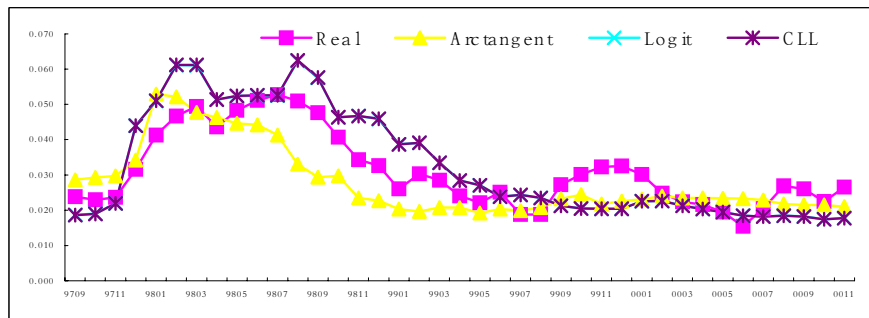
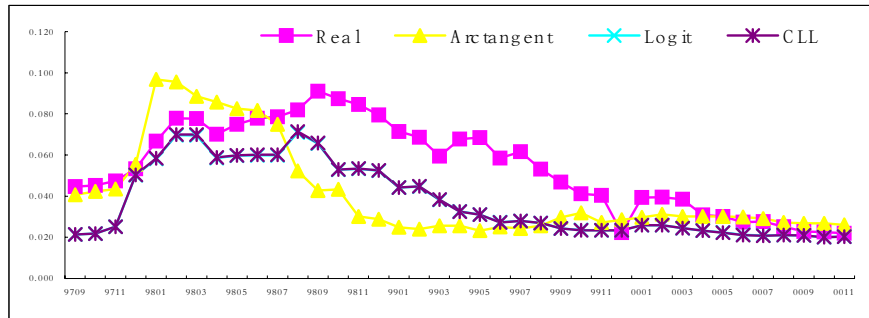


Figure A3. Continued

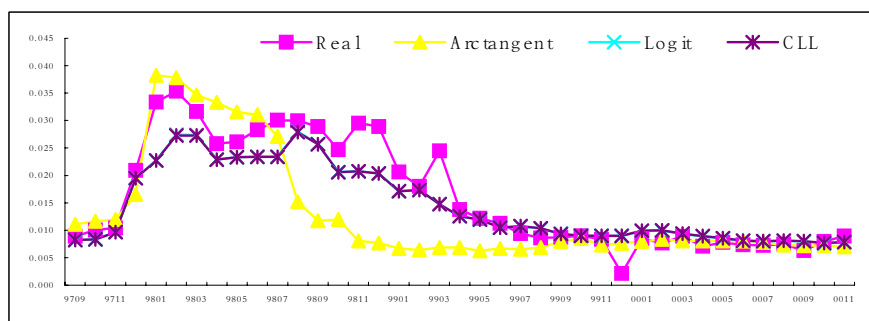
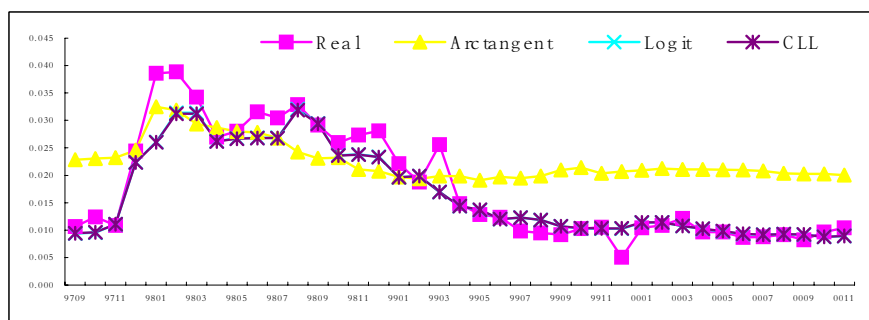
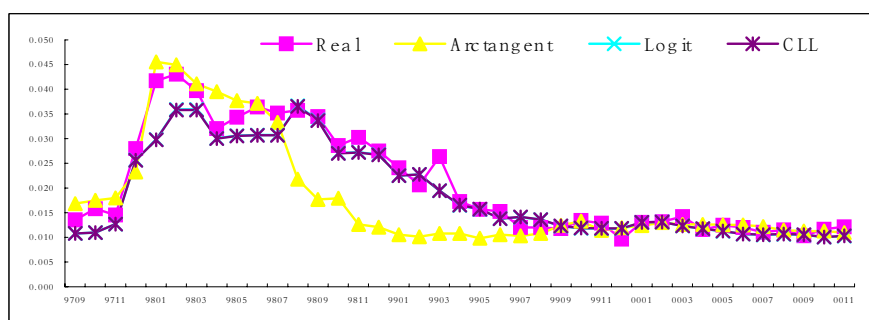
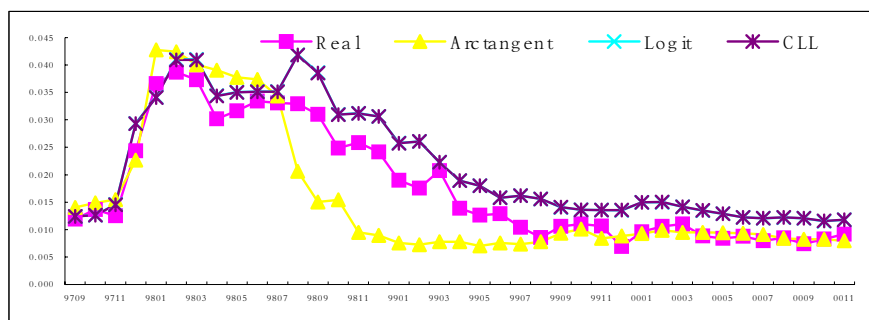
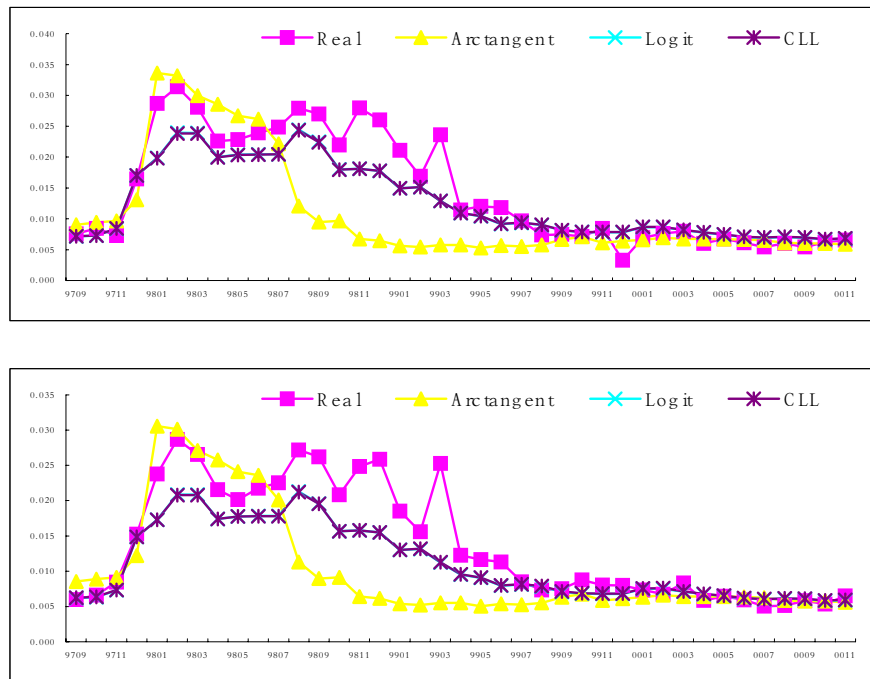


Figure A3. Continued



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