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Risk Based Capital and Capital Allocation in Insurance

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Abstract

The science of capital allocation has made significant advances in our understanding of allocation and use of risk based capital. Yet there is limited theoretical guidance on which risk measure is consistent with value maximisation and no well developed economic theory underlying the risk measures. Different firms use different risk measures and there is no agreement on the appropriate risk measure. Risk measures are applied inconsistently for different risks, different lines of business, products and divisions. For insurer pricing the price of risk should vary with the type of risk under consideration yet most risk based capital approaches implicitly use a common price of risk based on a firm wide expected cost of capital for pricing. Recent developments in capital allocation of risk capital for solvency and by-line pricing indicate a new direction is required. This paper highlights the importance of risk measure and discusses the insolvency default option value. It also discusses allocation by line and fair pricing, frictional costs and market imperfections and issues of risk based capital in a value maximizing framework.

Key words: *risk based capital, VaR, TailVaR, default put option, insurance pricing*

1. Introduction

Risk based capital has become a critical component of the management of financial service providers including banks, life insurers and non-life insurers. Prudential regulations in both banking and insurance have become more risk based and increasingly internal models are being used to determine capital requirements. The focus of this paper is insurance although the concepts apply equally to any corporation or financial service provider concerned with ensuring solvency in order to meet financial obligations. There is an extensive literature on risk based capital and capital allocation to insurance including Chandra and Sherris (2007), Cummins (2000), Cummins, Lin and Phillips (2006), Merton and Perold (1993), Myers and Read (2001), Sherris (2006) and Venter (2004).

Although risk based capital is available to meet the solvency requirements of the whole company, for many purposes capital is allocated to lines of business, to products or even to risk drivers that determine the exposure of the insurer to various risks. Capital allocation is used for many purposes. An important use is the determination of actual, or expected, returns on capital by line of business by determining actual cash flows, or expected cash flows, to equity, and deriving a return based on expected cash flows to allocated capital.

There are many different risk measures used in insurance including VaR, ruin probability, TailVaR, Expected Policyholder Deficit, and the Insolvency Default Put Option. A natural question that arises is which measure makes most economic sense? There are also many different approaches to allocating capital to lines of business including those proportional to risk measure used, proportional to liabilities, marginal allocations, and methods assuming equal expected returns to capital, as well as using covariance of losses. This also raises the question of how to determine an economically sensible measure?

Standard methods for capital allocation in practice generally consider lines of business or risks on an individual basis (which may be assets or liabilities) with no direct allowance for dependence between risks or business lines. Any diversification benefit is usually considered later at the aggregated level. Yet, capital is available to support all lines of business. This raises the question of how to allow for this in allocating capital to line of business?

Panning (2006), Yow and Sherris (2007a), and Zanjani (2002) develop value maximising models for an insurer allowing for different forms of market imperfections. The model of Yow and Sherris (2007a) was used to assess risk based capital determination at an enterprise level based on maximisation of shareholder value added (allowing for price elasticity of demand). The implications for pricing of lines of business will be reviewed. This approach can be compared with approaches based on minimisation of (expected) tax, agency and financial distress costs as in Chandra and Sherris (2007). The need to allocate capital, except for by-line contribution to insolvency risk, and implications for by-line market prices and profit margins will be discussed. Capital allocation should in practice reflect market factors such as policyholder price elasticity and need not be based on a perfectly competitive market assumption unless this is required by regulation or justified by market studies.

This paper aims to integrate recent research developments related to the determination of risk based capital and allocation of capital to lines of business into a consistent value maximising framework. The approach discussed uses a criterion of maximising the value added to an insurer allowing for market imperfections including frictional costs such as taxes, agency costs and financial distress costs as well as policyholder elasticity by line of business. This is consistent with modern financial theory where the goal of management is to maximise the value of the firm.

We bring together concepts in capital and capital allocation using a consistent framework for fair pricing and capital allocation. We highlight the importance of the insolvency put option value as a risk measure for solvency assessment. The value maximising approach incorporating frictional costs of capital and price elasticity is discussed and related to economic capital using VaR approaches. Recent developments in capital allocation for risk capital for solvency and by-line pricing indicate a new direction is required. We focus on the importance of risk measure, the allocation by line and fair pricing allowing for frictional costs and market imperfections based on value maximising and policyholder demand elasticity.

2. Risk Based Capital

Capital has many different definitions. In this paper financial capital will refer to the funding of productive assets or real capital and risk based capital will refer to the additional capital held to provide financial solvency and to manage volatility in business outcomes. In insurance the financial capital is generated mostly from the premiums charged to policyholders and is the expected claims cost or the loss reserves using a best estimate or market consistent valuation with no prudential margin. This paper focuses on risk based capital since this is critical in determining solvency, financial rating and capital costs for an insurer. Loss reserves are also an area that can impact on risk based capital. These should be determined on a market consistent valuation basis so that risk based capital reflects all available resources to absorb unexpected losses.

Risk based capital is also the focus of prudential regulations and solvency regulations under Basel II for banking and the European insurance equivalent, Solvency II. Economic capital is another term that refers to risk based capital. This is usually based on a Value-at-Risk criterion with a very low probability of insolvency consistent with the high credit ratings that insurers normally target in their capital management. Economic capital is based on risk and usually determined at the enterprise level and has an impact on pricing and financial management.

There are many different risk measures that can be used to determine risk based capital. These include the Value at Risk (VaR), the ruin probability from insurance risk theory, the Tail Value at Risk (TailVaR), the expected policyholder deficit and the insolvency (or default) put option. Since there are so many proposed methods for determining risk based capital it is natural to consider which measure makes the most economic sense.

In practice insurers hold levels of capital much higher than the minimum capital requirements of prudential regulations. The levels of capital held by insurers also usually exceed the level of economic capital determined on the basis of internal risk based models. An important factor is the ratings agencies such as Standard and Poor's and the level of capital required to maintain a target rating. Insurers generally hold levels of capital significantly in excess of regulatory capital levels. This can be a value-adding strategy since low levels of capital can put at risk the sales of future business if policyholders do not have confidence in the ability of the insurer to pay its claims.

An economic explanation for the level of capital held by insurers could be based on the minimisation of frictional costs of insurer capital. Chandra and Sherris (2007) developed a simple model of an insurer including agency, or frictional, costs and financial distress costs. Frictional costs arise because the insurer holds too high a level of capital. This can include deadweight losses due to taxation and also management perquisites. Holding too low a level of capital increases the expected costs of financial distress. As a result there is a trade-off between too high and too low a level of capital. Minimising the expected costs of capital produces a VaR criteria based on the probability that liability payments exceed assets and capital. Figure 1 gives the VaR probabilities based on the simple asset liability model and assumptions in Chandra and Sherris (2007). Even with realistic assumptions for the costs of capital the VaR probabilities are much higher than is observed in practice. This suggests that factors other than just minimisation of frictional costs are required to explain the level of capital held by insurers in practice.

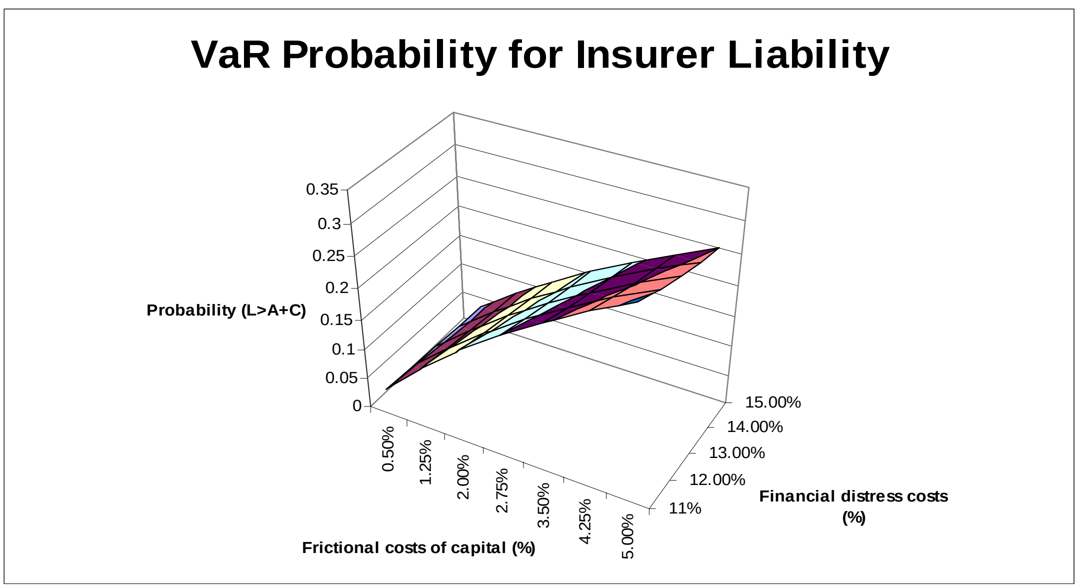

Figure 1: Optimal capital levels and VaR for Liabilities

Regulatory capital requirements have an important influence on risk based capital. The minimum level of capital for insurers, and banks, is prescribed by prudential capital requirements. These are often based on prescribed formula based but increasingly can be determined using an internal model. The approach increasingly adopted by prudential regulations is to determine capital based on specific risk categories including market, credit, insurance, and operational risks and these are then aggregated to assess enterprise wide solvency. The risk models used for these different risks often vary for the type of risk and include multivariate normal assumptions for log returns for market risks, models including frequency and severity for credit, operational and insurance risks and in some cases extreme value theory models particularly for catastrophic risks.

3. Capital Allocation

Although risk based capital is available to meet the solvency requirements of the whole company, for many purposes capital is allocated to lines of business, to products or even to risk drivers that determine the exposure of the insurer to various risks. Capital allocation is used for many purposes. An important use is the determination of actual, or expected, returns on capital by line of business by determining actual cash flows, or expected cash flows, to equity, and deriving a return based on expected cash flows to allocated capital.

Capital allocation can also be used to assess the value of acquiring or divesting a line of business or a particular division or company. The amount of capital required to support the acquisition, or freed up by a divestiture, can be used to assess expected return on capital and a decision made whether the purchase or sale is value generating.

Capital allocation is also used in determining line management compensation based on return on capital. Since capital can be a scarce resource and management are given the responsibility to manage risk and return, providing incentives through management compensation to maximize return on capital is an often used form of compensation. In practice management compensation may be based on executive options that depend on the overall performance of the company rather than a particular line of business or division.

The equity or capital of an insurer is the market value of the assets less the market-consistent value of liabilities including no allowance for prudential margins. Equity includes the insurer’s risk based capital. The equity value will also reflect other market based factors such as the value of the insolvency put option and the value of market determined profit margins from writing insurance business.

There are many different approaches to allocating capital to line of business. Cummins (2000) and Venter (2004) discuss the alternative approaches. The allocation of capital to lines of business can have a significant impact on financial decision making. By varying the allocation of capital according to different risk measures the expected return on capital for that line will also vary. Variation in allocated capital has a leveraged effect on the expected return on capital. Capital can be allocated to line of business so that the expected return on capital is equal across all lines of business and hence equal to the company wide expected cost of capital. This is one of many possible allocations. Capital can also be allocated to reflect the risk of the line of business. The question then arises as to how to quantify the risk. This is then a question of which risk measure to use and the commonly used candidates in insurance are the VaR and the TailVaR risk measures.

Risk based capital should be determined at an enterprise level to reflect the fact that capital is available to meet losses on all lines of business and is not just allocated to particular lines of business. When capital is determined by-line to reflect a range of risk factors, the total enterprise wide capital has to allow for a diversification benefit and a range of models including copula dependence models are required in order to determine this diversification benefit. Dependence models are also required if assessing risk based capital at an enterprise level.

Sherris (2006) presents a single period model framework for fair pricing, capital allocation and discusses the importance of the insolvency put option value as a risk measure, particularly for allocation of the impact of insolvency to by-line payoffs, and hence for pricing insurance contracts. Sherris (2006) also presents a single period example that illustrates the main concepts underlying the use of the insolvency put option. Sherris and van der Hoek (2006) present a model based on dependent log-normal distribution assumptions for determining the insolvency put option value by-line using closed form expressions. This model is implemented in Yow and Sherris (2007a) using a realistically calibrated single period model of an insurer in order to assess by-line pricing margins consistent with different levels of price elasticity for lines of business.

The following example is given in Sherris (2006). We review the example here to highlight the concepts of pricing, solvency and capital allocation.

Table 1 shows the payoffs in a simple insurer model. There is a single time period, a risky asset and a risk-free asset and two lines of business for the liabilities. There are 4 possible outcomes at the end of the period and these are referred to as states. The probabilities of the outcomes are given by the P-probs, usually referred to as the real-world or historical probabilities. In the Table we note that State 1 is where the risky asset has a minus 40% return, Liability 1 has a 200 loss, significantly larger than the expected loss for the line, and Liability 2 has a 40 loss, which is about equal to the expected loss. Thus State 1 is an extremely adverse outcome for the Example insurer. States 2 and 3 are both similar to each other with relatively modest asset returns and liability losses. State 4 has a 50% return on the risky asset, no loss on Liability 1 but a huge loss on Liability 2 well in excess of the expected loss.

The Q-probs are the pricing probabilities or the risk neutral probabilities. These are used to value the payoffs for the different states. This is done by multiplying the payoff in each state by the Q probability for that state, adding these together and dividing by one plus the risk free rate of return, which is 5% for this example. The time 0 values for the liabilities are shown in the Table using these Q-probs. Note that the returns on the assets are per dollar invested.

TABLE 1
Probabilities and Payoffs for Example Insurer

			Time 1 Payoffs			
State	*P*-probs	*Q*-probs	Risky Asset	Risk-Free Asset	Liability 1	Liability 2
1	0.1	0.1	0.6	1.05	200	40
2	0.6	0.4	1.1	1.05	4	10
3	0.2	0.4	1.0	1.05	2	4
4	0.1	0.1	1.5	1.05	0	310
Time 0 value			1.0	1.0	21.3333	38.6667

Table 2, also reproduced from Sherris (2006), shows the insurer balance sheet for the example insurer. The Time 0 assets have value of 200 and the Time 0 value of the total liabilities is 60. The Time 0 value of equity is the assets minus the liabilities which is 140. We assume this is an insurer has limited liability. Even though capital is available to absorb losses, in the event that the losses exceed the assets available including the shareholder funds these losses are absorbed by policyholders through reduced payouts on claims. Table 2 shows the shortfall in assets over liabilities for each of the states at Time 1. In State 1, because of the negative return on the risky asset and the very large loss on Liability 1, the insurer has a shortfall of 120 and is insolvent since it has 120 in assets to meet total liabilities of 240. For States 2 and 3 the insurer can meet its liabilities and it is not insolvent. In State 4, although it has a very high return on the risky asset producing 300 in asset value, the insurer also finds itself insolvent since the loss on Liability 2 is 310 and exceeds the assets by 10. This is not as large a shortfall as in State 1 even though the loss on Liability 2 in State 4 is much higher than the losses in State 1. Liability 2 appears to be the riskier line of business. The premium or Time 0 value for this Liability will reflect this variability in payoff.

TABLE 2
Insurer Balance Sheet Payoffs

	Time 1 Insurer Balance Sheet Payoffs				
State	Assets	L_1	L_2	Total L	$\max(L - A, 0)$
1	120	200	40	240	120
2	220	4	10	14	0
3	200	2	4	6	0
4	300	0	310	310	10
Time 0 value	200	21.3333	38.6667	60	12.381

It is important to recognise that if there were unlimited liability for shareholders in the insurer then there would be no shortfall in claim payments to the policyholders since the shareholders would provide the additional funds required to pay all claims. In this case the amount subscribed as capital at the start of the period to the insurer would have no bearing on the financial condition of the insurer. The guarantee to meet all claims that would be represented by the unlimited liability of the shareholders would have a value equal to the shortfall of the liability claim payments over the premiums paid by the policyholders accumulated at the rates of return on invested assets according to the assumed investment mix of the insurer. If an amount of capital is subscribed by shareholders at the start of the period then the value of this guarantee is the insolvency put option value. The insolvency put option value is the premium that the shareholders would have to pay if they were to purchase a guarantee by another party to meet excess of any policyholder claims in excess of the assets with investment returns at the end of the period. It is also the discount that the policyholders will require in their premiums to recognise the limited liability of the shareholders in order for them to pay a fair premium representing the expected payoffs at the end of the period allowing for insolvency.

Table 3 shows the value of the insolvency put option allocated to each line of business. Since the shortfall of assets over liabilities in the event of insolvency can be allocated to the liability lines of business assuming a priority for payment of liabilities. In this example we assume priority in proportion to the liability claim amount. This means if a liability is 30% of total liabilities in the event of insolvency then it receives 30% of the assets available. In State 1 the insurer is insolvent with a 120 shortfall. Liability 1 amounts to 120 and Liability 2 amounts to 40 so that with 120 in assets available this means there is a shortfall of 120 and sharing this proportionately to the amount owing on each liability gives a shortfall of 100 for Liability 1 and 20 for Liability 2. In the case of State 4 there is a shortfall of 10 and the only liability owing is in respect of claims on Liability 2. There are assets of 300 available to meet loss of 310. All of the shortfall is allocated to Liability 2.

TABLE 3
Liability Shortfalls in the Event of Insolvency

	Time 1 Liability Shortfalls	
State	$D_1 = L_1 \max(1 - \frac{A}{L}, 0)$	$D_2 = L_2 \max(1 - \frac{A}{L}, 0)$
1	100	20
2	0	0
3	0	0
4	0	10
Time 0 value	9.5238	2.8571

Table 4 shows the equity payoffs. As already noted, equity capital is available to meet all of the obligations of the insurer. It is not specifically allocated to Liability 1 or 2. The expected return to equity for

TABLE 4
Insurer Equity Payoffs

	Time 1 Insurer Equity Payoffs			
State	P-probs	Assets	Total L	Equity = max $(A - L, 0)$
1	0.1	120	240	0
2	0.6	220	14	206
3	0.2	200	6	194
4	0.1	300	310	0
Time 0 value		200	60	152.3810

4. Capital Structure and Financial Theory

The famous corporate finance theory on the irrelevance of capital structure, usually referred to as the Modigliani and Miller theory, determines the conditions for which capital structure is irrelevant for financial decision making. In insurance, determining the optimal capital structure is the equivalent to determining the amount of (risk-based) capital to hold for the insurer. In practice, if this capital level is irrelevant then we would not expect to observe the relatively high levels of capital held by insurers. Clearly the assumptions underlying this theory do not hold in practice and the assumptions underlying the theory identify the factors that we would expect to provide an explanation of the optimal level of capital held by insurers.

Financial theory generally assumes perfect markets and no frictional costs of capital such as agency costs or financial distress costs. Under these same assumptions a similar result also, almost, holds for capital allocation to line of business/division. Capital allocation to line of business is, mostly, irrelevant for financial decision making purposes. This is because when there are no frictional costs and a perfectly competitive insurance market, prices adjust for the level of capital held by the insurer. Prices reflect the (marginal) expected cost and a risk adjustment to the premium based on competitive market financial theory. The only direct impact that capital has on the payoffs on an insurance contract is on the payoff to the policyholder in the event of insurer default. In circumstances where there is no default, the level of capital the insurer holds does not impact the payoff on the insurance contract and the price of the insurance contract reflects only the economic risk from the insurance losses for the line of business.

In the event of insolvency the loss payments to the policyholders are reduced by the shortfall of assets compared to the loss payments and this shortfall reduces the expected payoffs to the policyholder. The value of this shortfall is equal to the insolvency put option value. The reduction in value can be determined for the insurer in the event of insolvency and, since the company defaults on all lines of business when it defaults on any one line of business, the insolvency put option value can be allocated to the losses on lines of business assuming they rank equally, to determine the by-line fair price of the contract.

On the standard assumptions of financial theory there is no value maximising optimum capital level without market imperfections. The most often considered market imperfections are the frictional costs of capital. These include agency costs, taxation and financial distress costs. There can also be an optimum capital level with no frictional costs. This arises when the insurance market is not a perfectly competitive market. In a perfectly competitive market insurers sell at market clearing risk adjusted premiums. If an insurer charges above the competitive market premium then it will not sell any business since other insurers will offer the competitive market premium. However where policyholders preferences influence their decision to purchase insurance from a particular insurance company, an insurer may charge a premium loading above the competitive market premium and still sell policies only fort a lower quantity. Policyholders may be willing to pay higher than the competitive market premium because of preferences for financial quality or other factors determining the elasticity of their preferences for a particular insurer.

5. Pricing allowing for costs of capital

An important use for risk based capital is to reflect the costs of capital in insurer by-line pricing. Premiums for lines of business normally include a risk loading to reflect the insurance risk for that line of business. More risky lines are expected to include higher risk loadings. It is important to distinguish between risk loadings that reflect the underlying risk factors of the line of business and a loading based on the costs of risk based capital. The risk loadings that reflect the risk in the line of business are often assessed using a theoretical pricing model such as the insurance capital asset pricing model. Traditional actuarial pricing methods take a cost based or supply-side approach and determine prices as expected costs plus a risk or profit loading. These methods do not directly allow for market conditions including policyholder preferences or market demand.

Insurance economics has developed pricing models based on financial pricing theory, including the capital asset pricing model and option pricing models. These models mostly assume insurers are price-takers in the insurance market. As a result, premium profit loadings reflect systematic risk factors. Fairley (1979), D'Arcy and Garven (1990), Cummins and Phillips (2005) consider financial pricing models. Myers and Cohn (1987) present discounted cash flow pricing models for by-line pricing. An allocation of capital to a line of business is required to implement these models since premiums are based on an expected return on allocated equity for each line of business.

Cummins and Danzon (1997) and Zanjani (2002) incorporate policyholder price elasticity and the policyholder default put option into their models of insurance pricing and capitalization, allowing a trade-off between price and quantity of insurance business reflecting policyholder demand. These models are more realistic and representative of insurance markets in contrast to the assumption of perfectly competitive financial asset markets.

Once capital has been allocated to lines of business it is not generally agreed as to how to determine a loading for the cost of capital. If the objective is to provide a fair rate of return for regulated lines of insurance business then the basis for determining the competitive rate of return on equity for the line of business must be considered. This is often a common enterprise wide cost of capital but it can also vary by line of business The extent to which frictional costs are included in the premium must also be considered as well as the allocation of these frictional costs to line of business.

Financial theory provides a basis for pricing insurance polices and determining risk loads that do not require an allocation of capital to line of business except for the by-line contribution to insolvency risk. In the case where the insurance market is not perfectly competitive, the market price will reflect policyholder preferences because of price elasticity and preference for financial quality which will mean that profit margins on policies will depend on the market demand at a given price offered by the insurer. The risk measure used to allocate capital to lines of business is important in determining the costs of capital for a line of business particularly where these depend directly on allocated capital.

6. Value Maximising Approach to Risk Based Capital and Capital Allocation

Financial models assuming fully competitive insurance markets do not produce optimal capital levels consistent with levels of capital held by insurers in practice. Ratings agency requirements for capital are at much higher levels than regulatory capital and insurers hold target levels of capital based on Board and shareholder decisions. In order to produce optimal capital levels it is necessary to include frictional costs, including taxes, agency and financial distress costs, and/or to assume imperfectly competitive insurance markets where profit loads for different lines depend on price elasticity or market demand including policyholder preferences for financial quality.

Yow and Sherris (2007a) and (2007b) develop and apply a value maximising approach incorporating frictional costs of capital and price elasticity for lines of business for a model insurer. They use a single period model and the enterprise, or economic, value added (EVA), defined as the difference between the value of equity at time 0 and the amount of initial capital subscribed allowing for frictional costs and insolvency. We briefly outline the model assumptions. Full details of the model are given in Yow and Sherris (2007a) and an application to insurer pricing margins and capital allocation given in Yow and Sherris (2007b).

For shareholders, allowing for corporate tax and agency costs, the time 1 model payoff is

$$E_1 = (V_1 - L_1 + D_1)(1 - \tau_1) + (\tau_1 - \tau_2)R_0,$$

where

- V_1 is the time 1 payoff from the assets of the insurer accumulated at random rate r_V,
- L_1 is the contractual time 1 liability payoffs of the insurer,
- D_1 is the contingent reduction in liability payoffs from insolvency with $D_1 = \max[L_1 - V_1, 0]$,
- R_0 is the initial shareholder cash capital subscribed at time 0,
- τ_1 is the tax rate applied to corporate insurer profits at time 1, and
- τ_2 is proportion of shareholder cash capital subscribed that is absorbed as agency costs at time 1.

The time 0 shareholder payoff value is

$$E_0 = (V_0 - L_0 + D_0)(1 - \tau_1) + e^{-r}(\tau_1 - \tau_2)R_0,$$

where

- $V_0 = R_0 + P_0 - c_0$ is the net cash available to invest at time 0 from shareholders and policyholders
- c_0 is the production cost for all policies issued
- L_0 is the fair value of the insurer liability obligations
- D_0 is the fair value of the insurer insolvency put option.

The insurer then selects by-line prices and capital subscribed to maximise the value added, or mathematically:

$$\max_{R_0, p_{i,0}} \{EVA_0\} = \max_{R_0, p_{i,0}} \{E_0 - R_0\}$$

$$= \max_{R_0, p_{i,0}} \left\{ \begin{array}{c} (P_0 - c_0 - L_0(1 - d_0))(1 - \tau_1) \\ -((1 - e^{-r})\tau_1 + e^{-r}\tau_2)R_0 \end{array} \right\},$$

where d_0 is the reduction in the time 0 fair value of the liabilities resulting from insolvency risk as a proportion of the liability.

It is important to note that allowance is made for demand with premium revenue at time 0 for sales from the N lines of business determined by

$$P_0 = \sum_{i=1}^{N} p_{i,0} q_{i,0}$$

where $p_{i,0}$ is the premium for a policy in the ith line and $q_{i,0}$ is the quantity sold in the ith line. The time 1 random loss payoff for a policy in the ith line is denoted by $L_{i,1}$ and the total random losses at time 1 is denoted by L_1 with

$$L_1 = \sum_{i=1}^{N} L_{i,1} q_{i,0}.$$

The fair value of total liabilities at time 0 are valued using a market based risk neutral valuation assumption. A risk-neutral Q probability measure is used to that value all cash flows in the model. We have

$$L_0 = e^{-r} \sum_{i=1}^{N} \mu_{i,1} q_{i,0},$$

where

$$\mu_{i,1} = E^Q[L_{i,1}]$$

is the fair, or market consistent, value of the insurance loss per policy for the ith line of business and r is the continuous compounding risk free rate of interest.

The value of the insurer default put option is given by

$$D_0 = e^{-r} E^Q[D_1].$$

The default ratio, d_0, is the default risk per dollar of liabilities with $D_0 = L_0 d_0$. The default ratio can be valued as a put option on the asset-liability ratio

$$d_0 = e^{-r} E^Q[d_1],$$

where

$$d_1 = \max[1 - \Lambda_1, 0]$$

and the asset-liability ratio is

$$\Lambda_1 = \frac{V_1}{L_1}.$$

Shareholder profit at time 1 is the shareholder payoff E_1 less the initial capital invested,

$$E_1 - R_0 = V_1 - L_1 + D_1 - R_0.$$

Corporate taxes, including the tax benefit from losses, are assumed to be

$$\tau_1(E_1 - R_0) = \tau_1(V_1 - L_1 + D_1 - R_0).$$

Shareholder agency costs of capital arising from management are assumed to be proportional to the amount of capital initially subscribed and equal to

$$\tau_2 R_0 .$$

Bankruptcy or financial distress costs are assumed to be zero if the insurer is solvent at time 1 otherwise they are assumed to be a percentage of the shortfall of assets over liabilities reflecting the size of the insolvency so they are

$$0 \text{ if } V_1 \geq L_1$$

or

$$f(L_1 - V_1) \text{ if } V_1 < L_1 .$$

with value at time 0 equal to

$$fD_0 .$$

This model can be used to assess optimal profit margins by line of business as well as risk based capital for the insurer.

The model was calibrated to be representative of a diversified multi-line insurer writing business in the Australian general insurance industry. The model incorporated frictional costs of capital including bankruptcy costs, policyholder price elasticities and policyholder preferences for insurer financial quality based on best estimate assumptions. Many of these require further research since there is limited guidance and no published empirical studies on these issues for Australian data. Australian Prudential Regulation Authority (APRA) data from the Half Yearly General Insurance Bulletin, December 1998 to December 2001 and data from the report Tillinghast-Towers Perrin, Research and Data Analysis Relevant to the Development of Standards and Guidelines on Liability Valuation for General Insurance, November 2001 was used to calibrate the distributions of losses and expense assumptions fro the model. Full details of the calibration are provided in Yow and Sherris (2007a).

The model was used to quantify the impact of varying assumptions for frictional costs on profit margins and capitalization. Assumptions were made for the frictional costs and policyholder preferences for financial quality based on the value of the insolvency put. The default ratio used is the default, or insolvency, option value as a percentage of the liabilities. It is assumed that policyholders have preferences for financial quality and price elasticity by-line that generates profit margin loadings in premiums in the range of 4% to 10% depending on the line of business. The lower the assumed elasticity the higher the profit margin since increases in premiums results in a lower percentage of sales reduction for lower price elasticity. Price elasticity assumed in the model is reflective of generally competitive insurance markets and vary by-line. They have not been determined based on empirical data, since no publicly available studies were available to quantify this assumption. However a range of values for different lines of business has been assumed.

Agency costs are deadweight losses that arise from the separation of ownership and control in a company. They include management perquisites and other costs arising from management making decisions that are not completely value maximising for shareholders. Figure 2 shows the effect of varying agency costs ranging from zero to 10%, assuming all other frictional costs are zero. Swiss Re (2005) suggest a level of 2% for agency costs is a reasonable assumption for an insurer. For this assumption in the model the value maximising capital level is 26% of liabilities and the default ratio is 0.2%. Agency costs have a significant impact on the levels of optimal capital in the model. High levels of agency costs result in dramatic reductions in capital levels to avoid paying these costs and a significant increase in default risk. Control of agency costs is clearly an important factor in any risk management strategy for an insurer since this will have significant value implications fro shareholders.

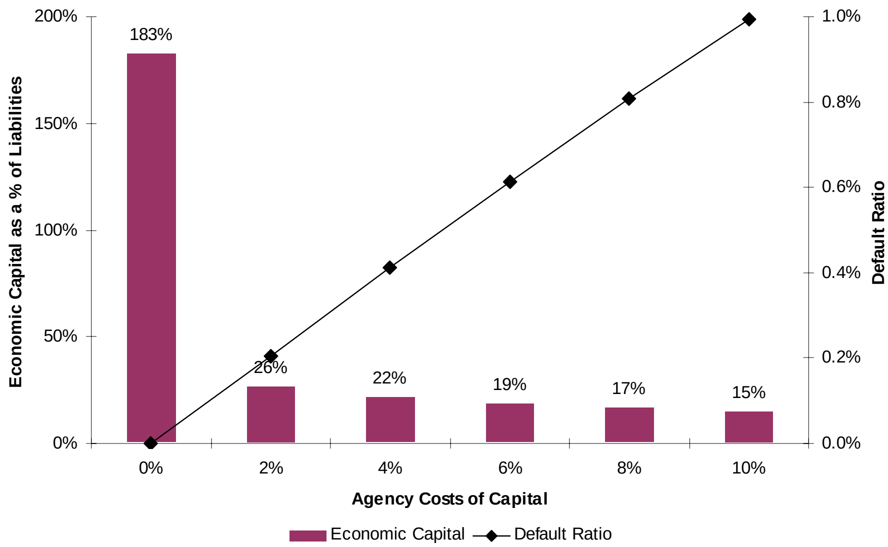

Figure 2. Optimal capital levels and default ratios for varying agency costs of capital

Figure 3 assumes that agency costs are 2% and considers the impacts of bankruptcy costs on capital and default risk. Higher levels of bankruptcy costs result in higher levels of capital since this reduces the probability of bankruptcy and, because policyholders are assumed to be willing to pay for financial quality through higher premiums, this is value maximising. As noted in Chandra and Sherris (2007), typical levels of bankruptcy costs are of the order of 20% although they have a wide range of variability. With this level of bankruptcy costs the model produces an optimal capital level of 29% of liabilities and a default ratio of about 0.1% which is around an A credit rating and lower than the AAA credit rating that is a common target in the insurance market. This does suggest that the model does not capture all the factors that determine optimal capital levels for insurers in practice. One factor that impacts the optimal levels of capital in the model is the policyholder preferences. Further research on actual market data and calibration of the model assumptions to higher levels of capital will provide a more definitive assessment of the costs and benefits of varying levels of capital in an insurer.

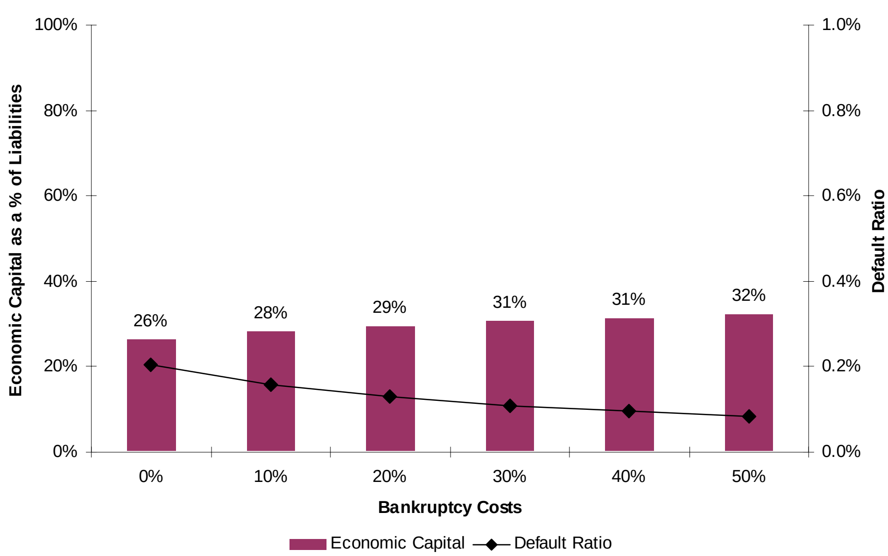

Figure 3. Optimal capital levels and default ratios for varying bankruptcy costs assuming 2% agency costs of capital

Figure 4 shows the profit margins for a number of approaches to determining premium loadings allowing for the cost of capital. The different approaches are referred to as strategies.

The first strategy is based on the maximization of insurer value added for the model insurer. The second determines capital at the insurer level using a VaR at a 99.5% probability of solvency over a one year horizon. Capital is then allocated to line of business in proportion to the VaR for individual lines of business. A constant cost of capital of 15% is assumed across all lines in order to determine premium loadings for the insurer cost of capital. Premium loadings also include frictional costs. These are added to fair values for expected losses and an allowance for the insolvency put is included. The quantity sold for the prices is then determined based on policyholder demand and the resulting financial quality of the insurer, and the optimal balance sheet derived. Strategy 3 is the same as for Strategy 2 except that a higher cost of capital is used based on an assumption that frictional costs increase the expected costs of capital and reflecting empirical studies of the costs of capital. Strategy 4 assumes different costs of capital by line reflecting the empirical results from Cummins and Phillips (2005).

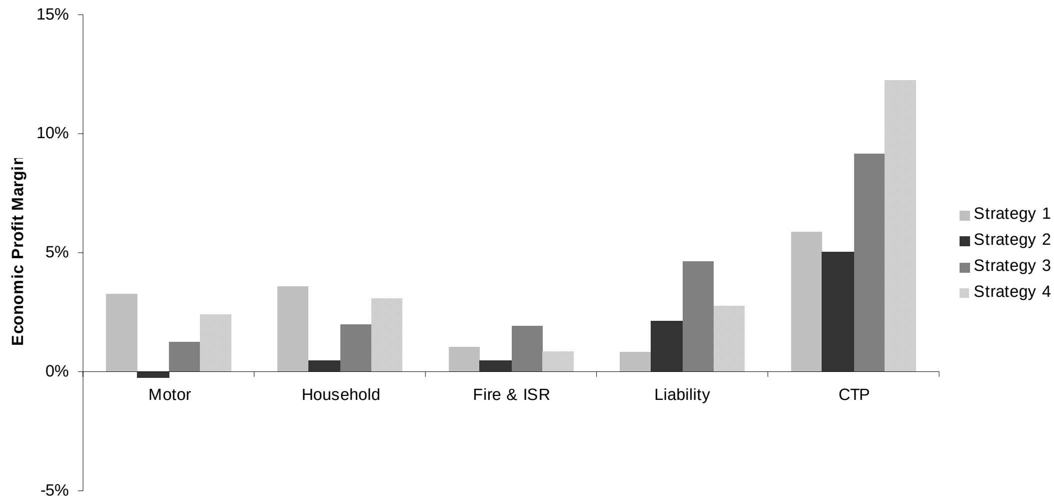

Figure 4. Profit margins for different allowance for economic capital in pricing

The model results illustrate how using VaR as a risk model can have shortcomings when compared with a value maximising model when determining premium loadings for the costs of capital. VaR does not readily translate into rating agency capital requirements since these reflect the insurance equivalent of credit spreads and are best captured by the insolvency put option value, as this is basically the insurer credit spread. VaR levels typically used by regulators translate into much lower credit spreads than the VaR probabilities once allowance is made for price elasticity and frictional costs on the by-line quantities. In the model these VaR assumptions result in higher levels of capital than is value maximising and result in higher levels of frictional costs.

In an imperfect insurance market where insurers can trade off price for volume, as determined by price elasticity assumption in the model, the profit margins do not reflect only the risk of the line of business, as assessed by VaR. Capital charges and prices based on VaR as a risk measure will not produce the profit maximising loading in premiums without taking into account market factors and the impact of frictional costs of value. The model also illustrates that if VaR, or similar risk measures, are used to allocate capital and determine capital charges by-line of business for pricing, then the capital charge by line should vary by line and should not be the total insurer cost of capital. Varying by-line costs of capital will produce profit margins closer to value maximising levels.

7. Conclusions

The science of capital allocation has made significant advances in our understanding of allocation and use of risk based capital, yet there is limited theoretical guidance on which risk measure is consistent with value maximisation and no well developed economic theory underlying the risk measures, different firms use different risk measures, there is no agreement on the appropriate risk measure, risk measures are applied inconsistently for different risks, different lines of business, products and divisions, and for insurer pricing the price of risk should vary with the type of risk under consideration yet most risk based capital approaches implicitly use a common price of risk based on a firm wide expected cost of capital for pricing.

Recent developments in capital allocation for risk capital for solvency and by-line pricing indicate a new direction is required. In order to assess risk based capital it is important to distinguish between the risk adjustment to premiums to reflect the underwriting risk of the line of business, usually regarded as a risk loading in a premium, and the capital that is held to ensure the insurer is able to meet its obligations under adverse circumstances. The premium risk loadings reflect market pricing of insurance risks and should also reflect the price elasticity of the line of business if a profit maximising assumptions is made. The risk based capital is usually held to provide a level of enterprise wide ability to meet obligations and can be regarded as a target capital level. Much of this capital is subscribed by equity investors in the case of a shareholder company. The risk measure that captures the economic value of this risk based capital is the insolvency put option value. This is similar in concept to a market based credit spread for risky debt. It reflects the probability of shortfall in assets to meet liability obligations, the severity of the shortfall and a price of risk based on market valuations.

Fair pricing of lines of business require the allocation of the default put option value by-line. Otherwise the risk loading by line reflects the underlying risk of each line of business and are determined by market factors that do not depend on the solvency or amount of capital held by the insurer. In fact these risk loadings, or profit loadings, should reflect market conditions including price elasticity for the case where insurance markets are not perfectly competitive.

The frictional costs and costs of market imperfections related to capital also impact on insurer pricing. These will in general be borne by the shareholder, or as a negative cost to equity value, and will only be recouped to the extent that value maximising premium loadings and demand elasticity in the insurer market allow a loading to be included in the premiums to recover these costs.

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