Valuing Surrender Options in Korean Interest Indexed Annuities

Changki Kim*

Abstract

We present surrender rate models with explanatory variables such as the difference between reference rates and crediting rates, policy age since issue, unemployment rates, and economy growth rates using the logit model.

We calculate the values of the surrender options in Korean interest indexed annuities. It is interesting to note that the values of the surrender options with surrender charges are negative numbers even though the surrender options are the rights given to the policy holders.

Key words:
Surrender Rate Models, Surrender Options, American Type Option Pricing.

1 Introduction

Surrender rates are one of the important factors of the interest rate sensitive cash flow movements. Kim (2005 b) shows that surrender rate fluctuations really affect the cash flows of the interest indexed annuities and investigates the surrender rate impacts on the value, the duration, and the convexity of interest indexed annuities.

Kuo, Tsai, and Chen (2003) uses the cointegration approach to reexamine the contending lapse rate hypotheses: the emergency fund hypothesis and the interest rate hypothesis. The paper shows that the interest rate hypothesis is favored against the emergency fund hypothesis in the sense that the interest rate is more economically significant than the unemployment rate in explaining the lapse rate dynamics.

Interest-indexed annuity (IIA) is one of the popular single premium deferred annuities (SPDA) sold in Korea. The distinctive features of IIA in Korea are the surrender options and annuitization options. The surrender options are American type put options given to the policy holders. In this paper we try to investigate the characteristics of policy holder surrender behaviors and calculate the value of the surrender options in Korean interest-indexed annuities.

There are a few examples on the surrender option pricing. Carriere (1996) calculates the early-exercise price for options using simulations and nonparametric regression. Albizzati and Geman (1994) calculates the value of the surrender option in life insurance policies. They derive a closed-form solution in the case of a single-premium policy. The price of the surrender option is computed in the case of French

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contracts using both the closed-form expression and Monte Carlo simulations. Bacinello (2003) calculates the price of a guaranteed life insurance participating policy, sold in the Italian market, which embeds a surrender option.

One of the main differences between our approach and the previous methods in calculating surrender options is the surrender rate model. We try to model the surrender rates with a few explanatory economic variables such as the difference between reference rates and crediting rates, policy age since issue, unemployment rates, and economy growth rates. We use a statistic model, called the logit model, in modeling surrender rates. The logit model enables us to use many explanatory variables for the surrender rates. And this logit model can be a useful tool for practical purposes.

Kim (2005 a) shows that the logit model and the complementary log-log model are generally better than the existing surrender rate models such as the arctangent model. The paper also shows that the surrender rate models should be different according to insurance policy types.

We calculate the values of the surrender options using two interest rate models, Black, Derman, and Toy (1990) model and Ho and Lee (1986) model to check if the values are independent on interest rate models.

One of the interesting things we find is that the values of the surrender option with surrender charges are negative numbers. These negative surrender option values may be some profits to the insurance companies but not to the policy holders who have the option (or the right!). One of the reasons for the negative surrender option value is that the policy holders do not use the surrender options properly for their profits. The irrational or un-optimal surrender behaviors of the policy holders may be due to bad economy conditions such as low economy growth rates and high unemployment rates. The surrender option is a right given to the policy holders and we may expect that the value of the surrender options be positive. If the policy holders use the surrender options for their profits (i.e. rationally or even optimally) then the value of surrender options can be positive and insurance companies may not have positive gains from surrender.

Another reason for the negative surrender option value is the high surrender charges. We notice that surrender charges really have an effect on the value of the surrender options of IIA. The insurance companies should find fair surrender charges to the policy holders considering the values of the surrender options.

2 The Structure of Korean Interest Indexed Annuities

In Korea, the annuity market is young and growing slowly. Many insurance companies are selling single premium deferred annuities (SPDA). But SPDA are sold with the primary focus on accumulation. Only a few of the policy holders purchase SPDA for the purpose of annuitization. Approximately less than 2% of deferred annuity values are annuitized each year in Korea.

The distinctive features of IIA in Korea are the surrender options and annuitization options. The purchasers of SPDA can surrender at any time before annuitization if the new money rates move to their advantage with reasonable surrender charges. At the date of annuitization, they can also select one type of annuity out of four choices: lump sum of their account value, whole life annuity, fixed term annuity, or
inheritance annuity. The discussion that follows will primarily address the basic annual
guaranteed crediting interest products, so called, interest indexed annuities (IIA).

2.1 Crediting Interest Rates of IIA

There are several kinds of interest indexed annuities (IIA) sold in Korea. Almost
all contracts guarantee a minimum interest rate below which the renewal crediting
interest rates will not fall. Currently the minimum guaranteed interest rate is 3% in most
cases. Historically, it has been higher in high interest environments. A majority of
companies announce their crediting interest rates based on the market rates, investment
income gain rates, and expected future investment income gain rates. Under this
approach, the interest/gain rates currently available on new investments will be reflected
in the initial crediting interest rate.

The crediting interest rates are announced every month based on current market
rates, current investment gain rates, and the expected future portfolio income gain rates.
Current market rates are usually based on the average of three year government bond
rates, three year major company bond rates, and one year major bank deposit rates. The
current investment gain rates are calculated by the average of company portfolio gains
during the previous six months. The expected future portfolio income gain rates are
estimated reflecting the trend of the market conditions. Also minimum level of crediting
rates is guaranteed.

2.2 Surrender Charges of IIA

Many contracts credit the full premium to the account value and assess surrender
charges when the policy holder surrenders. The amount of surrender charges are usually
from 7% to 10% of the account value and decreased to zero over a 6-10 year period. The
range of surrender charges of different companies may be higher or lower and the penalty
periods may run for shorter or longer.

The amount of new contract acquisition costs or commissions will be recovered
by the amount of surrender charges and future investment income. The surrender charges
will prevent the customers from terminating the contract early and cover the unamortized
acquisition costs or commissions given to the agents at the time of issue.

2.3 Free Partial Withdrawals/ Provisional Loans of IIA

A portion of the account value can be withdrawn at any time without surrender
charges to provide liquidity to the contract owner. The maximum level is 90% of the
account value at the time of partial withdrawal, but a few companies might limit the
maximum level much lower than 90% of the account value. The interest rate for this
partial withdrawal is usually crediting rate plus 1.5%. Often the policy holders can take
advantage of this partial withdrawal option several times a year. For example, when the

\[ \text{For various characteristics and valuation of SPDA, we may refer Society of Actuaries (1991),}
\] Cox, Laporte, Linney, and Lombardi (1992), and Asay, Bouyoucos, and Marciano (1993).\]
Stock markets show signs of an upward jump, the policy holders can draw out their savings from the account without any surrender charges and invest this amount of money in the stock markets. After enjoying the profits from the stock market, they can return to their insurance contracts paying relatively low interest. So this characteristic of high maximum level of partial withdrawal without surrender charges is a source that one might overuse the partial withdrawal option. Moreover the death benefit amount is still guaranteed during the partial withdrawal period.

2.4 Death Benefits in IIA

Usually the death benefit is the account value. A few variations of death benefits are considered according to the companies, for example, the account value plus 10% of premium, and another 10% of premium in the case of accidental death. Some contracts allow the spouse to take over ownership of the contract at the time of death of the owner if the spouse was a beneficiary.

2.5 Annuitization and Annuity Options in IIA

The policy holder can choose the initial annuitization date. The owner may change it before the chosen initial annuitization date. The range of the initial annuitization date is from age 45 to age 70 and usually 10 years after issue.

Guaranteed annuitization rates may be announced by the company, but these rates are really conservative. The crediting rates reflect the current market rates and portfolio income gain rates with minimum guaranteed rate of 3%. But the guaranteed annuitization rates may be based on the minimum guaranteed rate of 3% plus very conservative bonus. Some policy holders prefer minimum rate of return guaranteed products. The mortality may be mildly conservative reflecting annual improvement factors, in recognition of anticipated future mortality reductions.

There are several annuity options provided by the contract. The typical types of annuity options are (a) the lump sum withdrawal of the account value at the date of annuitization, (b) certain periods from 5 to 20 years, (c) life income with a guaranteed period of 10 years, and (d) inheritance annuity. The annuitant of inheritance annuity receives only the interest of the account value while he/she is alive and the account value will be given to the heir/heiress when the annuitant dies.

At the time of annuitization, the policy holder can choose the type of annuity considering the health condition of the annuitant and the interest rate level. So there may be potential mortality and interest rate risks to the companies with this annuity option.

3 Modeling Surrender Rates for Korean Interest Indexed Annuities

As discussed in Kim(2005 a), the surrender option is not a function of interest rate only. It depends on the policy age since the contract was issued. It also reflects the unemployment rate and the economy growth rate. For old examples of lapse studies, we may refer Richardson and Hartwell (1951), Buck (1960), and Brzezinski (1975).

We want to model the policy holder surrender behavior statistically. The variables considered are (a) the difference between reference new money rates and
product crediting rates with surrender charges, (b) the policy age since the contract was issued, (c) unemployment rates, (d) economy growth rates, and (e) seasonal effects. We will use a cascade method to generate the variables rather than use a full multivariate analysis, even though the explanatory variables are based on the methods of time-series analysis\(^2\). Figure 1 shows the cascade structure for the surrender rate modeling.

Figure 1. Cascade Structure for the Surrender Rate Modeling

3.1 Simulation of Basic Rates

We need a few basic rates to be used in modeling surrender rates and valuing the options in IIA contracts. The basic rates are short rates \(\{i(t,k), t=0,1, \ldots, T-1, \text{ and } k=0,1,2, \ldots, t\} \) for discounting the future cash flows, new money rates \(\{i_{m}(t,\omega), t=0,1, \ldots, T-1\} \) for reference rates, economy growth rates \(\{i_{EG}(t,\omega), t=0,1, \ldots, T-1\} \) for...

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..., T-1}, unemployment rates \( \{i_{UE}(t, \omega), t=0,1, \ldots, T-1\} \), announced rates \( \{i_a(t, \omega), t=0,1, \ldots, T-1\} \), IIA crediting rates \( \{i_c(t, \omega), t=0,1, \ldots, T-1\} \), minimum guaranteed rates \( i_g \), and surrender charges \( Sc(t) \). For the details and justifications on the simulation methods of the basic rates, refer Kim (2005 b). We restate the simulation results from Kim (2005 b) for our uses in this paper.

### 3.1.1 Short Rates

For a given term structure, we calculate the set \( \{P(0,t)\} \) of zero coupon bonds from a given interest rate model. \( P(0,t) \) is the time 0 price of a zero coupon bond paying 1 at maturity t. The set of short rates \( \{i(t,k)\} \), for \( t = 0,1,\ldots \) and for \( k=0,1,2, \ldots, t \) for each t, should be simulated satisfying the following equation,

\[
P(0,t) = \hat{P}(0,t),
\]

where \( \hat{P}(0,t) \) is the time 0 price of a zero coupon bond paying 1 at maturity t from the market.

We use Black-Derman-Toy (BDT) model and Ho-Lee (HL) model to generate short rates \( \{i_t, t=0,\ldots, T-1\} \).

In valuing a stream of path-dependent cash flows, we may face practical limitations since the total number of paths is really huge. For example, in valuing the surrender rate option prices of interest rate indexed annuities (IIA) with 10 year of surrender option maturity, we have \( 2^{120} \) paths, which is computationally infeasible. So we need to pick a subset of all interest rate paths with a computationally reasonable number, \( K \), of sample paths\(^3\).

Figure 2. Simulated Short Rate from a Binomial Lattice

For an example of the procedure t=5, see Ho (1992).
Let us denote the set of all interest rate paths as $\Omega$ and the number of elements of $\Omega$ as $|\Omega|$. Then $|\Omega| = 2^{120}$. We denote the subset of sample paths as $\hat{\Omega}$ with $|\hat{\Omega}| = K$, $\hat{\Omega} = \{ \omega_1, \omega_2, \ldots, \omega_K \}$.

Here we denote the k-th path of $\hat{\Omega}$ as $\omega_k$. For a path $\omega_k$, we can define a function, $\omega_k$, from time to state such that $\omega_k(t)$ is the state, s, at time t on path $\omega_k$,

$$\omega_k(t) = s,$$

where $k = 1, 2, \ldots, K$, $t = 0, 1, \ldots, T-1$, and $s = 0, 1, \ldots, t$.

For notational simplicity, let us denote $(t, \omega_k)$ to be the node at time t on path $\omega_k$, instead of $(t, \omega_k(t))$, on the binomial lattice. We show an example in Figure 2. Since the state is 3 at time t=5 on the path $\omega_k$, $(t, \omega_k)$ is (5,3) on the binomial lattice.

### 3.1.2 New Money Rates

We propose to use the maximum of the 3 year yield rates (under the increasing term structure) and the short rates (under the decreasing term structure) as the reference new money rates. We will use these reference rates to calculate the value from surrender of surrendered contracts under the assumption that the surrendered cash value is invested with the reference new money rates.4

At node $(t,k)$, to calculate the 3 year yield rates, $Y(t,k,t+3\text{year})$, for the next ten years, we can use interest rate models such as Black-Derman-Toy (BDT) or Ho-Lee (HL) models with the following formula,

$$P(t,k,u) = \sum_{j=0}^{u} A(t,k,u,j)$$

$$= \frac{1}{(1 + Y(t,k,u))^{u-t}},$$

where $P(t,k,u)$ is the price at node $(t,k)$ of the zero-coupon bond maturing at time $u>t$, and $A(t,k,u,j)$ is the Arrow-Debreu price.

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4 As one of examples, we choose 3 year rates for the reference new money rates. We may choose other reference rates such as 5 year or 10 year rates etc.

We denote $i^*_m(t)$ to be reference rates, reference market rates, and new money rates at time t with the same meaning.
For a given term structure, we can generate short rates by an interest rate model, \( \{ i(t, \omega_k), t = 0, 1, 2, \ldots , T, \text{ and } k = 1, 2, \ldots , K \} \) in the future. The resulting formula for new money rates, \( i_m(t, \omega_k) \), is
\[
  i_m(t, \omega_k) = \max \{ i(t, \omega_k), Y(t, \omega_k, 36) \}.
\] (4)

Note that as the reference rates, we use the short rates when the term structure is decreasing or the 3 year yield rates when the term structure is increasing at \( t, \omega_k \).

### 3.1.3 Economy Growth Rates

We collect past data on economy growth rates \( \{ i_{EG}(t) \} \) and past reference rates \( \{ i_m(t) \} \) of IIA, and induce a function \( h_{EG} \) such that
\[
  i_{EG}(t) = h_{EG}(i_m(t)) + e_{EG},
\] (5)
where \( e_{EG} \) is an error term. We observe that there are patterns on economy circulations with periods of 30 months. We consider this in modeling economy growth rates. We show the formula for the economy growth rates below,

\[
  i_{EG}(t) = 0.00767 - 0.095883 \times i_m(t) - 0.00565 \times \sin \left( \frac{2\pi t}{30} \right) + 0.013263 \times \cos \left( \frac{2\pi t}{30} \right) + \xi_t,
\] (6)

where
\[
  \xi_t = \xi_{t-1} - \phi_1 \xi_{t-1} - \phi_2 \xi_{t-2},
\] (7)
with \( \phi_1 = -0.733740, \phi_2 = 0.335031 \) and \( \xi_t \) follows the normal distribution, \( N(0, 0.0000842) \).

We use this function to simulate future economy growth rates,
\[
  i_{EG}(t, \omega_k) = h_{EG}(i_m(t, \omega_k)) + e_{EG},
\] (8)
for the given new money rate \( i_m(t, \omega_k) \).

### 3.1.4 Unemployment Rates

We collect past data on unemployment rates \( \{ i_{UE}(t) \} \) and observe that the unemployment rates are given by the following formula,
\[
  i_{UE}(t) = i_{UE}(t-1) \times \{ 1 + 0.11840 - 4.11360 i_{EG}(t) - 0.11440 \times DV_3 \\
  - 0.20997 \times DV_4 - 0.16229 \times DV_5 - 0.12605 \times DV_6 \\
  - 0.07518 \times DV_7 - 0.10894 \times DV_8 - 0.15145 \times DV_9 \\
  - 0.09962 \times DV_{10} \} + \xi_t,
\] (9)
where \( i_{EG}(t) \) is the economy growth rate at time \( t \), \( DV_j \) is 1 at month \( j \) and 0 otherwise, and \( \xi_t \) follows the normal distribution, \( N(0, 0.005231) \).

### 3.1.5 Announced Rates
The issuing companies usually announce crediting interest rates every month based on current market rates, current investment gain rates, and the expected future portfolio income gain rates. Considering simplicity and high correlation between reference market rates and announced rates, we collect past reference rates \( \{ i_m(t) \} \), and induce a function \( h_a \) such that

\[
i_a(t) = h_a(i_m(t)) + e_a,
\]

where \( e_a \) is an error term. Now we have the formula for the announced rates,

\[
i_a(t) = 0.100538 + 0.002527 \frac{-1}{i_m(t)} + \varepsilon_t,
\]

where \( \varepsilon_t \) follows the normal distribution \( \text{N}(0, 0.00001746) \).

We use this function to simulate future announced rates,

\[
i_a(t, \omega_k) = h_a(i_m(t, \omega_k)) + e_a,
\]

for the given reference rate \( i_m(t, \omega_k) \).

### 3.1.6 Crediting Rates of IIA

The crediting rate, \( i_c(t, \omega_k) \), is the maximum between the rates announced by the company and the guaranteed interest rate,

\[
i_c(t, \omega_k) = \max\{ i_a(t, \omega_k), i_g \},
\]

where the announced rate \( i_a(t, \omega_k) \) is given, and the guaranteed annual interest rate \( i_g \) is,

\[
i_g = 3\% \text{ annually.}
\]

In practice, considering the surrender behaviors of the policy holders, the crediting rate \( i_c(u, \omega_j) \) at time \( u \) on path \( \omega_j \) is dependent on the latest surrender time before time \( u \),

\[
i_c(u, \omega_j) = i_c(u, \omega_j, u^*),
\]

where

\[
u^* = 0, \text{ if there is no surrender before time } u
\]

= the latest surrender time before time \( u \).

And we may use the notation \( i_c(u, \omega_j) \) and \( i_c(u, \omega_j, u^*) \) together throughout the whole chapters. For computational purposes, we use the formula for the announced rates

\[
i_a(u, \omega_j) = i_a(u, \omega_j, u^*)
\]

\[
= 0.100538 + 0.002527 \frac{-1}{0.2i_m(u^*, \omega_j) + 0.8i_m(u, \omega_j)} + \varepsilon_u,
\]

where \( \varepsilon_u \) follows the normal distribution \( \text{N}(0, 0.00001746) \).

### 3.1.7 Surrender Charges
The surrender charges during the year $t$, $Sc(t)$, are

- $Sc(t) = 7\%$, $0 \leq t < 1$
- $Sc(t) = 6\%$, $1 \leq t < 2$
- $Sc(t) = 5\%$, $2 \leq t < 3$
- $Sc(t) = 4\%$, $3 \leq t < 4$
- $Sc(t) = 3\%$, $4 \leq t < 5$
- $Sc(t) = 2\%$, $5 \leq t < 6$
- $Sc(t) = 1\%$, $6 \leq t < 7$
- $Sc(t) = 0\%$, $7 \leq t < 10$.  

(17)

### 3.1.8 Surrender Rates

It is interesting to note that the probability of surrender depends on so many variables such as the difference between new money rates and crediting rates, economy growth rates, unemployment rates, policy-age since IIA policy was issued, and seasonal effects. The existing surrender rate models such as arctangent model or exponential model are using the difference between reference market rates and crediting rates only. But we have observed that surrender rates are dependent on diverse variables\(^5\).

For explanatory purposes we consider only 1 year of policy-age since IIA policy was issued. The seasonal effects are considered with the dummy-variable,

$$DV_j = \begin{cases} 1 & \text{on } j-th \text{ month} \\ 0 & \text{otherwise} \end{cases},$$

(18)

where 1-st month is January, 2-nd month is February, 3-rd month is March, and so on. So $DV_1 = 1$ on January, $DV_2 = 1$ on February and so on.

The probability of surrender between time $t$ and $t+1$ and on path $\omega_k$, $q_s(t, \omega_k)$, is given by

$$q_s(t, \omega_k) = \frac{d_s(t, \omega_k)}{l(t, \omega_k)} = f(i_m(t-j, \omega_k) - i_c(t-j, \omega_k), i_{UE}(t, \omega_k), i_{EG}(t, \omega_k), DV_t),$$

(19)

where $l(t, \omega_k)$ is the number of contracts survived, $d_s(t, \omega_k)$ is the number of contracts surrendered, and $DV_t$ is the seasonal effect dummy-variable.

Here, we use the Logit Model,

\(^5\) In Cox, Laporte, Linney, and Lombardi (1992), they examine the persistency of single-premium deferred annuities with various variables such as product characteristics, product distribution methods, credited interest rates, and age of the annuitant. Even though they do not include other variables such as federal tax penalty for early withdrawals, changes in family income or wealth, health of the annuitant, divorce, marriage, employment status, they mention that these variables may also be related to withdrawal activity. They quantify the relationship between withdrawal activity and variables associated with withdrawal activities.
\[
\ln \left( \frac{q_s(t, \omega_k)}{1-q_s(t, \omega_k)} \right) = \beta_0 + \sum_{j=0,2,4,6,8,10,12} \beta_j \cdot (i_m(t-j, \omega_k) - i_c(t-j, \omega_k)) \\
+ \beta_{UE} \cdot i_{UE}(t, \omega_k) + \beta_{EG} \cdot i_{EG}(t, \omega_k) + \sum_{j=1}^{11} \beta_{\text{month-j}} \cdot D\text{V}_j.
\]  

(20)

The parameter estimates are shown in Table 1.

It is interesting to note that the parameter \( \beta_{UE} \) for the unemployment rates is very large, 50.6348. It means that the surrender rates change very greatly according to the unemployment rate movements. But, considering the unemployment rate change ratio is not so radical as that of the reference market rates (new money rates), it is not strange for us to have a large \( \beta_{UE} \).

It seems also reasonable that the parameter \( \beta_{EG} \) for the economy growth rates is a negative number, -5.3360. We can guess that when the economy condition is good the policy holders may not surrender their IIA policies.

Now our final model for the IIA surrender rates, \( \{ q_s(t, \omega_k), t=0,1,2, \ldots, T \text{ and } k=1,2,\ldots,K \} \), is given by the following formula,

\[
q_s(t, \omega_k) = \frac{1}{1 + \exp(-\alpha)},
\]  

(21)

where

\[
\alpha = \beta_0 + \beta_{UE} \cdot i_{UE}(t, \omega_k) + \beta_{EG} \cdot i_{EG}(t, \omega_k) \\
+ \sum_{j=0,2,4,6,8,10,12} \beta_j \cdot [i_m(t-j, \omega_k) - i_c(t-j, \omega_k)] + \sum_{j=1}^{11} \beta_{\text{month-j}} \cdot D\text{V}_j.
\]  

(22)

We show the graph of the real and predicted (using Logit model) surrender rates of IIA policies below.

Table 1. Parameter Estimates with Logit Model (IIA)

<table>
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<th>Parameter</th>
<th>DF</th>
<th>Estimate</th>
<th>Error</th>
<th>Chi-Square</th>
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### Figure 3. Real and Predicted Surrender Rates of IIA

![Graph showing real and predicted surrender rates over time](image)

### 4 Valuing Surrender Options in Interest Indexed Annuities

As discussed above, there are several implicit options in interest indexed annuities (IIA) such as surrender option, minimum interest rate guarantees, minimum rate of return guarantees, and annuity selection option. Also there are death benefits before annuitization with the amount of the account value plus 10% of initial premium without the application of surrender charge. The main factor of risks among the implicit options
in IIA contracts is the surrender option given to the policy holders. The new acquisition costs are paid to the agents or brokers as commissions at the time of issue. If a customer surrenders early before the company makes enough profit to recover the amount of the new acquisition costs, there may be losses to the company. Also early surrender might cause the mismatch of asset and liability cash flows resulting in the duration mismatch.

In this section, we want to examine the surrender option of IIA and calculate the value of this option. The surrender option may not be exercised optimally. Even when the competitive market rates are high, some customers may not surrender their policies. As we see above, the policy holder surrender behavior depends not only on the difference between the reference market rates (new money rates) and crediting rates, but also on the policy age since the contract was issued, the unemployment rates, surrender charges, seasonal effects, and economic growth rates. At time \( t \) and on path \( \omega_k \), the probability of surrender, \( q_s(t, \omega_k) \), is given by

\[
q_s(t, \omega_k) = \frac{1}{1 + \exp(-\alpha)} ,
\]

where

\[
\alpha = \beta_0 + \beta_{i_c}^{\omega_k} * i_{c}(t, \omega_k) + \beta_{i_m}^{\omega_k} * i_{m}(t, \omega_k) \\
+ \sum_{j=0,2,4,6,8,12} \beta_j * \left( i_{m}(t-j, \omega_k) - i_{c}(t-j, \omega_k) \right) + \sum_{j=3}^{11} \beta_{month,j} * D_{V_j} .
\]

We consider the same IIA contract for a representative male client of age 40 with initial single premium of \( A' \) as the one in the previous section. We already defined the account values for a path \( \omega_k \), \( \{ AV(t, \omega_k) \} \), as follows. The account value at time 0 and on path \( \omega_k \), \( AV(0, \omega_k) \), is the initial single premium,

\[
AV(0, \omega_k) = A' .
\]

The account value at time \( t \) and on path \( \omega_k \) is given by

\[
AV(t, \omega_k) = AV(t-1, \omega_k) [1 + i_c(t-1, \omega_k)] ,
\]

where \( i_c(t-1, \omega_k) \) is the crediting rate at time \( t-1 \) and on path \( \omega_k \).

At time \( t \) and on path \( \omega_k \), the policy holder has the account value of \( AV(t, \omega_k) \). And he/she also has the option to surrender at time \( t \). If he/she thinks it would be advantageous to surrender IIA policy and accumulate it with high yielding new money rates, he/she might surrender the IIA contract at time \( t \). At time \( t \) on path \( \omega_k \), the policy holder has the information on the new money rate (reference rate) \( i_m(t, \omega_k) \), the crediting rate \( i_c(t, \omega_k) \), and the surrender charges \( S_c(t) \). At time \( t \) and on path \( \omega_k \), the policy holder has the account value \( AV(t, \omega_k) \) of IIA contract and can consider alternative value from surrender, \( SV(t, \omega_k) \), he/she may get when he/she surrenders the contract with the new money rate (reference rate) \( i_m(t, \omega_k) \) and surrender charge. The policy holder may also consider the keeping value, \( KV(t, \omega_k) \), the value that the policy holder may get when he/she does not surrender the contract. The exercise value for the policy holder who decides to surrender the IIA contract is calculated by the difference between the value from surrender, \( SV(t, \omega_k) \), and the keeping value, \( KV(t, \omega_k) \). To find the value
from surrender and keeping value at the node \((t, \omega_k)\) at time \(t\) on a path \(\omega_k\), we have to consider the future paths. Let us denote \(\Omega(t, \omega_k)\) to be the whole paths from the node \((t, \omega_k)\) and \(\hat{\Omega}(t, \omega_k)\) to be a subset of sample paths of \(\Omega(t, \omega_k)\). Then the number of elements or paths in \(\Omega(t, \omega_k)\), denoted by \(|\Omega(t, \omega_k)|\) is \(2^{T-t}\), which may be a large number we need to simulate a subset \(\hat{\Omega}(t, \omega_k)\) of sample paths of \(\Omega(t, \omega_k)\), with a computationally reasonable number of paths, \(|\hat{\Omega}(t, \omega_k)|\). We show a sample path \(\omega_j\) of \(\hat{\Omega}(t, \omega_k)\), i.e. \(\omega_j \in \hat{\Omega}(t, \omega_k)\) below\(^6\).

Let us denote \((s, \omega_j(s))\) to be the node at time \(s>t\) on path \(\omega_j\) on the binomial lattice. We show an example in Figure 4. Since the state is 5 at time \(s=9\) on the path \(\omega_j\), \((s, \omega_j(s))\) is \((9,5)\) on the binomial lattice. For simplicity, we denote the node as \((s, \omega_j)\) instead of \((s, \omega_j(s))\).

**Figure 4. A Sample Path from \((t, \omega_k)\)**

\[\begin{align*}
(s, \omega_j) &= (9,5) \\
(t, \omega_k) &= (5,3)
\end{align*}\]

\(^6\) We use Black-Derman-Toy model and Ho-Lee model for the binomial short rate lattice.

The sample paths can be selected using the linear path space model.
Generally, we describe a mathematical structure for the multiperiod model of a finite set of sample paths, \( \hat{\Omega} = \{ \omega_1, \omega_2, \ldots, \omega_K \} \), as follows. We assume that each \( \omega_k \in \hat{\Omega} \) has a positive probability of occurrence. We assume that a finite sequence of partitions, \( \{ P_0, P_1, \ldots, P_t, P_T \} \), of \( \hat{\Omega} \) is defined satisfying \( P_0 \subseteq P_1 \subseteq \ldots \subseteq P_t \subseteq \ldots \subseteq P_T \). We also assume that \( P_0 = \{ \hat{\Omega} \} = \{ \{ \omega_1, \omega_2, \ldots, \omega_K \} \} \), and \( P_T = \{ \{ \omega_1 \}, \{ \omega_2 \}, \ldots, \{ \omega_K \} \} \). The pair \( (\hat{\Omega}, \{ P_t \}) \) is called as a filtered space. The number of sets of \( \omega \)'s comprising the partition \( P_t \) is denoted by \( v_t \) so that \( v_t = |P_t| \) and \( P_t = \{ H_1^{(t)}, H_2^{(t)}, \ldots, H_{v_t}^{(t)} \} \). The elements \( \{ H^{(t)} \} \) of \( P_t \) are called to the time-\( t \) histories. The element of partition \( P_t \) which contains a path \( \omega \) is denoted by \( \omega^{(t)}(\omega) \). We denote \( \nu^{(t)} \) to be the number of elements of \( H^{(t)}(\omega) \). We also denote \( \hat{\Omega}^{(t)} \) to be a set consisting of one representative from each of the sets \( H_1^{(t)}, H_2^{(t)}, \ldots, H_{v_t}^{(t)} \) comprising \( P_t \). Then the number of elements of \( \hat{\Omega}^{(t)} \) is \( v_t \), \( |\hat{\Omega}^{(t)}| = v_t \). Note that the choice of representative sets \( \{ \hat{\Omega}^{(t)}, t=0,1,2, \ldots, T \} \) is not unique. We assume that \( \hat{\Omega}^{(0)} \subseteq \hat{\Omega}^{(1)} \subseteq \ldots \subseteq \hat{\Omega}^{(v)} \subseteq \ldots \subseteq \hat{\Omega}^{(T)} \).

**Figure 5. A Set of Sample Paths**

We show an example below. Suppose \( T=3 \). Then we have \( |\Omega| = 2^3 \). We assume a set of sample paths, \( \hat{\Omega} = \{ \omega_1, \omega_2, \ldots, \omega_7 \} \), with \( K=7 \). From Figure 5 we have
$P_0 = \{ \hat{\Omega} \}, K=7$

$P_1 = \{ H_1^{(1)}, H_2^{(1)} \}, v_1=2$, with

$H_1^{(1)} = \{ \omega_1, \omega_2, \omega_3, \omega_4 \}, v_1^{(1)} = 4,$

$H_2^{(1)} = \{ \omega_5, \omega_6, \omega_7 \}, v_2^{(1)} = 3,$

$P_2 = \{ H_1^{(2)}, H_2^{(2)}, H_3^{(2)}, H_4^{(2)} \}, v_2=4$, with

$H_1^{(2)} = \{ \omega_1, \omega_2 \}, v_1^{(2)} = 2,$

$H_2^{(2)} = \{ \omega_3, \omega_4 \}, v_2^{(2)} = 2,$

$H_3^{(2)} = \{ \omega_5 \}, v_3^{(2)} = 1,$

$H_4^{(2)} = \{ \omega_6, \omega_7 \}, v_4^{(2)} = 2,$

$P_3 = \{ \{ \omega_1 \}, \{ \omega_2 \}, \ldots, \{ \omega_7 \} \}, v_3 = 7 = K.$

We may have a particular choice of representative sets, $\hat{\Omega}^{(0)}, \hat{\Omega}^{(1)}, \hat{\Omega}^{(2)},$ and $\hat{\Omega}^{(3)}$ at each time such that $\hat{\Omega}^{(0)} \subseteq \hat{\Omega}^{(1)} \subseteq \hat{\Omega}^{(2)} \subseteq \hat{\Omega}^{(3)}$. For example

$\hat{\Omega}^{(0)} = \{ \omega_1 \},$

$\hat{\Omega}^{(1)} = \{ \omega_1, \omega_2 \},$

$\hat{\Omega}^{(2)} = \{ \omega_1, \omega_3, \omega_5, \omega_6 \},$

$\hat{\Omega}^{(3)} = \{ \omega_1, \omega_2, \ldots, \omega_7 \} = \hat{\Omega}.$

The policy holder has the option to surrender at any time on any path or to keep his/her policy. If he/she surrenders the policy, then he/she can decide to surrender again or keep his/her policy after one period. If he/she keeps the policy then he/she can also decide to surrender or keep his/her policy after one period.

Figure 6. Decision Trees
We use the reference rates (new money rates) to calculate the value from surrender of surrendered contracts under the assumption that the surrendered cash value is accumulated by the reference rates (new money rates) on a path.

And we use the crediting rates to calculate the accumulated value when the policy holder does not surrender. Note that the crediting rates are the function of current reference rate and the reference rate of latest surrender time. If there is no surrender before, then the crediting rate is the function of current reference rate and the reference rate of initial time. The crediting rate is greater than or equal to the minimum guaranteed rate.

If the policy holder surrenders an IIA contract issued at time 0 at time $t$ and on path $\omega_k$, he/she would have the account value reduced by surrender charges at time $t$,

$$AV(t, \omega_k)(1-Sc(t)).$$

(27)

After time $t$ on a path $\omega_j \in H(t)(\omega_k)$, the policy holder may surrender or keep the policy until the end of option period. We denote $C(t, \omega_k, T, \omega_j)$ to be the set of surrender behaviors of the policy holder from time $t$ to time $T-1$, given surrender at time $t$, on a path $\omega_j \in H(t)(\omega_k)$,

$$C(t, \omega_k, T, \omega_j) = \{ e_1^{(t,\omega_k,T,\omega_j)}, e_2^{(t,\omega_k,T,\omega_j)}, e_3^{(t,\omega_k,T,\omega_j)}, \ldots, e_{2^{T-1-t}}^{(t,\omega_k,T,\omega_j)} \},$$

(28)

where each $e_l^{(t,\omega_k,T,\omega_j)}$, $l = 1, 2, \ldots, 2^{T-1-t}$, is a surrender behavior of the policy holder from time $t$ to time $T-1$, given surrender at time $t$, on a path $\omega_j \in H(t)(\omega_k)$. Here we assume that the policy holders always reinvest.

We show an example of the set $C(T-3,\omega_k, T, \omega_j)$ of surrender behaviors given surrender at $T-3$,

$$C(T-3, \omega_k, T, \omega_j) = \{ e_1^{(T-3,\omega_k,T,\omega_j)}, e_2^{(T-3,\omega_k,T,\omega_j)}, e_3^{(T-3,\omega_k,T,\omega_j)}, e_4^{(T-3,\omega_k,T,\omega_j)} \},$$

(29)

with each surrender behavior is as follows

$$e_1^{(T-3,\omega_k,T,\omega_j)} = (s, s, s),$$

$$e_2^{(T-3,\omega_k,T,\omega_j)} = (s, k, s),$$

$$e_3^{(T-3,\omega_k,T,\omega_j)} = (s, s, k),$$

and

$$e_4^{(T-3,\omega_k,T,\omega_j)} = (s, k, k),$$

(30)

where $s$ denotes surrendering and $k$ denotes keeping.
If the policy holder invests the surrendered amount of money, \( AV(t, \omega_k)(1-Sc(t)) \), until the option maturity \( T \) under the surrender behavior \( e^{(t, \omega_k, T, \omega_t)} \), then the accumulated value from surrender \( SV(e^{(t, \omega_k, T, \omega_t)}) \) at time \( T \) and on a path \( \omega_j \in H^{(t)}(\omega_k) \) is denoted by \( SV(e^{(t, \omega_k, T, \omega_t)}) \), \( l = 1,2, \ldots, 2^{T-1-t} \). This value depends on the policy holder surrender behaviors such as the number of surrenders and surrender times.

As an example, if we assume that the surrendered cash value is invested with new money rates for two consecutive periods only, i.e. under \( e^{(t, \omega_k, T, \omega_t)} = (s,s,k,k, \ldots, k) \), then we have
\[
SV(e^{(t, \omega_k, T, \omega_t)}) = AV(t, \omega_k)(1-Sc(t))(1+i_m(t, \omega_j))(1+i_m(t+1, \omega_j)) \times \prod_{u=t+2}^{T-1}(1+i_c(u, \omega_j, u^*)),
\]
where
\[
u^* = 0, \text{ if there is no surrender before time } u
\]
\[= \text{the latest surrender time before time } u, \tag{32}
\]
and \( i_c(u, \omega_j, u^*) \) is the crediting rate at time \( u \), on path \( \omega_j \in H^{(t)}(\omega_k) \). Since we assume that surrender occurs at the end of period we apply the surrender charge \( Sc(1) \) in the above formula. We also assume that there is no surrender charge at time \( T \).

At time \( T \), the average of the accumulated value from surrender, \( \overline{SV}(t, \omega_k, T, \omega_j) \), given \( \omega_j \in H^{(t)}(\omega_k) \), is the conditional expected value of \( \{ SV(e^{(t, \omega_k, T, \omega_t)}) \}, l = 1,2, \ldots, 2^{T-1-t} \},
\[
\overline{SV}(t, \omega_k, T, \omega_j) = E^Q[SV(e^{(t, \omega_k, T, \omega_t)}) | H^{(t)}(\omega_k)] = \sum_{e^{(t, \omega_k, T, \omega_t)} \in C(t, \omega_k, T, \omega_j)} \Pr(e^{(t, \omega_k, T, \omega_t)}) SV(e^{(t, \omega_k, T, \omega_t)}), \tag{33}
\]
where \( \Pr(e^{(t, \omega_k, T, \omega_t)}) \) is a risk neutral probability of the surrender behavior \( e^{(t, \omega_k, T, \omega_t)} \in C(t, \omega_k, T, \omega_j) \).

---

7 Surrender option maturity \( T \) is the annuitization date of IIA contracts.

8 Here we assume that the surrendered cash values are accumulated by the reference new money rates on a given set of paths of new money rates under the simulation based valuation method. We may choose another reference rates and accumulation methods.

We also assume that the policy holders always reinvest. In practice, they may terminate their contracts with lump-sum withdrawals before time \( T \).

9 Note that we use a risk neutral probability of the surrender behavior. As an example see Moller (2001).
At current time $t$, the value from surrender, $SV(t, \omega_k)$, is the expected discounted value of \{ $SV(t, \omega_k, T, \omega_j)$, $\omega_j \in H^{(i)}(\omega_k)$ \} discounted by the short rates, \{ $i(u, \omega_j)$, $u=t, \ldots, T-1$, $\omega_j \in H^{(i)}(\omega_k)$ \};

$$SV(t, \omega_k) = E^Q \left[ \frac{SV(t, \omega_k, T, \omega)}{\prod_{u=t}^{T-1}(1 + i(u, \omega))} \right]$$

$$= \sum_{\omega \in H^{(i)}(\omega_k)} \Pr(\omega) \frac{SV(t, \omega_k, T, \omega)}{\prod_{u=t}^{T-1}(1 + i(u, \omega))}, \quad (34)$$

where $\Pr(\omega)$ is a risk neutral probability.

If the policy holder does not surrender the IIA contract at time $t$ and on path $\omega_k$, he/she would have the account value $AV(t, \omega_k)$. The account value will be increased with crediting rate $i_c(t, \omega_k, t')$. After one period, at time $u>t$ and on a path $\omega_j \in H^{(i)}(\omega_k)$, the policy holder may decide to surrender or not to surrender (i.e. keep) the policy. And the account value will be accumulated according to the surrender behavior of the policy holder.

We denote $F(t, \omega_k, T, \omega_j)$ to be the set of surrender behaviors of the policy holder from time $t$ to time $T-1$, given keeping at time $t$, on a path $\omega_j \in H^{(i)}(\omega_k)$,

$$F(t, \omega_k, T, \omega_j) = \{ f_1^{(t, \omega_k, T, \omega_j)}, f_2^{(t, \omega_k, T, \omega_j)}, f_3^{(t, \omega_k, T, \omega_j)}, \ldots, f_{2^{T-1}-t}^{(t, \omega_k, T, \omega_j)} \}, \quad (35)$$

where each $f_l^{(t, \omega_k, T, \omega_j)}$, $l = 1, 2, \ldots, 2^{T-1}-t$, is a surrender behavior of the policy holder from time $t$ to time $T-1$, given keeping at time $t$, on a path $\omega_j \in H^{(i)}(\omega_k)$.

We show an example of the set $F(T-3, \omega_k, T, \omega_j)$ of surrender behaviors given keeping at $T-3$,

$$F(T-3, \omega_k, T, \omega_j) = \{ f_1^{(T-3, \omega_k, T, \omega_j)}, f_2^{(T-3, \omega_k, T, \omega_j)}, f_3^{(T-3, \omega_k, T, \omega_j)}, f_4^{(T-3, \omega_k, T, \omega_j)} \}, \quad (36)$$

with each surrender behavior is as follows

$$f_1^{(T-3, \omega_k, T, \omega_j)} = (k, s, s),$$
$$f_2^{(T-3, \omega_k, T, \omega_j)} = (k, k, s),$$
$$f_3^{(T-3, \omega_k, T, \omega_j)} = (k, s, k),$$
and
$$f_4^{(T-3, \omega_k, T, \omega_j)} = (k, k, k), \quad (37)$$

where $s$ denotes surrendering and $k$ denotes keeping.
If the policy holder invests the amount of money, \( AV(t, \omega_k) \), until the option maturity \( T \) under the surrender behavior \( f_{i(t,\omega_k,T,\omega_j)} \), then the accumulated value at time \( T \) and on a path \( \omega_j \in H^{(t)}(\omega_k) \) is denoted by \( KV(f_{i(t,\omega_k,T,\omega_j)}) \), \( l = 1, 2, \ldots, 2^{T-1-t} \). This value depends on the policy holder surrender behaviors such as the number of surrenders and surrender times.

As an example, if we assume that the account value is not surrendered for two consecutive periods (time \( t \) and \( t+1 \)), surrendered for the next two consecutive periods (time \( t+2 \) and \( t+3 \)), and there is no surrender after time \( u > t+3 \), i.e. under \( f_{i(t,\omega_k,T,\omega_j)} = (k,k,s,s,k,k,k,k,k, \ldots, k) \), then we have

\[
KV(f_{i(t,\omega_k,T,\omega_j)}) = AV(t, \omega_k)*(1+i_c(t, \omega_j, u^*)) *(1-Sc(t+2))*(1+i_m(t+2, \omega_j))*(1-Sc(1))*(1+i_m(t+3, \omega_j)) \prod_{u=t+4}^{T-1}(1+i_c(u, \omega_j, u^*)),
\]

(38)

where \( \omega_j \in H^{(t)}(\omega_k) \), and \( i_c(u, \omega_j, u^*) \) is the crediting rate at time \( u \), on path \( \omega_j \).

At time \( T \), the average of the accumulated value, \( KV(t, \omega_k, T, \omega_j) \), given \( \omega_j \in H^{(t)}(\omega_k) \), is the conditional expected value of \( KV(f_{i(t,\omega_k,T,\omega_j)}) \), \( l = 1, 2, \ldots, 2^{T-1-t} \),

\[
KV(t, \omega_k, T, \omega_j) = E^Q[KV(f_{i(t,\omega_k,T,\omega_j)}) | H^{(t)}(\omega_k)]
= \sum_{f_{i(t,\omega_k,T,\omega_j)} \in F(t, \omega_k, T, \omega_j)} \Pr(f_{i(t,\omega_k,T,\omega_j)} \in F(t, \omega_k, T, \omega_j))KV(f_{i(t,\omega_k,T,\omega_j)}),
\]

(39)

where \( \Pr(f_{i(t,\omega_k,T,\omega_j)}) \) is a risk neutral probability of the surrender behavior \( f_{i(t,\omega_k,T,\omega_j)} \in F(t, \omega_k, T, \omega_j) \).

At current time \( t \), the keeping value, \( KV(t, \omega_k) \), is the expected discounted value of \( \{KV(t, \omega_k, T, \omega_j), \omega_j \in H^{(t)}(\omega_k)\} \) discounted by the short rates, \( \{i(u, \omega_j), u=t, \ldots, T-1, \omega_j \in H^{(t)}(\omega_k)\} \),

\[
KV(t, \omega_k) = E^Q \left[ \frac{KV(t, \omega_k, T, \omega)}{\prod_{u=t}^{T-1}(1+i(u, \omega))} | H^{(t)}(\omega_k) \right]
\]

Note that the crediting rates for the value from surrender and for the keeping value may be different at time \( u > t \), since they are dependent on past new money rates applied. \( u^* \) is defined to be the latest surrender time before \( u \) or 0 if there is no surrender before time \( u \).
\[
= \sum_{\omega \in H^T(\omega_k)} \Pr(\omega) \frac{KV(t, \omega_k, T, \omega_k)}{\prod_{u=t}^{T-1}(1 + i(u, \omega))},
\]

where \( \Pr(\omega) \) is a risk neutral probability.

At time \( t \) on a path \( \omega_k \), some policy holders surrender their IIA policies with positive probability \( q_s(t, \omega_k) \). Here we want to calculate the difference between the value from surrender \( SV(t, \omega_k) \) and the keeping value \( KV(t, \omega_k) \). This can be the exercise value, \( EV(t, \omega_k) \), for the policy holder who surrenders their IIA contract at time \( t \) on a path \( \omega_k \) even though they do not know the future. This exercise value can be positive or negative.

\[
EV(t, \omega_k) = p(t, \omega_k) \cdot q_s(t, \omega_k) \cdot \{SV(t, \omega_k) - KV(t, \omega_k)\}. \tag{41}
\]

Note that, at the annuitization time \( T \), the policy holder has the option to surrender IIA policy (take out the lump sum) or use the annuity option. But, right now, we do not have enough experience data on the use of the annuity option, and we do not have reliable statistics on the ratio of surrender at the time of annuitization \( T \), because much of the business is still young. Actually annuity option is another source of risks in IIA. Here we just consider the surrender cases before the annuitization time \( T \) and assume that the terminal exercise values are given by

\[
EV(T, \omega_k) = 0. \tag{42}
\]

Now we calculate the value of surrender option (VSO) for the total exercise values at time 0 as below,

\[
VSO = \sum_{k=1}^{K} \Pr(\omega_k) \sum_{t=1}^{T-1} \frac{EV(t, \omega_k)}{\prod_{u=0}^{T-1}(1 + i(u, \omega_k))}, \tag{43}
\]

where \( K \) denotes the number of paths at time 0.

We present the value of surrender option (VSO) in Table 2. Case 1 is for surrender charges \( Sc(0) = 7\% \) and decreased by 1\% each year until 0\%. Case 2 is for surrender charges \( Sc(t) = 0\% \) for all time \( t \). Here the initial single premium is 10,000.

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<th>Surrender Charge</th>
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<td>-156.63</td>
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<tr>
<td>Case 2</td>
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</tbody>
</table>

We notice that surrender charges really have an effect on the value of the surrender option of IIA.
We calculate the values of the surrender option using two interest rate models, Black-Derman-Toy (BDT) model, and Ho-Lee (HL) model to check if the values are independent on interest rate models. And the two values of the surrender option of IIA with BDT model and HL model are almost the same, but not exactly the same. So the values may be dependent on the particular choice of interest rate model.

It is also interesting to note that the values of the surrender option with surrender charges are negative numbers. These negative values may be some profits to the insurance companies and not to the policy holders who have the option (or the right!). The surrender option is a right given to the policy holders and we may expect that the value of the surrender options be positive. It may not be really surprising for someone who notice that some insurance companies get positive gains from surrender.

If the policy holders can use the surrender options for their profits (i.e. rationally or even optimally) then the value of surrender options can be positive and insurance companies may not have positive gains from surrender. We may consider, for future research topic, the irrational surrender behaviors of the policy holders probably caused by other economy conditions such as low economy growth rates, high unemployment rates, etc and the rational and optimal surrender behaviors11.

5 Fair Surrender Charges

As discussed above the values of the surrender option with surrender charges are negative numbers. The negative values of the surrender option are some profits to the insurance companies and not to the policy holders who have the option. But the policy holders may expect that the values of the surrender option should be positive. Even though the reasons for the negative values of the surrender option may be the irrational policy holder surrender behaviors due to unemployment or other factors, we still notice that the current surrender charges are very high.

Here we calculate the values of surrender option under four surrender charge scenarios: from 7% to 0% decreased by 1% each year for 7 years, from 5% to 0%

11 For mortgage-backed securities, Green and LaCour-Little (1999) show that borrower prepayment behavior appears to be highly irrational sometimes, in the sense that many borrowers prepay their mortgages when it is not optimal to do so and fail to prepay their mortgages when the prepayment option is substantially advantageous.

Richard (1991) mentions several reasons why people do not prepay when the pay off is positive. One reason is that the borrower has lost his job. Another reason is that the value of the house is lower than the mortgage value. There is also a group of people who are not rational and hold high-premium loans without prepaying them.

We may investigate the rational surrender behaviors, i.e. the policy holders surrender their contracts only when the exercise values are positive.

For rational prepayment of mortgage-backed securities, see Stanton (1995), and McConnell and Singh (1994).
decreased by 1% each year for 5 years, only 1% for 5 years, and no surrender charges. Using various surrender charge scenarios, the insurance company may find fair surrender charges not only for the company but also for the policy holders considering the economic factors in the surrender rate models.

Table 3. Finding Fair Surrender Charges

<table>
<thead>
<tr>
<th>Surrender Charges</th>
<th>VSO (BDT)</th>
<th>VSO (HL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7% - 0%</td>
<td>-158.07</td>
<td>-156.63</td>
</tr>
<tr>
<td>5% - 0%</td>
<td>-94.79</td>
<td>-94.12</td>
</tr>
<tr>
<td>1%</td>
<td>-25.60</td>
<td>-25.27</td>
</tr>
<tr>
<td>0%</td>
<td>4.77</td>
<td>4.81</td>
</tr>
</tbody>
</table>

6 Conclusion

In Korea, many insurance companies are selling single premium deferred annuities (SPDA). Especially interest-indexed annuities (IIA) are one of the most popular SPDA products. There are several options in IIA such as surrender option, minimum interest rate guarantees, minimum rate of return guarantees, and annuity selection option.

The surrender behaviors of IIA policy holders are complicated and sometimes irrational. It is really difficult to investigate and quantify the policy holder surrender behaviors with numerical methods. We have considered a model on the policy holder surrender behaviors statistically. The basic explanatory rates are modeled and the surrender rates are calculated using the Logit model. The variables considered are (a) the difference between reference rates (new money rates) and product crediting rates with surrender charges, (b) the policy age since the contract was issued, (c) unemployment rates, (d) economy growth rates, and (e) seasonal effects. We use a cascade method to generate the variables rather than use a full multivariate analysis.

We use Black-Derman-Toy model and Ho-Lee model to generate short rates. We notice that the choice of interest rate model is a consideration in valuing interest rate contingent cash flows. The values of the surrender option of IIA may depend on the choice of interest rate models.

We also notice that the surrender charges have an effect on the value of the surrender option of IIA. Finding fair surrender charges not only for the insurance companies but also for the policy holders should be considered.

It is interesting to note that the values of the surrender option with surrender charges are negative numbers, which can be some profits to the insurance companies not to the policy holders who have the option (or the right!). For the future research topic we may investigate the rational/irrational surrender behaviors of the policy holders.

References


