

Capital Management and Frictional Costs in Insurance¹

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Abstract

In this paper we develop a model of an insurer incorporating frictional costs of capital and assess and illustrate an optimal capital management strategy. Frictional costs of capital are the additional taxation on profits of shareholders from an insurer's business over and above those from direct equity investment in financial assets, transaction costs in raising equity capital, moral hazard and adverse selection costs from underwriting insurance risks, agency costs such as management perquisites, as well as the costs of financial distress and insolvency including legal costs and loss of new business opportunities. These costs are deadweight losses to equity investors in an insurer and need to be minimised in order to maximise shareholder returns. Frictional costs of capital do not include the expected return to shareholders, normally regarded as the nominal cost of capital. Fair pricing taking into account risk and optimal capital provides a fair expected rate of return to shareholders. A trade-off exists between the frictional costs of the solvent insurer and the contingent financial distress costs of the insolvent insurer. Higher levels of capital will lower expected financial distress costs but increase other frictional costs such as additional taxation, agency, moral hazard and adverse selection costs. By minimising total expected frictional costs, the model determines the optimal trade-off and the optimal level of insurer capital. The model includes adjustment costs of raising and shedding capital. The paper has implications for the long run sustainability of insurers. The efficient management of the frictional costs of capital should be a focus of the risk management strategy of an insurer and is essential for the long run survival of an insurer.

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Introduction

The capital management strategy of an insurer is critical to the long run financial sustainability of its business. In competitive markets for insurance premiums and investment returns, capital expects to earn a fair rate of return but the amount of capital held determines the level of solvency of the insurer. The amount of capital held has economic consequences because of frictional costs.

Holding capital in an insurer is costly because of additional taxation costs associated with profits to shareholders of an insurance company, transaction costs arising from equity financing, and agency costs arising from the informational asymmetries which exist between the shareholders and management of an insurance company. On the other hand, financial distress costs, another type of frictional cost, are reduced by an insurer holding higher amounts of equity capital. Financial distress costs include administration costs arising in the event of liquidation and loss of new business opportunities from the diversion of management resources from managing the business. Frictional costs are deadweight losses for an insurer's shareholders and in order to maximise shareholders' expected returns, risk management and capital management strategies should ensure they are a minimum.

We consider a model proposed in the banking literature in Estrella (2004) and develop a similar model of an insurer with frictional costs in order to derive an optimal capital management strategy. The objective is to minimise the expected value of the frictional costs by determining the level of capital the insurer should raise or shed. The model produces a trade-off between the frictional costs of capital and the costs of financial distress. Higher levels of capital will lower expected financial distress costs and increase the frictional costs of capital associated with equity financing.

The optimal capital management strategy is considered in both a single period and a multi-period model. In the single period model, the amount of capital an insurer should hold to minimise the frictional costs of capital and financial distress is determined by a Value at Risk (VaR) criterion based on the frictional costs. The model can be used to quantify the financial cost of regulatory capital requirements where the probability-of-ruin solvency measure for regulatory capital differs from the optimal level of capital.

The multi-period model incorporates a further cost of capital associated with external flows of capital, referred to as adjustment costs. These costs arise from the direct raising and shedding of external capital as opposed to the frictional costs of holding capital. The multi-period model incorporating adjustment costs demonstrates how the long run capital objective of the insurer is determined by the relative size of the frictional and adjustment costs and the current value of future insurer operating results.

The significance of capital and financial distress costs, as well as capital management strategies in both the banking and insurance industry is discussed. The single period model for an insurer is then developed and results discussed. The multi-period model is developed and analysed. The optimal capital management strategies are derived for the single period model and the infinite horizon model. Numerical results for the

optimal capital strategy using a range of reasonable values for frictional costs in both models are used to illustrate the models.

Although the model is not new, the application to an insurer and the demonstration of the implications of the model for capital management of an insurer is interesting.

Frictional Costs of Capital

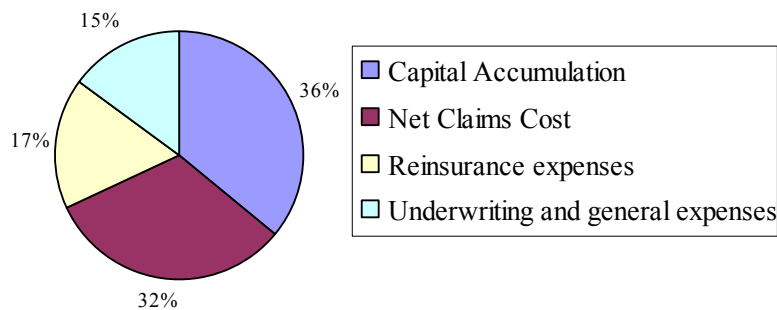
Capital is the lifeblood of a financial intermediary such as an insurer or bank. Without sufficient capital these intermediaries are not able to operate. Optimal capital structure is a cornerstone of modern financial theory. The financial services industry places great emphasis on capital due to the integral part that banks, insurers and other financial institutions play in the stability of an economy. In most economies, the financial sector is heavily regulated either through the imposition of risk management processes or capital requirements to ensure a low probability of insolvency. These requirements have increased in recent years following major losses in banks and insurer insolvencies. Never before has there been more focus on capital and risk management strategies.

Capital requirements are a dominant issue for the long run sustainability of the insurance industry. Costs related to capital impact on pricing, profitability and solvency. Capital management strategies must be developed in order to efficiently manage the costs of capital and ultimately ensure the long run viability of the institution. The companies that will survive and provide the expected returns to shareholders will be those with the optimal capital and risk management strategies.

Capital issues in the Australian insurance industry have emerged in recent years. A good example is the medical indemnity insurance business in Australia. A report by the Australian Competition and Consumer Commission (ACCC) in December 2003 indicated that 36 percent of the premiums charged by the industry were used for capital accumulation purposes. Figure 1 shows the break-up of the major factors in pricing in medical indemnity insurance. The proportion of premiums used to accumulate capital is significant.

Figure 1: Components of premium pool - Medical Indemnity

Source: ACCC Medical indemnity report, December 2003



Holding capital involves costs. Costs arise when too little capital is held by an insurer. These costs are the costs of financial distress and insolvency. In the medical indemnity insurance case, the significant collapse and provisional liquidation of United Medical Protection (UMP) in May 2002 highlighted the consequences of inadequate capital. UMP suffered from a severe case of under-reserving, with the level of claims being paid out exceeding liability valuations, resulting in financial distress costs. It is clear that the management of these costs should be a main focus of a risk and capital management strategy. Shareholder returns and payments to policyholders will be maximised if these capital and financial distress costs are minimised.

Capital costs

The cost of capital has traditionally been defined as the expected return to investors arising from the profits of the company. Increasingly the focus of capital management has turned to frictional costs. However, estimating frictional costs of capital is difficult especially for some costs such as agency costs. The cost of capital is usually estimated as a weighted average of the costs associated with different forms of capital used to finance the assets of the company. The cost of debt capital is usually available from traded markets or can be estimated using traded debt market data. The cost of equity capital is a more contentious issue which has led to the development of different techniques such as the Discounted Cash Flow (DCF) model (Brealey and Myers, (2003)), the Capital Asset Pricing Model (CAPM) (Sharpe, (1964)), and other factor models such as the Fama-French Three-Factor (FF3F) model (Fama and French, (1992)).

The cost of capital or WACC is used as a discount rate in determining the net present value of investments in real assets of a business. Often the investment is not actively traded and it is necessary to evaluate the net present value of the real asset. An investment will usually be made if the net present value is positive. The insurance business is different in that most investments by the insurer are in traded financial assets. Policyholders are the equivalent of debt providers for other firms. The insurance company's primary business is to pool and underwrite risks of loss for property and casualty risks. Capital is held as additional collateral against unexpected increases in the pool costs of meeting claim liabilities in order to ensure the insurer meets its promises to policyholders.

Kielholz (2000), in a recent study estimating the cost of capital in the insurance industry, estimates the cost of capital using the DCF and CAPM techniques for five major insurance markets (United States, United Kingdom, France, Germany and Switzerland) over the past twenty years.

Table 1 reports the average cost of equity capital for different countries and different businesses (i.e. life insurance versus non-life) over the period from 1991 to 1998. Kielholz (2000) finds that the cost of equity capital for non-life insurers in the five markets averaged 10.5% in 1998. The cost of equity capital has reduced significantly throughout the 1990s. In his analysis, Kielholz (2000) notes that one of the main factors that lead to the decline in the cost of equity capital has been the relatively low levels of nominal interest rates that have been prevalent which in turn track closely to the cost of equity capital in the insurance industry. Other factors such as the upturn in

investment markets and better underwriting performances have also been factors determining the rate of return shareholders obtain in insurance markets.

Table 1: Insurance industry average cost of equity capital estimates

	U.S.		U.K.		Switzerland		France		Germany	
	Life	Non-life	Life	Non-life	Life	Non-life	Life	Non-life	Life	Non-life
1991	n/a	16.20%	20.31%	20.25%	n/a	13.56%	16.09%	15.93%	14.44%	14.78%
1992	14.19%	15.70%	17.38%	17.52%	n/a	13.00%	16.41%	16.11%	15.14%	14.85%
1993	14.31%	14.06%	13.91%	14.48%	n/a	11.66%	17.29%	16.76%	14.32%	13.96%
1994	14.79%	13.90%	12.37%	13.09%	8.20%	9.49%	12.32%	11.50%	11.40%	10.93%
1995	15.31%	13.81%	13.51%	14.60%	7.44%	9.10%	12.31%	11.66%	10.88%	10.05%
1996	14.82%	13.33%	13.29%	14.73%	4.81%	6.49%	11.14%	10.60%	9.29%	8.39%
1997	15.49%	12.75%	13.13%	14.19%	4.96%	6.69%	8.51%	8.69%	8.28%	7.92%
1998	14.00%	13.18%	13.74%	15.23%	6.06%	6.85%	8.54%	8.43%	7.81%	8.81%
Average	14.70%	14.12%	14.71%	15.51%	6.29%	9.61%	12.83%	12.46%	11.45%	11.21%

Cummins and Phillips (2003) estimate insurer costs of capital and find that models with capital structure factors included produce higher expected costs of capital. An interpretation of these results is that frictional costs related to capital structure impact on costs of capital and that these costs are significant.

Costs of financial distress have been examined in Altman (1984). Bankruptcy costs can broadly be defined as either direct bankruptcy costs or indirect bankruptcy costs. Direct bankruptcy costs are "... those explicit costs paid by the debtor in the reorganization/liquidation process." These direct bankruptcy costs included legal, accounting, filing and other administrative costs related to the liquidation of the firm's assets. Indirect bankruptcy costs are defined by Altman (1984) as "the opportunity costs of lost managerial energies [which could lead to] ... lost sales, lost profits, the higher cost of credit, or possibly the inability of the enterprise to obtain credit or issue securities to finance new opportunities". Included as part of his definition of indirect costs of bankruptcy are, "... lost profits that a firm can be expected to suffer due to significant bankruptcy potential" so that a company in financial distress, and not necessarily insolvent, will experience these costs.

Altman (1984) estimates these indirect costs by explicitly measuring sample data of failed firms in the U.S. In estimating direct and indirect bankruptcy costs, he uses empirical data of eighteen corporate failures in the industrial and retail industries in the U.S. Direct costs are calculated by using the explicit costs recorded and documented in bankruptcy records of the sampled failed corporations.

A summary of Altman's results for direct bankruptcy costs is shown in Table 2. For the eighteen firms he sampled the average amount of direct bankruptcy costs relative to the value of the firm just prior to bankruptcy was 6.4%. They ranged from 23% down to 0.6%. This study confirms that direct bankruptcy costs can be substantial. Altman (1984) also estimated indirect bankruptcy costs.

Table 2: Altman (1984) analysis of direct bankruptcy costs

<i>Bankrupt Company</i> <i>(Year)</i>	<i>Direct Bankruptcy</i>		<i>% of Direct Bankruptcy Costs</i> <i>to Firm Value</i>		
	<i>Costs</i> <i>(\$000)</i>	<i>Firm Value (\$000,000)</i> <i>t-3</i>	<i>t[†]</i>	<i>t-3</i>	<i>t[†]</i>
Abercrombie and Fitch (1976)	471	10.7	10.5	4.40%	4.49%
Ancorp National Service (1975)	523	104.2	49.8	0.50%	1.05%
Reck Industries (1970)	650	47.7	108.4	1.36%	0.60%
Fishman, M. H. (1974)	703	41.2	8.6	1.71%	8.17%
Food Fair (1978)	n/a	376.2	416.9	n/a	n/a
Grant, W. T. (1975)	2000	1393.0	917.0	0.14%	0.22%
Interstate Stores (1974)	1664	269.1	98.2	0.62%	1.69%
Kenton (1974)	950	47.0	29.7	2.02%	3.20%
Mangel Stores (1974)	9019	47.5	38.6	18.99%	23.37%
National Bellas Hess (1972)	255	42.7	40.0	0.60%	0.64%
Neisner Bros. (1977)	1630	86.9	91.0	1.88%	1.79%
United Merchants & Manufacturing (1977)	9513	407.6	203.6	2.33%	4.67%
Bowmar Instruments (1975)	1950	35.7	11.3	5.46%	17.26%
Drew National (1975)	2278	32.5	11.1	7.01%	20.52%
Frier Industries (1978)	297	6.3	6.9	4.71%	4.30%
Precision Polymers (1976)	468	13.9	3.6	3.37%	13.00%
Universal Container (1978)	500	11.7	16.0	4.27%	3.13%
Valley Fair (1977)	541	8.4	17.7	6.44%	3.06%
Winston Mills (1978)	335	9.1	8.2	3.68%	4.09%
Average				3.86%	6.40%

† t represents the year of bankruptcy

Estimating indirect bankruptcy costs is more difficult due to the ‘opportunity cost’ nature of these costs. As a proxy for indirect bankruptcy costs, Altman (1984)

employed two approaches using forgone sales and profits. He determined the amount of indirect bankruptcy costs a company experienced as the difference between the level of profits the corporation would achieve with no financial distress, and the level of profits the corporation experiences because of financial distress.

The first approach to measure this difference utilises a linear regression of each of the sampled failed firms on the sales of each sampled corporation for the ten-year period thirteen years prior to bankruptcy. From the linear regression, the expected sales figures for the three years prior to bankruptcy are extrapolated and assumed to be the expected amount of sales the corporation would achieve with no financial distress. By multiplying the expected sales figures by the average profit margin of the industry of the corporation, the expected profits of the corporation were calculated. The difference between the expected profits calculated and the actual profits which eventuated from the company in distress then became Altman's proxy for indirect bankruptcy costs. These results are summarised in Table 3.

Altman's analysis shows that the indirect costs of bankruptcy are approximately 11% of the value of the firm just prior to bankruptcy. In one case they were as high as 62%. This is significantly higher than the direct bankruptcy costs estimated by Altman (1984) and indicates their relevance for capital management.

The second approach Altman (1984) used to estimate indirect bankruptcy costs was through the use of 'expert opinion' of securities analysts to determine the expected profits under no financial distress on a sample of failed companies. Subtracting the actual profits gave a proxy for indirect bankruptcy costs. The resulting indirect bankruptcy costs were higher than the results in Table 3. Compared to the average indirect bankruptcy cost of 11% just prior to bankruptcy in the regression technique, Altman's second technique resulted in 17.7% average indirect bankruptcy costs.

Altman argues that a possible reason for this large difference in results may be due to the differing sample of firms used in each analysis. The conclusion however, remains the same in that the size of these indirect bankruptcy costs is large even though methods to calculate these costs are imprecise.

Indirect bankruptcy costs are an important cost to be considered in capital management. In the insurance context, Smith et al (2003) discuss the effects that the costs of financial impairment and financial distress have on the pricing of insurance contracts. They discuss the heightened cost of financial impairment in the insurance industry because of the high credit sensitivity of insurance policyholders. As the financial soundness of an insurer deteriorates, policyholders may withdraw or not renew their contracts and the loss of sales and profits an insurer experiences can be significant. An insurer's future sales and growth opportunities is referred to as 'franchise value' by Smith et al (2003) and the loss of this franchise value can be seen as an example of indirect bankruptcy costs in insurance.

Table 4 summarises the total direct and indirect bankruptcy costs from the sample of eighteen firms used in the Altman (1984) analysis. Figure 2 (a) and Figure 2 (b) show the constituent break-up of the Altman (1984) analysis between direct and indirect bankruptcy costs respectively.

Table 3: Altman (1984) analysis of indirect bankruptcy costs

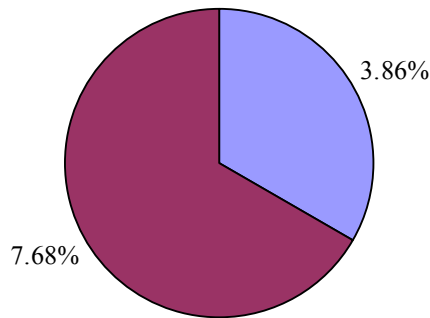
<i>Bankrupt Company</i> (Year)	<i>Indirect</i> <i>Bankruptcy Costs</i> (\$000)	<i>Firm Value (\$000,000)</i>		<i>% of Indirect Bankruptcy</i> <i>Costs to Firm Value</i>	
		<i>t-3</i>	<i>t</i> [†]	<i>t-3</i>	<i>t</i> [†]
Abercrombie and Fitch (1976)	2312	10.7	10.5	21.61%	22.02%
Ancorp National Service (1975)	2383	104.2	49.8	2.29%	4.79%
Reck Industries (1970)	0	47.7	108.4	0.00%	0.00%
Fishman, M. H. (1974)	1267	41.2	8.6	3.08%	14.73%
Food Fair (1978)	6058	376.2	416.9	n/a	n/a
Grant, W. T. (1975)	2703	1393.0	917.0	0.19%	0.29%
Interstate Stores (1974)	22294	269.1	98.2	8.28%	22.70%
Kenton (1974)	7029	47.0	29.7	14.96%	23.67%
Mangel Stores (1974)	587	47.5	38.6	1.24%	1.52%
National Bellas Hess (1972)	2269	42.7	40.0	5.31%	5.67%
Neisner Bros. (1977)	415	86.9	91.0	0.48%	0.46%
United Merchants & Manufacturing (1977)	9652	407.6	203.6	2.37%	4.74%
Bowmar Instruments (1975)	0	35.7	11.3	0.00%	0.00%
Drew National (1975)	2018	32.5	11.1	6.21%	18.18%
Frier Industries (1978)	816	6.3	6.9	12.95%	11.83%
Precision Polymers (1976)	117	13.9	3.6	0.84%	3.25%
Universal Container (1978)	243	11.7	16.0	2.08%	1.52%
Valley Fair (1977)	0	8.4	17.7	0.00%	0.00%
Winston Mills (1978)	5131	9.1	8.2	56.38%	62.57%
Average				7.68%	11.00%

† t represents the year of bankruptcy

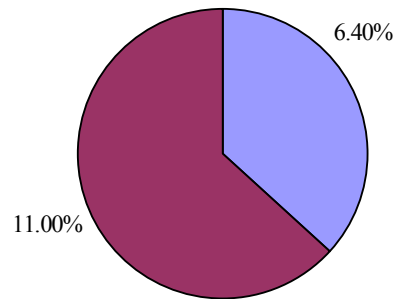
Table 4: Altman (1984) analysis of average total bankruptcy costs

	<i>Years prior to bankruptcy</i>	
	3	0
Direct bankruptcy costs/Firm Value	3.86%	6.40%
Indirect bankruptcy costs/Firm Value	7.68%	11.00%
Total bankruptcy costs/Firm Value	11.54%	17.40%

**Figure 2 (a):
Direct vs. Indirect Bankruptcy costs
3 years prior to bankruptcy**



**Figure 2 (b): Direct vs. Indirect
Bankruptcy costs
just prior to bankruptcy**



■ Direct bankruptcy costs/Firm Value
■ Indirect bankruptcy costs/Firm Value

Modigliani and Miller (1958) were the first to demonstrate that, without frictional costs, risk and capital management strategies are irrelevant. If markets operate to provide fair returns and prices, the capital structure and financing policy of the firm does not create additional value. Changing the capital structure of a firm is a way of shifting risk between the owners and debt providers that will be fairly compensated in financial markets. Managing the frictional costs of capital will be value creating.

A Banking Model with Costs of Capital

Estrella (2004) develops an optimal capital management strategy of a simplified bank through the minimisation of three costs – the cost of capital, the cost of financial distress and the cost of adjusting the level of external capital, referred to as adjustment costs. The traditional view of the cost of capital, as estimated in studies by Kielholz (2000) and Cummins and Phillips (2003), is that it is the expected return to shareholders arising from dividends and capital appreciation. This is actually an expected return to capital arising from the profits of the business. Capital is a residual claimant and expected returns are derived from the profitability of the underlying business. The costs of capital that are relevant for capital and risk management are in fact the frictional costs associated with financing through equity.

Estrella (2004) defines the costs of capital as "... not necessarily the nominal return on capital, but may be the difference between the cost of capital funding and funding through other means such as debt". Therefore, the cost Estrella (2004) aims to minimise is not the expected return demanded by equity providers, the 'nominal return on capital', but the additional costs of equity funding which include additional taxation on profits to shareholders, transaction costs in raising equity capital and agency costs involved between shareholders and management. The nominal return on capital is determined in financial markets by competitive supply and demand and if markets price fairly then this represents the fair risk adjusted expected rate of return to shareholders. For any given risk level, this cost is market determined whereas the frictional costs of capital can be optimised through a risk and capital management strategy.

Estrella (2004) considers the cost of financial distress defined as, '... costs of bankruptcy, including loss of charter value, reputational loss, and legal costs'. Estrella (2004) also considers the costs from adjusting the level of capital in the bank, referred to as adjustment costs, which occur when a firm is raising new external capital or when it is shedding external capital, for example through the payment of dividends. Adjustment costs when raising capital occur due to monitoring costs of equity, arising from agency costs and informational asymmetry. Exit costs of capital may arise from 'round-trip' costs of having to raise equity again in the future. These costs are described in Myers and Majluf (1984), Winter (1994) and McNally (1999).

The Estrella (2004) model is based on a simplified balance sheet of a bank and models the costs of capital and of financial distress as a function of the bank's end of period capital. The optimisation strategy is solved by determining the optimal level of capital to raise or shed at the beginning of each period that will minimise these costs.

Estrella (2004) develops both a single period model and a multi-period model. In the single period model the bank determines the optimal level of capital to raise or shed at the beginning of the period in order to minimise the frictional costs of capital and financial distress. The second model extends the single period model to an infinite horizon, multi-period model which also includes the adjustment costs associated with raising and shedding capital. In both models, the objective is to minimise expected frictional costs which are expressed as functions of the bank's end of period capital.

The balance sheet model includes assets, liabilities and equity. The liabilities are the deposits held by the bank. The assets are broken up into two components – a risk-free portfolio where the return earned on these assets are fixed, for example bank loans, and a risky portfolio whose rate of return is stochastic, for example investments made by the bank. Equity is the residual of assets less liabilities. This includes current equity plus new capital generated, or less any capital shed, at the beginning of the current period.

The model determines a net loss variable as the difference between the return on liabilities, assumed to be fixed, and the return on total assets, both risky and risk-free. The net loss variable, L , is a random variable because of the random nature of the returns on the risky portfolio of assets. Since L is a random variable, Estrella (2004) models the loss variable as an expected loss amount $E[L]$, and an unexpected loss

amount u , where the distribution of u is not explicitly specified to maintain generality other than requiring $E[u] = 0$.

The end of period capital is expressed as a function of the net loss variable L , the current capital and any new external capital generated at the beginning of the current period. That is $K_t = K_{t-1} + R_t - L_t$ where K_t is the end of period t capital, K_{t-1} is the current (or end of period $t-1$) capital, R_t is the amount of capital raised or shed at the beginning of time period t , and L_t is the net loss random variable during period t . The end of period capital is also a random variable due to the random nature of the losses. The amount of capital at the beginning of the period, K_{t-1} , is known and the amount R_t is the control variable in the optimisation problem. The objective function is to determine the optimum amount of capital, R_t , in order to minimise the expected frictional costs of capital and the expected costs of financial distress.

In the single-period model the optimal amount of capital to be raised or shed at the beginning of the period (R_t) is solved using standard optimisation techniques. Estrella (2004) determines the solution for R_t which minimises the frictional costs of capital and financial distress as a function of a probability-of-ruin/VaR risk measure. He shows that under the assumption of no adjustment costs, the expected level of capital at the end of each period is constant.

A similar technique is used by Estrella (2004) when adjustment costs are taken into account in the infinite horizon, multi-period model. The objective of the multi-period model is to determine the optimal path of capital required to be raised or shed at the beginning of each period in order to minimise all three costs – adjustment costs, the cost of financial distress and the frictional costs of capital.

To solve for a closed-form solution in the multi-period model, a functional form for the expected frictional costs of capital and financial distress defined in the single period model is used. Adjustment costs are modelled proportional to the square of the amount of external capital raised or shed so that adjustment costs are expressed as $c_a R_t^2$, where c_a is a constant. The solution for the multi-period model with adjustment costs indicates how the optimal amount of capital is dependent on the present value of the bank's future losses, and in general, is not the same as in the single period model with no adjustment costs.

An Insurance Company Model

Based on Estrella (2004), an insurance company model will be developed for the purposes of estimating the optimal capital management strategy for an insurer allowing for frictional costs of capital and financial distress. The aim of the model is to understand the factors determining optimal capital decisions made by insurers over time in both a single period and multi-period setting. We aim to illustrate the impact of these costs on capital management strategies in both the single period and multi-period models.

The marginal tax costs associated with the capital investments made by an insurer represents a frictional cost of capital. Myers and Cohn (1987) discuss how, due to the taxation of income derived from the capital investments made by an insurer, shareholders are often subject to double taxation when placing funds into corporate investments. This additional taxation cost would not arise if funds were directly invested in other financial assets.

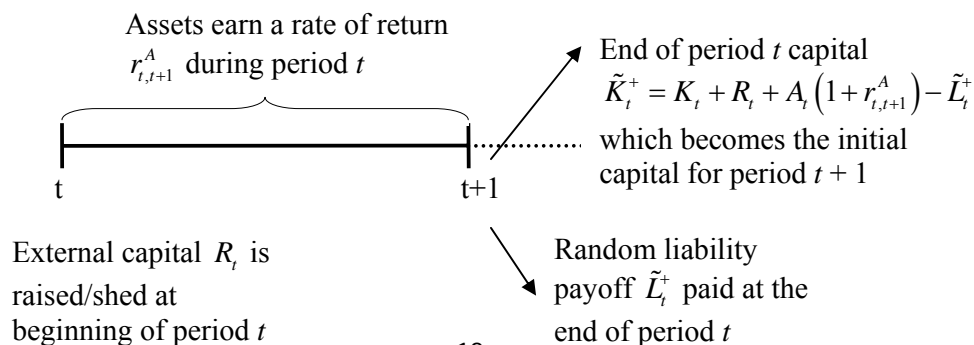
Additional transaction costs occur from equity financing in the capital markets. These additional transaction costs arise from factors such as brokerage fees or the requirement to produce disclosure statements or a prospectus. The marginal transaction costs borne by the shareholders of the insurer are deadweight losses and another form of frictional cost.

Agency costs are the costs that arise from the separation of ownership and control between management and shareholders. Jensen and Meckling (1976) and Jensen (1986) discuss agency costs. The value of the insurer falls due to management making decisions that are inconsistent with the maximisation of shareholders' wealth. For higher levels of capital, the probability of these agency costs arising increases because of the higher informational asymmetries that exist in equity funding. These costs are another example of a frictional cost of capital.

Both moral hazard and adverse selection costs arising from underwriting of insurance risks are borne by the shareholders of the insurer since these are not covered by the premiums charged to policyholders. The marginal costs borne by the shareholders of the insurance company will be a form of frictional cost associated with the use of capital for an insurer.

In the model of an insurer balance sheet we will develop subscripts to denote a point in time or a period of time. In general, a term with a subscript only will denote the value at the beginning of a time period. For example, A_t denotes the total premium income, included in the insurer's assets, at the beginning of time period t . A term with a time subscript and a superscript + symbol will be at the end of a time period. For example, the amount of capital at the end of time period t will be denoted by \tilde{K}_t^+ . The "~" symbol on K indicates it is a random variable. Finally, a term with two subscripts separated by a comma will denote a time period. For example, the term $r_{t,t+1}^A$ will denote the return on assets earned during the period t where the superscript A illustrates that it is the return received on A_t . Diagram 1 summarises the model structure for the insurer.

Diagram 1: Timing of cash flows for the insurer in period t



The liabilities comprise the outstanding claims provision, the unearned premiums liability and the equity capital. It is assumed that the insurer is financed by equity. We ignore the unearned premiums liability and consider only the random claims liability payoff. We assume that the insurer has only a single line of business – a short-term, single period renewable contract where all claims from previous time periods have been settled at the end of the coverage period. This line of business is equivalent to a short-tail general insurance product, where the claim payoffs are settled in a relatively short period of time.

The payment of claims on the liabilities of this line of business, denoted by \tilde{L}_t^+ , is a random variable. To obtain a numerical solution to the optimisation problem a parametric distribution for \tilde{L}_t^+ is not required. To illustrate the properties of the solution to the model we will assume that \tilde{L}_t^+ has a lognormal distribution.

The insurance company's assets include the premiums received from policyholders and the shareholder capital from equity owners of the insurer. The assets are invested into a portfolio of financial investments. In the model, at the start of time period t , the insurance company will hold existing capital from the prior period comprising retained profits and existing shareholder capital. This becomes a part of the initial capital for the current period t . The value of initial capital at the start of period t is denoted by K_t .

The insurer receives premium income from existing policyholder renewals and the acquisition of new business at the start of period t . Each policy acquired or renewed by the insurer will attract a fixed premium P_t paid by the policyholder at the start of the period. Q_t is the total number of policies sold and renewed at the beginning of period t . Combining the price and demand of the insurer's business, the total premium revenue the insurer will generate at the beginning of period t will be $P_t Q_t$. In the single period model, the total amount of premium revenue is $A_t = P_t Q_t$. A more sophisticated version of the model would allow price and quantity to depend on the insurer's capital. We do not address this issue.

The assets are invested in a portfolio of financial instruments. To highlight the role of liability uncertainty it is assumed that the assets earn a deterministic rate of interest denoted by $r_{t,t+1}^A$ during period t . We can easily allow the return on assets to be stochastic in order to produce a model with dependence between the return on assets and the distribution of liability claim payouts. This does not alter the intuition of the results.

The insurer has, at the beginning of time period t , current equity plus premiums received from existing and new business. The management of the insurance company must decide if this amount of initial capital is optimal. By increasing capital, the insurer's frictional costs of capital will increase as factors such as the agency costs associated with management perquisites increase. By decreasing capital, the insurance company will increase the expected financial distress costs. There is a trade-off that determines the optimal amount of capital to hold at the beginning of each period. The

insurance company will need to decide either to raise more capital, in the case where too little capital is being held, or shed some capital, in the case where too much capital is being held. Capital can be raised through new shares and capital can be shed either through the payment of dividends or the repurchase of shares at the start of each period.

The amount of capital generated or shed at the beginning of the period in order to obtain the optimal level of capital is denoted by R_t . If $R_t > 0$, then this represents the additional capital raised at the start of period t . If $R_t < 0$, then this represents the dividends paid or shares repurchased at the start of period t . After the adjustment of capital by R_t , the level of capital at the beginning of the period t will be $K_t + R_t + A_t$ which is the sum of the initial shareholder capital rolled over from the prior period and the total premium income received at the beginning of the current period t , allowing for capital adjustments R_t .

The premium income generated at the beginning of the current period is assumed to be invested in a portfolio of financial instruments earning a rate of return equal to $r_{t,t+1}^A$ over the period. The end of period capital is the initial assets plus the investment return, less the random claims liability payoff paid at the end of the period.

Denoting the amount of capital at the end of period t by \tilde{K}_t^+ , we have:

$$\tilde{K}_t^+ = K_t + R_t + A_t (1 + r_{t,t+1}^A) - \tilde{L}_t^+$$

In the single period model we assume that there are no costs associated with adjusting to the optimal level of capital and consider only the frictional costs of capital and the cost of financial distress. The model includes frictional costs of capital as a fixed and constant proportion, denoted by c_c , of the end of period capital amount \tilde{K}_t^+ provided the insurer is solvent. If we denote C_c as the frictional costs of capital, then $C_c = \max(c_c \tilde{K}_t^+, 0)$ and if the insurer is insolvent the frictional costs of capital are assumed to be zero. Because \tilde{K}_t^+ is a random variable, the frictional costs of capital are random. To minimise this cost we minimise the expected costs. The expected value of the frictional costs of capital can be expressed as:

$$E_t(C_c) = c_c \int_0^{K_t + R_t + A_t (1 + r_{t,t+1}^A)} (K_t + R_t + A_t (1 + r_{t,t+1}^A) - \tilde{L}_t^+) f(\tilde{L}_t^+) d\tilde{L}_t^+$$

where $f(\tilde{L}_t^+)$ is the probability density function of \tilde{L}_t^+ and $E_t(\cdot)$ is the expectation operator conditioned on the beginning of period t .

The second cost is the expected costs of financial distress. These costs include the administration and legal costs for a company in liquidation, as well as other indirect costs such as the loss of franchise value and future sales. Following Estrella (2004), the cost of financial distress is modelled in a similar fashion to the frictional costs of capital by assuming it is a fixed and constant proportion of the end of period capital. We denote this proportion by c_f and model the cost of financial distress, denoted by C_f as $C_f = \max(-c_f \tilde{K}_t^+, 0)$ with $c_f \geq 0$. The costs of financial distress take on non-

zero values if \tilde{K}_t^+ is negative and the insurer is insolvent at the end of the period t . The expected value of the cost of financial distress can be expressed as

$$E_t(C_f) = -c_f \int_{K_t + R_t + A_t(1+r_{t,t+1}^A)}^{\infty} (K_t + R_t + A_t(1+r_{t,t+1}^A) - \tilde{L}_t^+) f(\tilde{L}_t^+) d\tilde{L}_t^+.$$

Denoting the sum of the expected frictional costs over a single time period by C , we have $C = E_t(C_c + C_f)$ and the optimisation problem is then to solve the problem $\min_{R_t} E_t(C_c + C_f)$. The control variate in the minimisation problem is the capital adjustment amount R_t required at the beginning of period t to minimise the expected frictional costs of capital and the expected cost of financial distress. We assume that there are no costs associated with either obtaining or shedding this net level of external capital flow R_t .

We then have,

$$\begin{aligned} C &= E_t(C_c + C_f) \\ &= c_c \left\{ \int_0^{K_t + R_t + A_t(1+r_{t,t+1}^A)} (K_t + R_t + A_t(1+r_{t,t+1}^A) - \tilde{L}_t^+) f(\tilde{L}_t^+) d\tilde{L}_t^+ \right\} \\ &\quad - c_f \left\{ \int_{K_t + R_t + A_t(1+r_{t,t+1}^A)}^{\infty} (K_t + R_t + A_t(1+r_{t,t+1}^A) - \tilde{L}_t^+) f(\tilde{L}_t^+) d\tilde{L}_t^+ \right\} \\ &= c_c \left\{ \int_0^{K_t + R_t + A_t(1+r_{t,t+1}^A)} (K_t + R_t + A_t(1+r_{t,t+1}^A) - \tilde{L}_t^+) f(\tilde{L}_t^+) d\tilde{L}_t^+ \right\} \\ &\quad - c_f \left\{ \int_0^{\infty} (K_t + R_t + A_t(1+r_{t,t+1}^A) - \tilde{L}_t^+) f(\tilde{L}_t^+) d\tilde{L}_t^+ \right. \\ &\quad \left. - \int_0^{K_t + R_t + A_t(1+r_{t,t+1}^A)} (K_t + R_t + A_t(1+r_{t,t+1}^A) - \tilde{L}_t^+) f(\tilde{L}_t^+) d\tilde{L}_t^+ \right\} \end{aligned}$$

Differentiating with respect to R_t , using Leibnitz rule, and setting the differential to zero gives

$$\frac{\partial C}{\partial R_t} = c_c \left\{ \int_0^{K_t + R_t + A_t(1+r_{t,t+1}^A)} f(\tilde{L}_t^+) d\tilde{L}_t^+ \right\} - c_f \left\{ \int_0^{\infty} f(\tilde{L}_t^+) d\tilde{L}_t^+ - \int_0^{K_t + R_t + A_t(1+r_{t,t+1}^A)} f(\tilde{L}_t^+) d\tilde{L}_t^+ \right\} = 0$$

By rearranging the first-order condition we obtain

$$\begin{aligned} c_c \left[1 - \Pr(\tilde{L}_t^+ > K_t + R_t + A_t(1+r_{t,t+1}^A)) \right] &= c_f \left[1 - \Pr(\tilde{L}_t^+ \leq K_t + R_t + A_t(1+r_{t,t+1}^A)) \right] \\ \Leftrightarrow 1 - \Pr(\tilde{L}_t^+ > K_t + R_t + A_t(1+r_{t,t+1}^A)) &= \frac{c_f}{c_c} \left[\Pr(\tilde{L}_t^+ > K_t + R_t + A_t(1+r_{t,t+1}^A)) \right] \\ \Leftrightarrow \frac{1}{\Pr(\tilde{L}_t^+ > K_t + R_t + A_t(1+r_{t,t+1}^A))} - 1 &= \frac{c_f}{c_c} \end{aligned}$$

$$\Leftrightarrow \Pr(\tilde{L}_t^+ > K_t + R_t + A_t(1 + r_{t,t+1}^A)) = \frac{c_c}{c_c + c_f}$$

$$\Leftrightarrow \Pr(K_t + R_t + A_t(1 + r_{t,t+1}^A) - \tilde{L}_t^+ \leq 0) = \frac{c_c}{c_c + c_f}$$

The second-order condition is satisfied ensuring R_t is a minimum since:

$$\frac{\partial^2 C}{\partial R_t^2} = (c_c + c_f) f(K_t + R_t + A_t(1 + r_{t,t+1}^A)) \geq 0 \quad \forall R_t$$

Denoting $F_{\tilde{L}_t^+}(\cdot)$ as the cumulative distribution function of the random liability payoff

\tilde{L}_t^+ , the optimal R_t in the single period model, denoted by R_t^* , is given by:

$$R_t^* = F_{\tilde{L}_t^+}^{-1}\left(1 - \frac{c_c}{c_c + c_f}\right) - K_t - A_t(1 + r_{t,t+1}^A)$$

where $F_{\tilde{L}_t^+}^{-1}(\cdot)$ is the inverse of the cumulative distribution function of the random liability payoff \tilde{L}_t^+ .

The solution of the single period optimisation problem is both intuitive and interesting. The trade-off between high levels of capital and low levels of capital is determined by the relative size of the frictional costs of capital and the costs of financial distress. If we consider the case where there are no frictional costs of capital by taking c_f to positive and $c_c = 0$, then because $F_{\tilde{L}_t^+}^{-1}(1) \rightarrow \infty$, the optimal amount of external capital required at the beginning of period is infinite. The optimal capital strategy of the insurance company is to take on as much capital as possible to negate the possibility of any financial distress costs.

Considering the case where there are no financial distress costs, $c_f = 0$ but holding capital is costly with $c_c > 0$, the optimal amount of external capital required to shed at the beginning of the period will equal $K_t + A_t(1 + r_{t,t+1}^A)$. The optimal strategy of the insurer is to hold only enough capital to pay the expected value of the liabilities. Any additional capital held will incur frictional capital costs and since there are no costs associated with financial distress or insolvency, the optimal strategy would be to hold a minimum level of capital so that the frictional costs of capital are minimised. Because both of these costs are experienced in practice it is the trade-off between them that determines the optimal capital.

The results also shows that the amount of capital required to minimise the expected frictional costs of capital and financial distress, can be expressed in terms of a Value at Risk (VaR) requirement. Because R_t^* is determined by

$$\Pr(\tilde{L}_t^+ > K_t + R_t^* + A_t(1 + r_{t,t+1}^A)) = \frac{c_c}{c_c + c_f},$$

we see that this is a VaR requirement on the insurer loss distribution. This provides an economic basis for risk measures such as VaR and probability of ruin and may help explain why these have become important in practice. Frictional costs provide a motivation for considering VaR risk measures.

The ratio $\frac{c_c}{c_c + c_f}$ determines the VaR probability that results in an optimal level of capital for the insurer. It is of interest to assess the likely values of such a ratio to compare with regulatory requirements. Assume that a regulator requires a probability-of-ruin α , not equal to the ratio of costs $\frac{c_c}{c_c + c_f}$. More than the optimal economic level of capital will be required if $\alpha < \frac{c_c}{c_c + c_f}$ and less capital will be required if $\alpha > \frac{c_c}{c_c + c_f}$. The expected cost of these regulatory requirements will be the difference between the frictional costs of capital and financial distress for the optimal start of period capital, R_t^* , and the frictional costs of capital and financial distress when the start of period capital is that required by regulation.

To illustrate the practical implications of this result we use some simplifying assumptions. We assume the random payoff of the claims liabilities at the end of the period has a Lognormal distribution with mean claim amount 20 and standard deviation (volatility) of 4. The initial capital K_t is taken as 1. Three different premium amounts are considered equal to 10, 20 and 25 corresponding to low premiums, expected claims and higher premiums. The interest rate is assumed to be 12%.

Table 5 summarises the balance sheet data of the insurance company in this illustration.

Table 5: Insurance company data

Expected liability payoff at end of period	$E_t[\tilde{L}_t^+]$	20
Standard deviation of liability payoff	$\sigma_{\tilde{L}_t^+}$	4
Initial capital	K_t	1
Premium income: scenario 1	$A_t(1)$	10
Premium income: scenario 2	$A_t(2)$	20
Premium income: scenario 3	$A_t(3)$	25
Return on premium income	$r_{t,t+1}^A$	12%

There is very little data available for estimating the frictional costs of capital and the cost of financial distress, so a range of values for each cost is taken to determine the ratio $\frac{c_c}{c_c + c_f}$. For the case of financial distress costs, the range used in this numerical

illustration are based on Altman's estimates of direct and indirect bankruptcy costs which approximately ranged between 11% and 15%. The frictional costs of capital, assumed ranged from 0.5% to 5%. Table 6 shows the resulting values. These values are then used to determine optimal capital levels in the single period model for

varying premiums. These results are shown in Table 7, for the market premiums of 10, 20 and 25. The Table displays the optimal adjustment capital amount R_t^* for varying ratios of $\frac{c_c}{c_c + c_f}$ as well as the optimal expected end of period capital amount.

Table 6: Values for $\frac{c_c}{c_c + c_f}$ based on different assumed values for c_f and c_c

c_c	c_f				
	11%	12.000 %	13.000 %	14.000 %	15.000 %
0.50%	0.043	0.040	0.037	0.034	0.032
1.25%	0.102	0.094	0.088	0.082	0.077
2.00%	0.154	0.143	0.133	0.125	0.118
2.75%	0.200	0.186	0.175	0.164	0.155
3.50%	0.241	0.226	0.212	0.200	0.189
4.25%	0.279	0.262	0.246	0.233	0.221
5.00%	0.313	0.294	0.278	0.263	0.250

Table 7: Optimal Capital Adjustment and End of Period Expected Capital

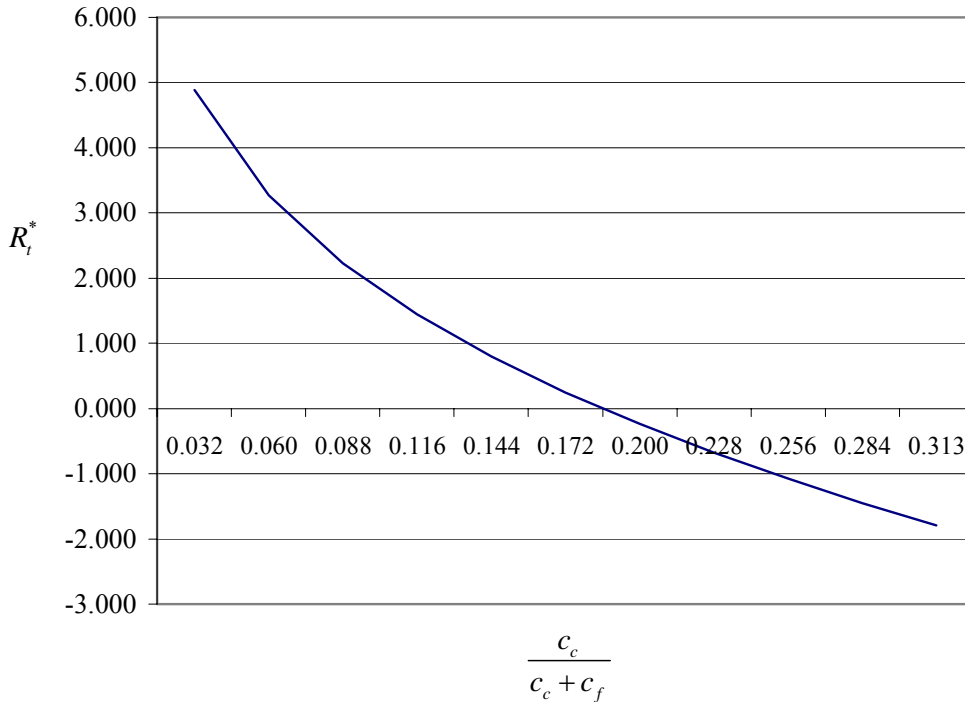
$\frac{c_c}{c_c + c_f}$	Premium = 10		Premium = 20		Premium = 25	
	R_t^*	K^*	R_t^*	K^*	R_t^*	K^*
0.032	8.282	16.082	4.882	8.282	-0.718	8.282
0.060	6.671	14.471	3.271	6.671	-2.329	6.671
0.088	5.629	13.429	2.229	5.629	-3.371	5.629
0.116	4.841	12.641	1.441	4.841	-4.159	4.841
0.144	4.197	11.997	0.797	4.197	-4.803	4.197
0.172	3.647	11.447	0.247	3.647	-5.353	3.647
0.200	3.162	10.962	-0.238	3.162	-5.838	3.162
0.228	2.725	10.525	-0.675	2.725	-6.275	2.725
0.256	2.325	10.125	-1.075	2.325	-6.675	2.325
0.284	1.953	9.753	-1.447	1.953	-7.047	1.953
0.313	1.605	9.405	-1.795	1.605	-7.395	1.605

As the ratio $\frac{c_c}{c_c + c_f}$ increases, representing either an increase in the expected frictional costs of capital or a decrease in the expected financial distress costs, the optimal amount of adjustment capital required at the beginning of the period decreases. More capital is raised when the expected frictional costs of capital are low and more capital is shed when expected frictional capital costs are high. As the market premium increases, the capital required decreases.

Figure 3 illustrates the optimal adjustment capital for the case with premiums of 20 as the VaR criteria changes. As the value at risk probability decreases the adjustment capital becomes increasingly sensitive to changes in the probability level. At levels

typical of regulatory requirements, small changes in the insurer's business outcomes can have significant impacts on the capital requirements.

Figure 3: Optimal adjustment capital under different scenarios for $\frac{c_c}{c_c + c_f}$



The cost of regulatory requirements can be assessed. Assume that the ratio $\frac{c_c}{c_c + c_f}$ is equal to 20% with $c_c = 3.5\%$ and $c_f = 14\%$, and the total premium income received by the insurance company is 20. The optimal adjustment capital for the insurer when no regulatory requirement is imposed is -0.238, resulting in an expected end of period capital equal to 3.162. When a probability-of-ruin of 5% is imposed on the insurer, the amount of capital that is required to be raised by the insurer will increase to 3.763, resulting in an expected end of period capital amount equal to 7.163. The extra regulatory capital generates an additional amount of expected frictional capital costs equal to $3.5\% \times (7.163 - 3.162) = 0.14$. Compared to the expected amount of frictional costs at the optimal point K^* of $3.5\% \times 3.162 = 0.11$, the additional cost borne by the insurer in expected frictional costs due to regulatory capital increase by over 100%.

A Multi-period Insurer Model

We now extend the insurer model to a multi-period infinite horizon setting, where adjustment costs associated with raising and shedding external capital are explicitly modelled. The single-period model assumed that there were no costs associated with raising or shedding external capital to meet the optimal level of capital required to minimise the expected frictional costs of capital and financial distress. Frictional costs

of capital also arise from the raising and shedding of capital. These costs of raising and shedding external capital are referred to as adjustment costs.

When a firm is raising new external capital, entry costs are incurred and when it is shedding capital externally, exit costs are incurred. Apart from the direct transaction costs there are other costs involved. Entry costs can arise from asymmetric information between equity providers and the management of an insurance company. Due to the high monitoring costs of equity and the informational advantage that the insurance company management has over its investors about the true value of the equity, investors are assumed to demand a premium for providing capital as discussed in Myers and Majluf (1984). Winter (1994) also considers entry costs from raising new external capital in the property-liability insurance market. Winter (1994) suggests that a proposal to raise external equity from the market may signal that profits are expected to be lower. In the case of shedding capital, McNally (1999) contends that a firm may incur a premium due to the market viewing the repurchase as an indication of an undervaluation of a firm's stock price. This would cause a stock price increase, resulting in an increase in the share buyback cost.

The optimal capital is selected so that the level of adjustment capital will minimise the frictional costs of capital, the cost of financial distress and adjustment costs. For a single period, this would be to find $\min_{\{R_t\}} C + C_a$ where C is the sum of the expected frictional costs of capital and cost of financial distress and C_a is the adjustment cost associated with raising or shedding external capital.

In order to provide an analytical solution we adopt a functional form for the cost function $C = E_t(C_c + C_f)$ by noting that C is a convex function of R_t , denoted by $C(R_t)$. Using a second-order Taylor series expansion around the optimal level of adjustment capital R_t^* we have

$$C(R_t) \approx k + (R_t - R_t^*) \left. \frac{\partial C}{\partial R_t} \right|_{R_t^*} + \frac{1}{2} (R_t - R_t^*)^2 \left. \frac{\partial^2 C}{\partial R_t^2} \right|_{R_t^*}$$

where k is a constant. The constant k does not affect the optimal value of R_t and since

at the optimum $\left. \frac{\partial C}{\partial R_t} \right|_{R_t^*} = 0$ the first-order term disappears and we obtain the

expression

$$C(R_t) \approx \frac{1}{2} (R_t - R_t^*)^2 \left. \frac{\partial^2 C}{\partial R_t^2} \right|_{R_t^*}$$

Using the fact that

$$\left. \frac{\partial^2 C}{\partial R_t^2} \right|_{R_t^*} = (c_c + c_f) f(K_t + R_t^* + A_t(1 + r_{t,t+1}^A))$$

we then have

$$C(R_t) \approx \frac{1}{2} (c_c + c_f) f(K_t + R_t^* + A_t(1 + r_{t,t+1}^A)) (R_t - R_t^*)^2$$

Expressing R_t^* in terms of the (constant) optimal expected end of period capital K^* we obtain

$$C(R_t) \approx \frac{1}{2}(c_c + c_f) f\left(K^* + E_t(\tilde{L}_t^+)\right) \left(K_t + R_t + A_t(1 + r_{t,t+1}^A) - E_t(\tilde{L}_t^+) - K^*\right)^2$$

We will use this as the functional form for the expected sum of the frictional costs of capital and cost of financial distress in a multi-period model.

Consistent with the quadratic expression for expected frictional costs, adjustment costs, denoted by C_a , are assumed to take the form $C_a = \frac{1}{2}c_a R_t^2$ where c_a is a constant.

The aim of the multi-period problem is identical to that of the single period model except that the insurance company will minimise costs over an infinite horizon by choosing an optimal path of capital, R_{t+i} for $i = 0, 1, 2, \dots, \infty$, that will be adjusted at the beginning of each period.

The objective function is

$$\min_{\{R_{t+i}\}} E_t \sum_{i=0}^{\infty} \beta^i \left\{ \frac{1}{2}(c_c + c_f) f\left(K^* + E_t(\tilde{L}_t^+)\right) \left(K_{t+i} + R_{t+i} + A_{t+i}(1 + r_{t+i,t+i+1}^A) - \tilde{L}_{t+i}^+ - K^*\right)^2 + \frac{1}{2}c_a R_{t+i}^2 \right\}$$

where $\beta = \frac{1}{1+r_f}$ is a time discount factor. The insurance company will determine the

optimal level of adjustment capital for each period that will minimise the present value of expected frictional costs associated with holding capital, financial distress and capital adjustments.

If we divide by the constant term $(c_c + c_f) f\left(K^* + E_t(\tilde{L}_t^+)\right)$ and set

$a = \frac{c_a}{(c_c + c_f) f\left(K^* + E_t(\tilde{L}_t^+)\right)}$ then the objective becomes:

$$\min_{\{R_{t+i}\}} E_t \sum_{i=0}^{\infty} \beta^i \left\{ \frac{1}{2} \left(K_{t+i} + R_{t+i} + A_{t+i}(1 + r_{t+i,t+i+1}^A) - \tilde{L}_{t+i}^+ - K^*\right)^2 + \frac{a}{2} R_{t+i}^2 \right\}$$

As the end of period capital is a function of the level of adjustment capital R_t , given by $\tilde{K}_t^+ = K_t + R_t + A_t(1 + r_{t,t+1}^A) - \tilde{L}_t^+$ the objective can be written as:

$$\min_{\{\tilde{K}_{t+i}^+\}} E_t \sum_{i=0}^{\infty} \beta^i \left\{ \frac{1}{2} \left(\tilde{K}_{t+i}^+ - K^*\right)^2 + \frac{a}{2} \left(\tilde{K}_{t+i}^+ - K_{t+i} - A_{t+i}(1 + r_{t+i,t+i+1}^A) + \tilde{L}_{t+i}^+\right)^2 \right\}$$

Differentiating with respect to \tilde{K}_{t+i}^+ for all values of $i = 0, 1, 2, \dots, \infty$ gives an infinite set of first-order conditions to solve for \tilde{K}_{t+i}^+ for $i = 0, 1, 2, \dots, \infty$. The first-order conditions are formally derived in Appendix 1. The infinite set of first-order conditions are a set of second-order difference equations in \tilde{K}_t^+ . They can be written as:

$$E_t \left\{ \begin{aligned} & \tilde{K}_{t+i+1}^+ - \frac{\gamma}{\beta} \tilde{K}_{t+i}^+ + \frac{1}{\beta} \tilde{K}_t^+ \\ & - \frac{1}{\beta} \tilde{L}_{t+i}^+ + \tilde{L}_{t+i+1}^+ + \frac{1}{\beta} A_{t+i}(1+r_{t+i,t+i+1}^A) - A_{t+i+1}(1+r_{t+i+1,t+i+2}^A) + \frac{K^*}{a\beta} \end{aligned} \right\} = 0$$

for $i = 1, 2, \dots, \infty$, where $\gamma = \frac{1}{a} + 1 + \beta$ and for $i = 0$, the \tilde{K}_t^+ term is replaced by K_t .

These first order conditions can be rearranged to obtain:

$$\begin{aligned} & E_t \left\{ \tilde{K}_{t+i+1}^+ - \frac{\gamma}{\beta} \tilde{K}_{t+i}^+ + \frac{1}{\beta} \tilde{K}_t^+ \right\} \\ & = E_t \left\{ \frac{1}{\beta} \tilde{L}_{t+i}^+ - \tilde{L}_{t+i+1}^+ - \frac{1}{\beta} A_{t+i}(1+r_{t+i,t+i+1}^A) + A_{t+i+1}(1+r_{t+i+1,t+i+2}^A) - \frac{K^*}{a\beta} \right\} \end{aligned}$$

The lag operator \mathbf{L} is then applied to the left-hand side to obtain:

$$\begin{aligned} & E_t \left\{ \left(1 - \frac{\gamma}{\beta} \mathbf{L} + \frac{1}{\beta} \mathbf{L}^2 \right) \tilde{K}_{t+i+1}^+ \right\} \\ & = E_t \left\{ \frac{1}{\beta} \tilde{L}_{t+i}^+ - \tilde{L}_{t+i+1}^+ - \frac{1}{\beta} A_{t+i}(1+r_{t+i,t+i+1}^A) + A_{t+i+1}(1+r_{t+i+1,t+i+2}^A) - \frac{K^*}{a\beta} \right\} \end{aligned}$$

The characteristic polynomial for this difference equation is $\lambda^2 - \frac{\gamma}{\beta} \lambda + \frac{1}{\beta} = 0$ with roots λ_1 and λ_2 given by

$$\lambda_1 = \frac{1 + a + a\beta - \sqrt{(1 + a\beta + a)^2 - 4a^2\beta}}{2a\beta} \quad \text{and} \quad \lambda_2 = \frac{1}{\beta\lambda_1} \quad \text{where} \quad 0 \leq \lambda_1 < 1 \quad \text{and}$$

$$\lambda_2 = \frac{1}{\beta\lambda_1} > 1 \quad \text{if} \quad \beta < 1 \quad (\text{or equivalently } r_f > 0).$$

We then have

$$\begin{aligned} & E_t \left\{ (1 - \lambda_1 \mathbf{L})(1 - \lambda_2 \mathbf{L}) \tilde{K}_{t+i+1}^+ \right\} \\ & = E_t \left\{ \frac{1}{\beta} \tilde{L}_{t+i}^+ - \tilde{L}_{t+i+1}^+ - \frac{1}{\beta} A_{t+i}(1+r_{t+i,t+i+1}^A) + A_{t+i+1}(1+r_{t+i+1,t+i+2}^A) - \frac{K^*}{a\beta} \right\} \end{aligned}$$

using

$$(1 - \lambda_1 \mathbf{L})(1 - \lambda_2 \mathbf{L}) = \left(1 - \frac{\gamma}{\beta} \mathbf{L} + \frac{1}{\beta} \mathbf{L}^2 \right).$$

Using the identity $\frac{\mathbf{L}}{1 - \lambda_2 \mathbf{L}} = \frac{-\beta\lambda_1}{1 - \beta\lambda_1 \mathbf{L}^{-1}}$ we obtain

$$\begin{aligned}
& E_t \left\{ (1 - \lambda_1 \mathbf{L}) \tilde{K}_{t+i}^+ \right\} \\
&= E_t \left\{ \frac{\beta \lambda_1}{1 - \beta \lambda_1 \mathbf{L}^{-1}} \left[\tilde{L}_{t+i+1}^+ - A_{t+i+1} (1 + r_{t+i+1, t+i+2}^A) \right] \right. \\
&\quad \left. - \frac{\lambda_1}{1 - \beta \lambda_1 \mathbf{L}^{-1}} \left[\tilde{L}_{t+i}^+ - A_{t+i} (1 + r_{t+i, t+i+1}^A) \right] + \frac{\lambda_1}{1 - \beta \lambda_1 \mathbf{L}^{-1}} \frac{K^*}{a} \right\}
\end{aligned}$$

The lag operator on the left-hand side is expanded and the difference equation on the right-hand side is evaluated in the backward direction since $\beta \lambda_1 < 1$. Note that

$$\frac{1}{1 - \phi \mathbf{L}} = \sum_{j=0}^{\infty} \phi^j \mathbf{L}^j \text{ for } |\phi| < 1 \text{ and we then have:}$$

$$\begin{aligned}
& E_t \left\{ \tilde{K}_{t+i}^+ - \lambda_1 \tilde{K}_{t+i-1}^+ \right\} \\
&= E_t \left\{ \sum_{j=0}^{\infty} (\beta \lambda_1) (\beta^j \lambda_1^j \mathbf{L}^{-j}) \left[\tilde{L}_{t+i+1}^+ - A_{t+i+1} (1 + r_{t+i+1, t+i+2}^A) \right] \right. \\
&\quad \left. - \lambda_1 \sum_{j=0}^{\infty} (\beta^j \lambda_1^j \mathbf{L}^{-j}) \left[\tilde{L}_{t+i}^+ - A_{t+i} (1 + r_{t+i, t+i+1}^A) \right] + \frac{\lambda_1}{1 - \beta \lambda_1} \frac{K^*}{a} \right\} \\
&= \lambda_1 E_t \left\{ \sum_{j=0}^{\infty} \beta (\beta^j \lambda_1^j) \left[\tilde{L}_{t+i+j+1}^+ - A_{t+i+j+1} (1 + r_{t+i+j+1, t+i+j+2}^A) \right] \right. \\
&\quad \left. - \sum_{j=0}^{\infty} (\beta^j \lambda_1^j) \left[\tilde{L}_{t+i+j}^+ - A_{t+i+j} (1 + r_{t+i+j, t+i+j+1}^A) \right] \right\} + (1 - \lambda_1) K^* \\
&= \lambda_1 E_t \left\{ \sum_{j=0}^{\infty} (\beta^j \lambda_1^j) \left[\beta \tilde{L}_{t+i+j+1}^+ - \tilde{L}_{t+i+j}^+ + A_{t+i+j} (1 + r_{t+i+j, t+i+j+1}^A) \right] \right. \\
&\quad \left. - \beta A_{t+i+j+1} (1 + r_{t+i+j+1, t+i+j+2}^A) \right\} + (1 - \lambda_1) K^*
\end{aligned}$$

The identity $\frac{\lambda_1}{1 - \beta \lambda_1} = 1 - \lambda_1$ is used to determine the transformed coefficient of K^* .

This then gives the optimal expected end of period capital as:

$$\begin{aligned}
E_t \left(\tilde{K}_{t+i}^+ \right) &= \lambda_1 E_t \left\{ \tilde{K}_{t+i-1}^+ + \sum_{j=0}^{\infty} (\beta^j \lambda_1^j) \left[\beta \tilde{L}_{t+i+j+1}^+ - \tilde{L}_{t+i+j}^+ + A_{t+i+j} (1 + r_{t+i+j, t+i+j+1}^A) \right] \right\} \\
&\quad + (1 - \lambda_1) K^*
\end{aligned}$$

for $i = 1, 2, \dots, \infty$. For the initial period with $i = 0$, the optimal amount of expected end of period capital is:

$$E_t \left(\tilde{K}_t^+ \right) = \lambda_1 \left\{ K_t + E_t \sum_{j=0}^{\infty} (\beta^j \lambda_1^j) \left[\beta \tilde{L}_{t+j+1}^+ - \tilde{L}_{t+j}^+ + A_{t+j} (1 + r_{t+j, t+j+1}^A) \right] \right\} + (1 - \lambda_1) K^*$$

The optimal adjustment capital at the beginning of the current period is determined from $R_t = E_t \left(\tilde{K}_t^+ \right) - K_t - A_t (1 + r_{t, t+1}^A) + E_t \left(\tilde{L}_t^+ \right)$ as

$$R_t = (1 - \lambda_1) (K^* - K_t) + E_t \sum_{j=0}^{\infty} (1 - \lambda_1) (\beta^j \lambda_1^j) \left(\tilde{L}_{t+j}^+ - A_{t+j} (1 + r_{t+j, t+j+1}^A) \right)$$

The optimal expected end of period capital in the multi-period model incorporating adjustment costs is a weighted average of K^* and a term involving the capital at the beginning of the current period and the expected present value of the future operating results. If adjustment costs are zero, $c_a = 0$, then $a = \frac{c_a}{(c_c + c_f) f(K^* + E_t(\tilde{L}_t))}$ is zero

and, using L'Hospital's rule:
$$\lim_{a \rightarrow 0} \lambda_1 = \lim_{a \rightarrow 0} \frac{1 + a + a\beta - \sqrt{(1 + a\beta + a)^2 - 4a^2\beta}}{2a\beta} = 0.$$

Thus when adjustment costs are zero, $\lambda_1 \rightarrow 0$ and the optimal expected end of period capital in the multi-period model is equal to that in the single-period model K^* . As adjustment costs increase, the optimal end of period capital deviates away from K^* as more weight is placed on the expected present value of the insurer's operating results.

An insurer will determine the level of capital to hold each period allowing for expected losses into the future, as well as the extent of costs associated with financial distress and capital. The relative size of frictional costs of capital, the cost of financial distress and adjustment costs influences the optimal solution. For example when adjustment costs are high compared to the frictional costs of capital financial distress, $a \rightarrow \infty \Rightarrow \lambda_1 = 1$, the optimal end of period capital will be exclusively driven by the expected present value of the insurer's future operating results. At the other extreme, when adjustment costs are zero, the optimal end of period capital of the insurer is K^* , the level of capital which minimises the frictional costs of capital and financial distress.

With adjustment costs, the insurer finds it costly to achieve the optimal level of capital K^* to minimise the expected frictional costs of capital and financial distress. A compromise must be made that minimises the frictional costs of capital and financial distress, as well as the cost of raising and shedding external capital.

The optimal amount of adjustment capital allowing for adjustment costs is
$$R_t = (1 - \lambda_1)(K^* - K_t) + E_t \sum_{j=0}^{\infty} (1 - \lambda_1) (\beta^j \lambda_1^j) (\tilde{L}_{t+j}^+ - A_{t+j}(1 + r_{t+j,t+1+j}^A)),$$
 and when $\lambda_1 = 0$ the optimal solution is the same as for the single-period model R_t^* . The second term is a long run weighted average of the present value of the insurer's future operating results where the weights of $(1 - \lambda_1) (\lambda_1^j)$ sum up to one. As λ_1 increases from zero, adjustment costs increase, and the expected present value of future losses increasingly influence the optimal solution which deviates further away from R_t^* . As adjustment costs become increasingly large and $\lambda_1 = 1$, the cost of raising external capital becomes excessive and the second term reduces to zero. When adjustment costs are extremely high, the optimal amount of external capital to raise at the beginning of each period approaches zero.

To illustrate the multi-period results we use a 5,000 period model to proxy the infinite horizon setting. The balance sheet assumes premium income is 20 for the multi-period illustration. It is assumed that the premium income of the insurer equals 20 for all periods. A constant return on assets of 12% per period is used along with an initial

capital of 1 so that $K_t=1$ to begin. The assumptions for frictional costs of capital and costs of financial distress are summarised in Table 8.

Cost of capital	c_c	0.5%
Cost of financial distress	c_f	11%
Optimal level of adjustment capital	R_t^*	4.125
Expected end of period capital	K^*	7.525

The discount parameter used is $\beta = \frac{1}{1.05}$ and four assumptions for varying sizes of adjustment costs c_a are assumed for comparison. The corresponding values of a and λ_1 are given in Table 9.

Discount factor	β	0.952
Adjustment cost 1	$c_a(1)$	0.001%
Adjustment cost 2	$c_a(2)$	1%
Adjustment cost 3	$c_a(3)$	10%
Adjustment cost 4	$c_a(4)$	60%
a under $c_a(1)$	$a(1)$	0.006
a under $c_a(2)$	$a(2)$	5.534
a under $c_a(3)$	$a(3)$	55.343
a under $c_a(4)$	$a(4)$	332.058
λ_1 under $c_a(1)$	$\lambda_1(1)$	0.005
λ_1 under $c_a(2)$	$\lambda_1(2)$	0.668
λ_1 under $c_a(3)$	$\lambda_1(3)$	0.892
λ_1 under $c_a(4)$	$\lambda_1(4)$	0.964

The assumption that the expected liability payoffs are constant over time results in an optimal capital amount converging to a constant. To allow for the future expected liability payoffs to vary through time we assume that the liability payoff for the current period is log-normally distributed with mean 20 and standard deviation 4 and the future expected liability payoffs are random with parameters equal to the parameters of the lognormal distribution for the current year's liability payoff.

Figure 4 gives a comparison between the expected liability payoffs of the insurer over a sample period of time against the optimal adjustment capital required when adjustment costs are 1%. This shows the impact the multi-period model has on the optimal path. It shows that the optimal adjustment capital path tracks the random expected liability path in a similar fashion over each period, but with a one-period lag,

since large losses in one period, require the insurer to raise additional capital in the next period, to reduce the costs of financial distress.

Figure 4: The optimal capital adjustment path versus the expected liability payoff

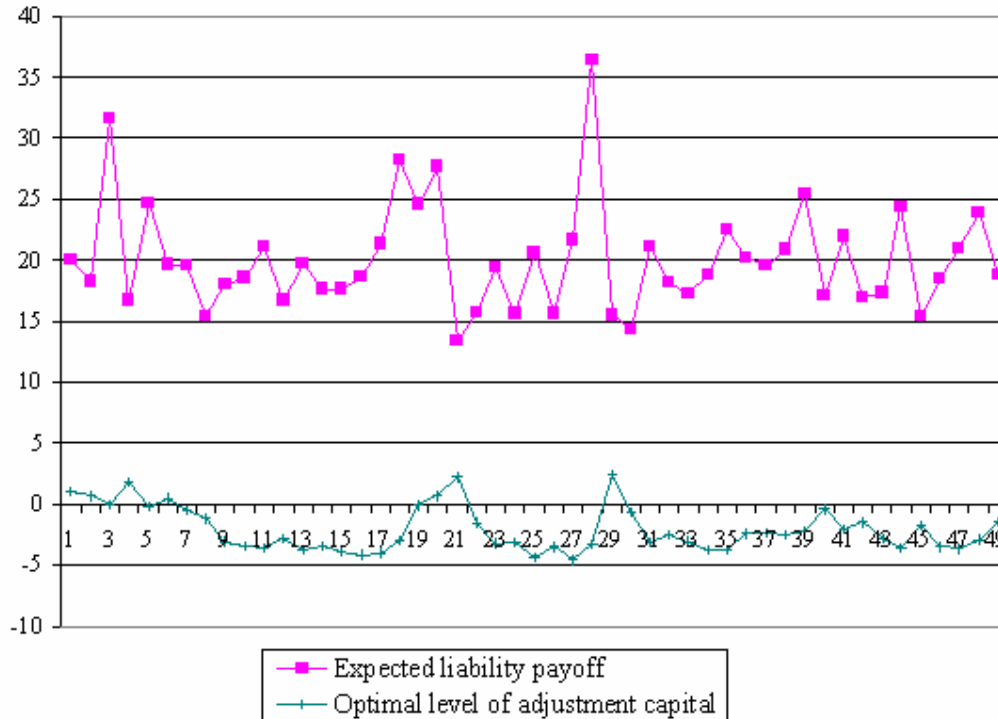


Figure 5 shows the optimal level of adjustment capital for the 4 different assumptions for adjustment costs listed in Table 9. The lower capital adjustment costs means the insurer can more flexibly adjust to the level of capital that will minimise the frictional costs of capital and financial distress. For the case where the adjustment costs are close to zero, $c_a(1) = 0.001\%$, the variability in the optimal level of adjustment capital from period to period is extremely high. As these adjustment costs increase, the variability reduces because of the increased importance of the adjustment costs.

Figure 6 shows the expected end of period capital path for each of the four levels of adjustment costs. We assume that the insurer will not cease business if the end of period capital is negative at any time. The assumed strategy for the insurer at the beginning of the next period after insolvency is to raise enough capital to restore the solvency of the insurer and eliminate the cost of financial distress. This is the reason for the negative end of period capital figures in some of the periods in Figure 6.

As capital adjustment costs increase, the end of period capital deviates further away from the optimal capital amount $K^* = 7.525$ which minimises the frictional costs of capital and financial distress.

The results correspond with intuition and demonstrate the usefulness of the model in understanding the tradeoffs between the various expected costs of capital.

Figure 5: Optimal level of adjustment capital under different levels of adjustment costs

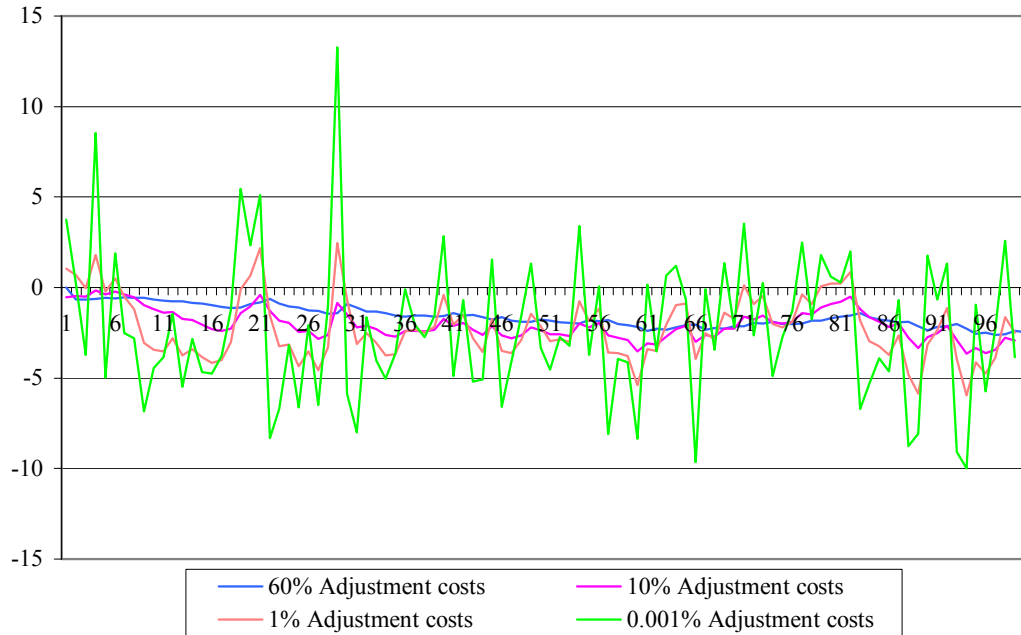
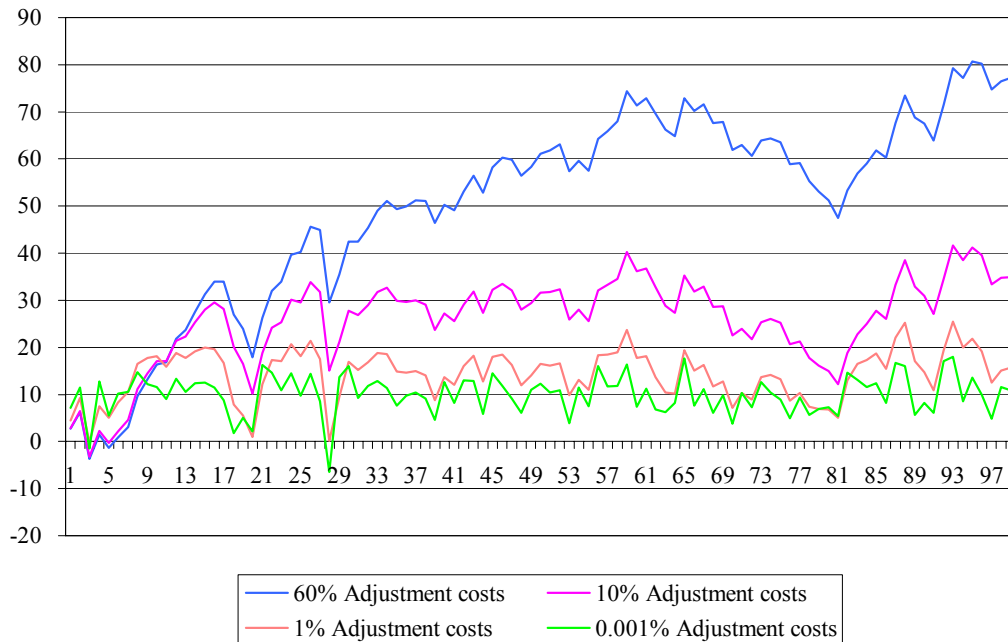


Figure 6: Optimal expected end of period capital amounts under different levels of adjustment costs



Conclusions

The model given in this paper considers the frictional capital costs, financial distress costs and capital adjustment costs and their impact on the optimal level of capital for

an insurer. There is a trade-off for an insurer between the expected frictional costs of capital when it operates in a solvent financial position, and the expected financial distress costs when an insurer faces insolvency. In a single period model, the optimal economic level of capital when adjustment costs are zero satisfies a Value at Risk (VaR) or probability of ruin criterion. This provides an economic justification for using a VaR or probability-of-ruin requirement for determining an insurer's capital.

The model can be used to estimate the financial costs of regulatory capital requirements based on VaR or probability-of-ruin measures where they differ from the economic optimal level of insurer capital. If the regulatory probability-of-ruin criteria is more stringent than the optimal required based on minimising expected frictional capital costs and financial distress costs, an insurer will hold additional regulatory levels of capital, thus incurring additional frictional costs.

A multi-period model is used to assess the impact of adjustment costs associated with raising and shedding capital on optimal capital. The multi-period model shows that in determining the optimal level of capital required each period, the insurer will factor into account the expected present value of future operating results. When the adjustment costs are high, the optimal amount of adjustment capital each period remains low since costs associated with attaining the level of capital to minimise the frictional costs of capital and financial distress are high. The optimal level of insurer capital is influenced more by the expected future operating results of the insurer. When adjustment costs are low, the insurer will adjust its capital to the level which minimises frictional costs.

The models used to illustrate the results use a range of premium values. In theory, premiums should be fairly priced and reflect the risk-adjusted present value of the losses and insolvency and frictional costs. It is possible to incorporate inelastic demand into the multi-period model following Cummins and Danzon (1997) where policyholders include an allowance for the financial quality of the insurer. This would allow the model to determine the relationship between premiums, capital and frictional costs more directly.

The return on the assets of the insurer was assumed to be deterministic. The model can be made more realistic by introducing a stochastic distribution for the asset return to represent the risks associated in asset-liability mismatching. Despite these limitations, the models and the results demonstrate an approach to optimal capital determination for an insurer that can be calibrated to data and used to develop an understanding of capital management of an insurer.

We have aimed to highlight the importance of frictional costs in capital and risk management in insurance and to provide a foundation for more sophisticated models.

Appendix 1:

Derivation of the first-order conditions in the multi-period setup

The optimisation requires the values of \tilde{K}_{t+i}^+ for $i = 0, 1, 2, \dots, \infty$, that minimise the expression:

$$\begin{aligned} & E_t \sum_{i=0}^{\infty} \beta^i \left\{ \frac{1}{2} (\tilde{K}_{t+i}^+ - K^*)^2 + \frac{a}{2} (\tilde{K}_{t+i}^+ - K_{t+i} - A_{t+i}(1+r_{t+i,t+i+1}^A) + \tilde{L}_{t+i}^+)^2 \right\} \text{ which is} \\ & = E_t \left\{ \frac{1}{2} (\tilde{K}_t^+ - K^*)^2 + \frac{a}{2} (\tilde{K}_t^+ - K_t - A_t(1+r_{t,t+1}^A) + \tilde{L}_t^+)^2 \right. \\ & \left. + \beta \left[\frac{1}{2} (\tilde{K}_{t+1}^+ - K^*)^2 + \frac{a}{2} (\tilde{K}_{t+1}^+ - \tilde{K}_{t+1} - A_{t+1}(1+r_{t+1,t+2}^A) + \tilde{L}_{t+1}^+)^2 \right] \right. \\ & \left. + \beta^2 \left[\frac{1}{2} (\tilde{K}_{t+2}^+ - K^*)^2 + \frac{a}{2} (\tilde{K}_{t+2}^+ - \tilde{K}_{t+2} - A_{t+2}(1+r_{t+2,t+3}^A) + \tilde{L}_{t+2}^+)^2 \right] + \dots \right\} \end{aligned}$$

Notice that \tilde{K}_{t+1}^+ , the initial capital at the beginning of period $t+1$, is equal to the end of period t capital \tilde{K}_t^+ since the end of period capital for period t is rolled over to the start of period $t+1$. In general, $\tilde{K}_{t+i}^+ = \tilde{K}_{t+i+1}^+$ for $i = 0, 1, 2, \dots, \infty$. For the case $i = 0$, the first-order condition is derived by differentiating with respect to \tilde{K}_t^+ and setting to zero to get

$$E_t \left\{ \left(\tilde{K}_t^+ - K^* \right) + a \left(\tilde{K}_t^+ - K_t - A_t(1+r_{t,t+1}^A) + \tilde{L}_t^+ \right) \right\} + \beta \left[-a \left(\tilde{K}_{t+1}^+ - \tilde{K}_t^+ - A_{t+1}(1+r_{t+1,t+2}^A) + \tilde{L}_{t+1}^+ \right) \right] = 0$$

using the fact that $\tilde{K}_{t+i}^+ = \tilde{K}_{t+i+1}^+$ for $i = 0, 1, 2, \dots, \infty$. Rearranging gives:

$$E_t \left\{ \tilde{K}_t^+ (1+a+a\beta) + a \left(\tilde{L}_t^+ - K_t - A_t(1+r_{t,t+1}^A) \right) - a\beta \left(\tilde{K}_{t+1}^+ - A_{t+1}(1+r_{t+1,t+2}^A) + \tilde{L}_{t+1}^+ \right) - K^* \right\} = 0$$

and after dividing by $a\beta$ gives:

$$E_t \left\{ \tilde{K}_t^+ \left(\frac{1}{a\beta} + \frac{1}{\beta} + 1 \right) + \frac{1}{\beta} \tilde{L}_t^+ - \frac{1}{\beta} K_t - \frac{1}{\beta} A_t(1+r_{t,t+1}^A) \right\} + \left\{ -\tilde{K}_{t+1}^+ + A_{t+1}(1+r_{t+1,t+2}^A) - \tilde{L}_{t+1}^+ - \frac{1}{a\beta} K^* \right\} = 0$$

With $\gamma = \frac{1}{a} + 1 + \beta$ the first-order condition for $i = 0$ can be written as

$$E_t \left\{ \tilde{K}_{t+1}^+ - \frac{\gamma}{\beta} \tilde{K}_t^+ + \frac{1}{\beta} K_t - \frac{1}{\beta} \tilde{L}_t^+ + \tilde{L}_{t+1}^+ + \frac{1}{\beta} A_t(1+r_{t,t+1}^A) - A_{t+1}(1+r_{t+1,t+2}^A) + \frac{K^*}{a\beta} \right\} = 0$$

The infinite system of first-order equations in the multi-period problem become:

$$E_t \left\{ \begin{aligned} & \tilde{K}_{t+i+1} - \frac{\gamma}{\beta} \tilde{K}_{t+i} + \frac{1}{\beta} \tilde{K}_{t+i} - \frac{1}{\beta} \tilde{L}_{t+i} + \tilde{L}_{t+i+1} \\ & + \frac{1}{\beta} A_{t+i} (1 + r_{t+i,t+i+1}^A) - A_{t+i+1} (1 + r_{t+i+1,t+i+2}^A) + \frac{K^*}{a\beta} \end{aligned} \right\} = 0$$

for $i = 0, 1, 2, \dots, \infty$. For $i = 0$, \tilde{K}_t is replaced by K_t thus \tilde{K}_t denotes the amount of capital at the beginning of period t and is not a random variable.

For $i = 1, 2, \dots, \infty$; the identity $\tilde{K}_{t+i-1}^+ = \tilde{K}_{t+i}^-$ can be used to give

$$E_t \left\{ \begin{aligned} & \tilde{K}_{t+i+1}^+ - \frac{\gamma}{\beta} \tilde{K}_{t+i}^+ + \frac{1}{\beta} \tilde{K}_t^+ - \frac{1}{\beta} \tilde{L}_{t+i}^+ + \tilde{L}_{t+i+1}^+ \\ & + \frac{1}{\beta} A_{t+i} (1 + r_{t+i,t+i+1}^A) - A_{t+i+1} (1 + r_{t+i+1,t+i+2}^A) + \frac{K^*}{a\beta} \end{aligned} \right\} = 0 \quad \square$$

Appendix 2: Characteristic polynomial for the second-order difference equation

We illustrate how to derive the result

$$E_t \left\{ \left(1 - \frac{\gamma}{\beta} \mathbf{L} + \frac{1}{\beta} \mathbf{L}^2 \right) \tilde{K}_{t+i+1}^+ \right\} = E_t \left\{ (1 - \lambda_1 \mathbf{L})(1 - \lambda_2 \mathbf{L}) \tilde{K}_{t+i+1}^+ \right\}$$

by use of the characteristic polynomial $\lambda^2 - \frac{\gamma}{\beta} \lambda + \frac{1}{\beta} = 0$ with roots λ_1 and λ_2 .

$$\text{We have that } \lambda_1 = \frac{\frac{\gamma}{\beta} - \sqrt{\left(\frac{\gamma}{\beta}\right)^2 - 4\left(\frac{1}{\beta}\right)}}{2} = \frac{\gamma - \sqrt{\gamma^2 - 4\beta}}{2\beta}.$$

$$\text{Substituting } \gamma = \frac{1}{a} + 1 + \beta \text{ gives } \lambda_1 = \frac{1 + a + a\beta - \sqrt{(1 + a + a\beta)^2 - 4a^2\beta}}{2a\beta}.$$

The product of the roots for the quadratic characteristic polynomial equal $\frac{1}{\beta}$ so that

$$\lambda_1 \lambda_2 = \frac{1}{\beta} \Leftrightarrow \lambda_2 = \frac{1}{\beta \lambda_1}. \square$$

To prove $(1 - \lambda_1 \mathbf{L})(1 - \lambda_2 \mathbf{L}) = 1 - \frac{\gamma}{\beta} \mathbf{L} + \frac{1}{\beta} \mathbf{L}^2$, evaluate the left-hand side as follows:

$$\begin{aligned} & (1 - \lambda_1 \mathbf{L})(1 - \lambda_2 \mathbf{L}) \\ &= 1 - (\lambda_1 + \lambda_2) \mathbf{L} + \lambda_1 \lambda_2 \mathbf{L}^2 \\ &= 1 - \frac{\gamma}{\beta} \mathbf{L} + \frac{1}{\beta} \mathbf{L}^2 \end{aligned}$$

since the characteristic polynomial has the sum of the roots equal to $\frac{\gamma}{\beta}$ and the

product of the roots equal to $\frac{1}{\beta}$. \square

Finally, the identity $\frac{\mathbf{L}}{1 - \lambda_2 \mathbf{L}} = \frac{-\beta \lambda_1}{1 - \beta \lambda_1 \mathbf{L}^{-1}}$ can be verified by rearranging to get

$$\mathbf{L}(1 - \beta \lambda_1 \mathbf{L}^{-1}) = -\beta \lambda_1 (1 - \lambda_2 \mathbf{L})$$

The left-hand side equals $1 - \beta \lambda_1$ and the right-hand side is

$$\beta \lambda_1 \lambda_2 - \beta \lambda_1 = \beta \lambda_1 \left(\frac{1}{\beta \lambda_1} \right) - \beta \lambda_1 = 1 - \beta \lambda_1. \square$$

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