Enhancing Insurer Value Via Reinsurance Optimization

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Agenda

- Motivation:
  - What do financial companies optimize?
  - Reinsurance! Is it relevant to the maximization of insurance firm’s economic value?
- Models of reinsurance optimization
- Results
Maximization of the shareholders’ value

- Stock insurers resemble financial corporations: they leverage themselves by issuing risky debt, i.e. insurance policies;
Maximization of the shareholders’ value

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- Why issuing insurance debt? Insurers have competitive advantage in creating value by borrowing in insurance (not capital) market;
Maximization of the shareholders’ value

- Stock insurers resemble financial corporations: they leverage themselves by issuing risky debt, i.e. insurance policies;
- Why issuing insurance debt? Insurers have competitive advantage in creating value by borrowing in insurance (not capital) market;
- Insurers are financed by their principals (shareholders);
Maximization of the shareholders’ value

- Shareholders (equity) capital is used to satisfy solvency requirements imposed by a regulator;
Maximization of the shareholders’ value

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- Shareholders of the insurance company are well diversified in a capital market;
Maximization of the shareholders’ value

- Shareholders (equity) capital is used to satisfy solvency requirements imposed by a regulator;
- Shareholders of the insurance company are well diversified in a capital market;
- Conclusion: the main (natural) insurer’s objective is to maximize the shareholders’ value under solvency constraints imposed by a regulator.
Incentives to reinsurance

- Frictional costs: reinsurance may create an additional layer of synthetic equity capital to mitigate expected financial distress costs;
Incentives to reinsure

- Frictional costs: reinsurance may create an additional layer of synthetic equity capital to mitigate expected financial distress costs;
- Overcapitalization does not mean high return on equity: there is tradeoff between purchase of reinsurance and required risk capital in the maximization of shareholders’ value.
Models of reinsurance optimization

Model M1 (maximization of return on equity using two control variables: change-loss reinsurance and risk capital):

\[
\begin{aligned}
\text{maximize} & \quad 1 + \rho(u; a, b) = \frac{V(u,a,b)}{u} = \mathbb{E}\left[\max\{0, u + P(a,b) - I_{a,b}(X)\}\right], \\
\text{subject to} & \quad u \geq u_{\min} \text{ and } (a, b) \in [0, 1] \times [0, \infty),
\end{aligned}
\]

where

\[I_{a,b}(X) = X - a(X - b)_+ \] is the retained risk;

\[u = u_{\min} + v = \text{VaR}_\alpha[X] - P + v \] is the risk capital
Models of reinsurance optimization

Model M2 (maximization of return on equity through reinsurance):

\[
\begin{align*}
\text{maximize} & \quad 1 + \rho(a, b) = \frac{V(a, b)}{u_{\min}(a, b)} = \frac{\mathbb{E}[\max\{0, u_{\min}(a, b) + P(a, b) - I_{a, b}(X)\}]}{u_{\min}(a, b)}, \\
\text{subject to} & \quad (a, b) \in [0, 1] \times [0, \infty),
\end{align*}
\]

where

\[
u_{\min}(a, b) = \text{VaR}_\alpha[I_{a, b}(X)] - P(a, b)
\]

is the minimal value of risk capital altered by reinsurance
M1 and M2 with corporate tax

The shareholders’ expected after-tax terminal value is equal to:

- within the model M1

\[
\tilde{V}(u, a, b) = \mathbb{E}_i \left[ \mathbb{E}_{I_{a,b}(X)} \left[ \max \left\{ (1 + i) \left( u + P(a, b) \right) - I_{a,b}(X); 0 \right\} \mid i \right] \right] \\
- \tau \mathbb{E}_i \left[ \mathbb{E}_{I_{a,b}(X)} \left[ \max \left\{ i \left( u + P(a, b) \right) + P(a, b) - I_{a,b}(X); 0 \right\} \mid i \right] \right]
\]
The shareholders’ expected after-tax terminal value is equal to:

- within the model M1

\[
\tilde{V}(u, a, b) = \mathbb{E}_i \left[ \mathbb{E}_{I_{a,b}(X)} \left[ \max \{ (1 + i) (u + P(a, b)) - I_{a,b}(X); 0 \} \mid i \right] \right]
\]

\[
- \tau \mathbb{E}_i \left[ \mathbb{E}_{I_{a,b}(X)} \left[ \max \{ i (u + P(a, b)) + P(a, b) - I_{a,b}(X); 0 \} \mid i \right] \right]
\]

- within the model M2

\[
\tilde{V}(a, b) = \mathbb{E}_i \left[ \mathbb{E}_{I_{a,b}(X)} \left[ \max \{ (1 + i) (u(a, b) + P(a, b)) - I_{a,b}(X); 0 \} \mid i \right] \right]
\]

\[
- \tau \mathbb{E}_i \left[ \mathbb{E}_{I_{a,b}(X)} \left[ \max \{ i (u(a, b) + P(a, b)) + P(a, b) - I_{a,b}(X); 0 \} \mid i \right] \right]
\]
Results of M1&M2

- **Result 1.** The model M1 does not induce demand for reinsurance in maximization of return on equity. For every predetermined level of return on equity the excess of risk capital decreases with the amount of reinsurance.
Results of M1&M2

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• **Result 2.** The model M1 induces demand for reinsurance in maximization of return on equity in the presence of corporate tax.
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- **Result 2.** The model M1 induces demand for reinsurance in maximization of return on equity in the presence of corporate tax.

- **Result 3.** An optimal tradeoff between the required minimal level of the risk capital and purchase of reinsurance occurs in the model M2.
Results of M1&M2

Graphical illustrations of the results:

Assumptions:

- Claims are exponentially distributed $F(x) = 1 - e^{-\gamma x}$, $\gamma = 0.01$;
- $\text{VaR}_{\alpha}[X] = -\frac{\ln(1-\alpha)}{\gamma}$ with $\alpha = 0.975$;
- The mean value premium principle is applied:

$$P = (1 + \theta) \mathbb{E}_F[X] = \mathbb{E}_G[X],$$

where $G(x) = F(kx)$ and $k = \frac{1}{1+\theta} \in (0.1)$ is a risk adjustment coefficient. Given insurer’s risk loading $\theta$ (there is no unique $\theta$ in incomplete insurance market) reinsurer’s loading for change-loss reinsurance contract is

$$\eta(b, \theta) = \frac{1}{k} \left( \int_{b}^{\infty} (1 - F(x))dx \right) / \left( \int_{b}^{\infty} (1 - F(x))dx \right) - 1 > \frac{1}{k} - 1 = \theta;$$

$\theta = 0.4$. 
Results of M1&M2

Graphical illustration of the Result 2:

Graphical illustration of the total return $1 + \rho(u_{\text{min}}, a, b)$ in the model M1 with corporate tax $\tau = 30\%$
Results of M1&M2

Illustration of the Result 2:

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>Optimal reinsurance</th>
<th>Maximal return on equity $\rho^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15%</td>
<td>$b^* = \infty$ or $a^* = 0$</td>
<td>26.83%</td>
</tr>
<tr>
<td>20%</td>
<td>$b^* = \infty$ or $a^* = 0$</td>
<td>25.492%</td>
</tr>
<tr>
<td>25%</td>
<td>$b^* = 99.31$ and $a^* = 1$</td>
<td>26.47%</td>
</tr>
<tr>
<td>30%</td>
<td>$b^* = 93.73$ and $a^* = 1$</td>
<td>26.01%</td>
</tr>
<tr>
<td>35%</td>
<td>$b^* = 87.69$ and $a^* = 1$</td>
<td>25.302%</td>
</tr>
<tr>
<td>40%</td>
<td>$b^* = 82.07$ and $a^* = 1$</td>
<td>24.58%</td>
</tr>
</tbody>
</table>
Graphical illustration of the Result 3:

Graphical illustration of the total return on equity as the function $1 + \rho(a, b) = \frac{V(a,b)}{u(a,b)}$

defined on $\{a \in [0, a_1]\} \cap \{b \leq \text{VaR}_{\alpha}[X]\}$ (domain of feasible reinsurance contracts)
Results of M1&M2

Remarks:

1) Model M2 induces demand for reinsurance in frictionless environment (without tax) under the assumption that gross premiums are not dependent on capital or reinsurance of the insurer (i.e. the gross premium does not reflect the effect of insolvency on policy payoff);

2) Model M2 should not induce any demand for reinsurance when the gross premium is adjusted with respect to the value of insolvency exchange option, unless frictional costs such as taxes and costs of financial distress are included. The adjusted gross premium $P$ is a solution to the equilibrium system of two equations

$$P = e^{-r}E_Q \left[ L_1 - (L_1 - A_1) 1_{\{A_1<L_1\}} \right]$$
$$E_0 = e^{-r}E_Q \left[ (A_1 - L_1) 1_{\{A_1>L_1\}} \right],$$

where $A_1 = (1 + r_A)(P + E_0)$ is the terminal value of assets; $E_0$ - present value of equity.
Yet another model: M3

Maximization of shareholders value in the presence of financial distress costs

Consider an insurance company over the period of time $[0, T]$ (the period between two consecutive audits), and three economic states of an insurer:

- “financially distressed ($m_{ST}^S \leq D$) & solvent ($F(S_T) > S^*$)”;  
- “healthy ($m_{ST}^S > D$) & solvent ($S_T > S^*$)”;  
- “insolvent”,

where $S_t$ - the company’s surplus at time $t \in [0, T]$;

$m_{ST}^S = \min_{t \in [0,T]} S_t$;  
$D$ ($0 < D < S^*$) - financial distress barrier;

$S_T - F(S_T)$ - financial distress (FD) costs;

$S^*$ - the company’s minimal capitalization level (regulatory capital).
Model M3 (contd)

Model insurer’s surplus (net worth) by geometric Brownian motion (M. Powers, 1995):

\[ dS_t = \mu S_t dt + \sigma S_t dW_t, \]

where \( \mu = \alpha \lambda \left( \pi_L (1 - \varepsilon P) - (\varepsilon L + \varepsilon P) + r_I \right) + r_I, \)
\[ \sigma = \sqrt{\alpha^2 \lambda^2 \sigma_L^2 + (\alpha \lambda + 1)^2 \sigma_I^2}; \]

company’s assets consist of insurance loss reserves \( (L) \) and surplus \( (S) \), which are invested in the capital market.
Model M3 (contd)

The terminal value of company’s surplus net of regulatory capital is:

\[ V_0 = \mathbb{E}_Q \left[ (S_T - S^*) \mathbf{1}_{\{\text{NO - FinDisress & Solvent}\}} \right. \]
\[ + \left. (F(S_T) - S^*) \mathbf{1}_{\{\text{FinDistress & Solvent}\}} \right] \]
\[ = \mathbb{E}_Q \left[ (S_T - S^*)^+ \mathbf{1}_{\{m_T^S > D\}} + (F(S_T) - S^*)^+ \mathbf{1}_{\{m_T^S \leq D\}} \right] \]
\[ = \mathbb{E}_Q \left[ (S_T - S^*) - (S_T - F(S_T)) \mathbf{1}_{\{m_T^S \leq D\} \cap \{F(S_T) > S^*\}} \right. \]
\[ + \left. (S^* - S_T) \left( \mathbf{1}_{\{S_T \leq S^*\}} + \mathbf{1}_{\{m_T^S \leq D\} \cap \{S_T > S^* \geq F(S_T)\}} \right) \right] \]

where \( Q \) is an equivalent martingale measure (we use the Numeraire Invariance Theorem and set risk-free rate to 0)
Model M3 (contd)

Possible forms of FD costs:
1) deadweight losses are proportional to the terminal value of company’s surplus with proportionate coefficient $1 - w$, $w \in (0, 1)$:

$$F_1(S_T) = w S_T,$$

(empirical studies show that in practice $1 - w$ is 10%-20% for production firms and 15%-25% for insurance companies);

2) deadweight losses are in the form of lost upside potential of terminal value of company’s surplus:

$$F_2(S_T) = (S_T - S^*)^+ - (S_T - (S^* + U))^+,$$

where $U > 0$ is the parameter of FD costs.
Model M3 (contd)

We consider $F(S_T) = F_2(S_T)$ (i.e. model with FD costs that come in the form of lost upside potential of surplus), and maximize the value $V_0$, w.r.t. company’s risk $\sigma(\alpha, \sigma_I)$, as a value of two different barrier options:

$$V_0 = \mathbb{E}_Q \left[ (S_T - S^*)^+ \mathbf{1}_{\{m_T^S > D\}} + (F(S_T) - S^*)^+ \mathbf{1}_{\{m_T^S \leq D\}} \right]$$

$$= \mathbb{E}_Q \left[ (S_T - S^*)^+ \mathbf{1}_{\{m_T^S > D\}} + (S_T - S^*)^+ \mathbf{1}_{\{m_T^S \leq D\} \cap \{S_T \leq S^* + U\}} + U \mathbf{1}_{\{m_T^S \leq D\} \cap \{S_T > S^* + U\}} \right]$$
Results of the Model M3

**Result 4.** There are risk-management incentives in maximization of the value $V_0$ (i.e. shareholders value, since solvent company pays dividends from the value $V_0$). At time $0$ risk managers optimally choose a level of the company’s risk $\hat{\sigma}(\alpha, \sigma_I)$ to maximize the value $V_0$:

$$\hat{\sigma}^2 = \frac{1}{T} \ln \left( \frac{D^2}{S^* (S^* + U)} \right) \ln \left( \frac{D^2 S^*}{S_0 (S^* + U)} \right) \ln \left( \frac{S^* + U}{S^*} \right)$$
Results of the Model M3

Result 5.

- For intermediate FD costs (i.e. \( \exists U' : \forall U > U' \)) the optimal company’s risk decreases with an increase in the FD costs;
- In this case risk managers can decrease optimal value of company’s (integrated investment-underwriting) risk

\[
\hat{\sigma} = \hat{\sigma}(\alpha, \sigma_I) = \sqrt{\alpha^2 \lambda^2 \sigma_L^2 + (\alpha \lambda + 1)^2 \sigma_I^2}
\]

by both the quota share \( \alpha \) of proportional reinsurance and the investment risk \( \sigma_I \).
Conclusion

- In the model M1 (conservative model) an insurer is well-capitalized (overcapitalized?). It does not allow insurer to reduce the minimum level of capitalization through purchasing reinsurance. In this model there is no demand for reinsurance in frictionless environment. However, there is demand for reinsurance in this model with reasonably high value of frictional costs, such as corporate tax;

- The model M2 imposes the demand for reinsurance in the frictionless environment (without tax), which is due to the assumption that the gross premium is not adjusted with respect to the value of insolvency put. For the same level of frictional costs the demand for reinsurance in the model M2 is higher than the one in the model M1;

- There are incentives to control company’s risk in the model M3 of maximization of shareholders value in the presence of FD costs;

- The decision to reinsure can be treated as both a risk-management and a capital-structure tool in shareholders’ value creation.