Optimal stochastic problems in consumption, investment and insurance

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Overview

1. Motivation
2. Literature
3. Richard (1975) model
4. Enriching the Richard (1975) model
1 Motivation

Institutional background

- Ageing populations
- Many governments considering
  - shifting from pay-go (DB) to funded (DC)
  - winding back social security
- Companies shifting from DB to DC
  \[ \rightarrow \text{Higher demand for financial planning} \]
Important issues in lifetime consumption, investment and insurance

- Life cycle consumption
  - life cycle theory, buffer stock theory
- Life cycle investment
  - age-phasing
- Life cycle insurance demand
  - human life value concept
- Life cycle annuity demand
2 Literature

- Yaari (1965)
- Merton (1969, 1971)
- Richard (1975)
- Bodie, Merton & Samuelson (1992)
- Fleming & Zariphopoulou (1991)
- Koo (1998)
- Viciera (2001)
- Campbell & Viciera (2002)
3 Richard (1975) model

\[
\max_{C, \pi, Z} E \left[ \int_{\tau}^{T} U(C(t), t) dt + B(Z(T), T) \right] \tag{1}
\]

\[
\frac{dQ(t)}{Q(t)} = \alpha dt + \sigma dq(t) \tag{2}
\]

\[
dW(t) = -C(t) dt - P(t) dt + Y(t) dt + rW(t) dt + (\alpha - r)\pi(t)W(t) dt + \sigma\pi(t)W dq(t) \tag{3}
\]

\[
P(t) = \mu(t)\{Z(t) - W(t)\} \tag{4}
\]
Dynamics of Richard model
Expected lifetime consumption, $\gamma$ varying
Household consumption and income over the life cycle.
Expected wealth components, $\gamma$ varying.
Expected investment in risky—as a proportion of financial wealth, $W(t)$
Expected insurance premia paid (+ values) and annuity payments received (− values)
4 Enriching the model

- Borrowing constraint
- Thin annuities market
- Risky income
Borrowing constraint
Investment in risky assets over time with costly borrowing.
Thin annuities market
Expected consumption with/without annuities.
Stochastic income
Expected investment path.
Expected consumption path.
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