A stochastic control model for individual asset-liability management

Sachi Purcal
School of Actuarial Studies
University of New South Wales
Sydney, Australia

November 2003

E-mail: s.purcal@unsw.edu.au
Overview

• Motivation

• Richard (1975) model

• Extension: risky income

• Solution method

• Results
Motivation

Institutional background

• Ageing populations

• Many governments considering
  – shifting from pay-go (DB) to funded (DC)
  – winding back social security

• Companies shifting from DB to DC

→ Higher demand for financial planning
Important issues in lifetime consumption, investment and insurance

- Life cycle consumption
  - life cycle theory, buffer stock theory

- Life cycle investment
  - age-phasing

- Life cycle insurance demand
  - human life value concept

- Life cycle annuity demand
Richard (1975) model

\[
\max_{C,\pi,Z} \mathbb{E} \left[ \int_\tau^T U(C(t), t) dt + B(Z(T), T) \right] \quad (1)
\]

\[
\frac{dQ(t)}{Q(t)} = \alpha dt + \sigma dq(t) \quad (2)
\]

\[
dW(t) = -C(t)dt - P(t)dt + Y(t)dt \\
+ rW(t)dt \\
+ (\alpha - r)\pi(t)W(t)dt \\
+ \sigma\pi(t)Wdq(t) \quad (3)
\]

\[
P(t) = \mu(t)\{Z(t) - W(t)\} \quad (4)
\]
Extension: risky income

Try to improve relevance of Richard model

Income important component of wealth

Model income as stochastic process with expected exponential path:

\[ \frac{dY}{Y} = \hat{\alpha} dt + \hat{\sigma} d\hat{q} \]  \hspace{1cm} (5)

Two controlled processes are now:

\[
\begin{pmatrix}
  dW \\
  dY
\end{pmatrix}
= \begin{pmatrix}
  -C - P + Y + rW + (\alpha - r)\pi W \\
  \hat{\alpha} Y
\end{pmatrix} dt + \begin{pmatrix}
  \sigma \pi W & 0 \\
  0 & \hat{\sigma} Y
\end{pmatrix} \begin{pmatrix}
  dq \\
  d\hat{q}
\end{pmatrix} \]  \hspace{1cm} (6)
Solution method

Unable to find closed-form solution

Numerical solution: Markov Chain approximation technique of Kushner & Dupuis (2001)
Results

Consumption and wealth relationship, salary fixed.

Investment and wealth relationship, salary fixed.
Premium and wealth relationship, salary fixed.

Consumption and salary relationship, wealth fixed.
Labour income (¥, ten millions)

Percentage of W(t) invested in risky assets

Investment and salary relationship, wealth fixed.

Premium (¥, ten millions)

Premium and salary relationship, wealth fixed.
Expected wealth path.

Expected consumption path.
Expected investment path.

Expected premium path.